CS 323H
Midterm Examination

1. DISCUSS THIS EXAM WITH NO ONE EXCEPT ALAN CLINE.

2. USE MOLER'S BOOK, ANY MATLAB TUTORIAL, OR ANY NOTES YOU HAVE TAKEN IN THIS CLASS. DO NOT USE ANY ONE ELSE'S CLASS MATERIALS. ASK IF IN DOUBT.

3. SUBMIT YOUR SOLUTIONS AS FILES: `problem1.xxx`, `problem2a.m`, `problem2b.xxx`, `problem2c.xxx`, `problem2d.xxx`, `problem3.xxx`, `TriLowerTriangularSolve.m`, and `problem5.m`, TO cline@cs.utexas.edu BY 10:30 PM, MONDAY, FEBRUARY 25, 2008.

4. QUICKLY READ THE PROBLEMS. I WILL READ EMAIL EXCEPT BETWEEN 9:00 and 10:30. I WILL ANSWER CLARIFYING QUESTIONS BUT PLEASE ASK ME NOTHING OF THE SORT "IS THIS CORRECT?".

[5] 1. . Evaluate $(1.2347 \cdot 10^7 \times 56.782 \cdot 10^{-4}) - (.9876 / .00000123)$ simulating 4 decimal digit rounding floating point arithmetic with exponent range -100 to +100. Put your solution on in a text file named `"problem1.xxx"` where xxx is a suffix such as txt or doc that is associated with your text processor.

[20] 2. . Consider solving the linear system:

$$\begin{bmatrix} 10^{-6} & 2 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$$

a. Using Matlab and `format long`, determine a solution x and y to the standard double precision accuracy. (For the purposes of the problem, we call call this the "exact solution".) Place all of the code necessary to solve this problem into a Matlab function named "`problem2a.m`".

b. Simulating 5 decimal digit rounding floating point arithmetic with exponent range -50 to +50, determine x and y using the **original** Gaussian algorithm (i.e. use the diagonal elements as you find them for pivots). Put your solution on in a text file named `"problem2b.xxx"` where xxx is a suffix such as txt or doc that is associated with your text processor.

c. Again simulating 5 decimal digit rounding floating point arithmetic with exponent range -50 to +50, determine x and y using the Gaussian algorithm with partial pivoting. Put your solution on in a text file named `"problem2c.xxx"` where xxx is a suffix such as txt or doc that is associated with your text processor.

d. Comment on the accuracies you have determined in parts b. and c. as compared with the exact solution. Explain the differences. Put your solution on in a text file named `"problem2d.xxx"` where xxx is a suffix such as txt or doc that is associated with your text processor.

[10] 3. You are seeking to solve a system of linear equations $Ax = b$, but because of uncertainties in measurements, you realize that the right hand side b will actually be some slightly perturbed vector $\bar{b}$ and that this will result in some perturbed solution $\bar{x}$ (i.e., $A\bar{x} = \bar{b}$). The largest component of $\bar{b}$ is about 20 in absolute value. When you solved $A\bar{x} = \bar{b}$ you obtained an $\bar{x}$ whose largest component was about .5 in absolute. The (infinity norm) condition number of A was reported by Matlab to be 2000. You require a solution is good to three decimal places (i.e. x and $\bar{x}$ must differ in all components by no more than .001). How accurate must the right hand side be measured – that is, what is the largest *difference* in absolute value you could possibly have between the components of b and those of $\bar{b}$? (The "infinity norm" of a vector is the largest element in absolute value.) Put your solution on in a text file named `"problem3.xxx"` where xxx is a suffix such as txt or doc that is associated with your text processor.

[15] 4. Consider the tridiagonal, lower triangular system:

$$a_1 x_1 = d_1$$
$$b_1 x_1 + a_2 x_2 = d_2$$

and for $k = 3,...,n$

$$c_k x_{k-2} + b_k x_{k-1} + a_k x_k = d_k.$$

Represented as with a matrix, this is:

$$
\begin{bmatrix}
a_1 & 0 & 0 & 0 & 0 & 0 & 0 \\
b_1 & a_2 & 0 & 0 & 0 & 0 & 0 \\
c_1 & b_2 & a_3 & 0 & 0 & 0 & 0 \\
0 & \bullet & \bullet & \bullet & 0 & 0 & 0 \\
0 & 0 & \bullet & \bullet & \bullet & 0 & 0 \\
0 & 0 & 0 & \bullet & \bullet & \bullet & 0 \\
0 & 0 & 0 & 0 & c_{n-2} & b_{n-1} & a_n
\end{bmatrix}
\begin{bmatrix}
x_1 \\ x_2 \\ x_3 \\ \bullet \\ \bullet \\ \bullet \\ x_n
\end{bmatrix}
=
\begin{bmatrix}
d_1 \\ d_2 \\ d_3 \\ \bullet \\ \bullet \\ \bullet \\ d_n
\end{bmatrix}
$$

Notice that this system has a unique solution if and only if all diagonal entries of the matrix are non-zero. Also notice that x_1 is easily determined, and from that x_2 is easily determined, and from those x_3, and so forth until x_n is determined Write a Matlab function named `"TriLowerTriangular-Solve.m"` that has input arguments vectors a, b, c, and d and output argument x, where x solves the linear system above. You may assume that $n = length(d)$, $n \geq 2$, and all elements of the array a are non-zero. (Deal with the equations in the original order – do not try to swap rows. The function should call no other Matlab functions.)

[10] 5. Determine $a_1, a_2, a_3,$ and a_4 so that

$$f(x) = a_1 + a_2 e^x + a_3 x^2 + a_4 \sin(x)$$

satisfies

$$f(2) = 3, f'(1) = 1, f''(2) = 0, \text{and } f(-2) = f(1).$$

Place all of the code necessary to solve this problem into a Matlab function named "problem5.m". (Hint: don't try to be overly fancy using Matlab to set up the equations. Do as much of the problem on paper as you like but put everything in which Matlab was used into the file.)