CS 232H
Midterm Examination Preparation

1. USE MOLER'S BOOK, ANY MATLAB TUTORIAL, OR ANY NOTES YOU HAVE TAKEN IN THIS CLASS. DO NOT USE ANY ONE ELSE'S CLASS MATERIALS. ASK IF IN DOUBT.

2. FOR THE ACTUAL EXAM, YOU WILL SUBMIT YOUR SOLUTIONS AS A DIRECTORY NAMED "`midterm.yyy`" (WHERE `yyy` IS YOUR LAST NAME) WITH FILES WITH NAMES TO BE GIVEN DURING THE EXAM TO cline@cs.utexas.edu BY 10:30 PM, MONDAY, FEBRUARY 25, 2008.

[5] 1. Evaluate $(1.26\cdot 10^{-6}\times(8{,}126{,}749.3-1.397))-(\frac{531\cdot 10^{30}}{532\cdot 10^{31}})$ simulating 5 decimal digit rounding floating point arithmetic with exponent range -70 to +70. Put your solution on in a text file named "`problem1.xxx`" where xxx is a suffix such as txt or doc that is associated with your text processor.

[10] 2. . Consider the expression $(f(z+h)-f(z))/h$ as an approximation to $f'(z)$ for small values of h. For the function f(x) = sin (x), the point z = .4, and values of $h=10^{-k}$ for k = 1, 2, …, 17 apply this approximation. Specifically, produce a table with rows having the three quantities:

h, $(\sin(.4+h)-\sin(.4))/h$, and the error $(\sin(.4+h)-\sin(.4))/h-\cos(.4)$

(Use formats `%14.2e, %25.17e, and %16.8e, respectively.)` Notice the behavior of the error and explain it in comments in comments. Place all of the code necessary to solve this problem into a Matlab function named "`problem2.m`".

[10] 3. You are seeking to solve a system of linear equations $Ax=b$, but because of uncertainties in measurements, you realize that the right hand side b will actually be some slightly perturbed vector $\bar{b}$ and that this will result in some perturbed solution $\bar{x}$ (i.e., $A\bar{x}=\bar{b}$). The largest component of $\bar{b}$ is about 5 in absolute value and the largest *difference* in components of b and $\bar{b}$ is about .001 in absolute value. When you solved $A\bar{x}=\bar{b}$ you obtained an $\bar{x}$ whose largest component was about .01 in absolute. The (infinity norm) condition number of A was reported by Matlab to be 300. What is the largest *difference* in absolute value you could possibly have between the components of x and those of $\bar{x}$? (The "infinity norm" of a vector is the largest element in absolute value.) Put your solution on in a text file named "`problem3.xxx`" where xxx is a suffix such as txt or doc that is associated with your text processor.

[15] 4. Consider the lower triangular system:

$$\begin{aligned} A_{1,1}x_1 &= b_1 \\ A_{2,1}x_1 + A_{2,2}x_2 &= b_2 \\ &\vdots \\ A_{n,1}x_1 + A_{n,2}x_2 + \cdots + A_{n,n}x_n &= b_n \end{aligned}$$

Notice that this system has a unique solution if and only if all diagonal entries of A are non-zero. Write a Matlab function named " `LowerTriangularSolve.m`" that has input arguments lower triangular matrix A and vector b and output argument x, where x satisfies $Ax = b$. (The function should call no other Matlab functions.)

[10] 5. Determine a_1, a_2, a_3 and a_4 so that

$$f(x) = 5 + a_1 \sin(x) + a_2 e^{2x} + a_3 x + a_4 / x$$

satisfies

$$f'(1) = f(3) + 2, f'(2) = 0, \int_1^2 f(x)dx = 4 \text{ and } f''(1) = 1 .$$

Place all of the code necessary to solve this problem into a Matlab function named "`problem5.m`".