

**Homework 13**  
**CS 336**

Name \_\_\_\_\_  
Seating Section: **R M L**

**The important issue is the logic you used to arrive at your answer.**

1. Consider the functions  $f$  and  $g$  defined on  $\mathbf{N}$  by  $f(n) = \begin{cases} n^2 & \text{for } n \text{ even} \\ 2n & \text{for } n \text{ odd} \end{cases}$  and  $g(n) = n^2$ . Show that  $f = O(g)$  but that  $f \neq o(g)$  and  $g \neq O(f)$ .

2. Display a function  $f: N \rightarrow R$  that is  $O(1)$  but is not constant.

3. Define the relation " $\leq$ " on functions from  $\mathbf{N}$  into  $\mathbf{R}$  by  $f \leq g$  if and only if  $f = O(g)$ . Prove that  $\leq$  is reflexive and transitive. (Recall: to be *reflexive*, you must have  $f \leq f$  for all functions  $f$ ; to be *transitive*, you must have that  $f \leq g$  and  $g \leq h$  implies  $f \leq h$  for all functions  $f$ ,  $g$ , and  $h$ .)