Asymptotic Dominance Theory

- **Definition 1**: Given the functions $f: N \to R$ and $g: N \to R$, f is *asymptotically dominated by g* if there exist non-negative constants M and N such that for all $n \geq N$, $|f(n)| \leq M|g(n)|$. This is denoted by $f = O(g)$.

- **Definition 2**: Given the functions $f: N \to R$ and $g: N \to R$, $f = o(g)$ if for every positive ε, there exists a non-negative constant N such that for all $n \geq N$, $|f(n)| \leq \varepsilon|g(n)|$.

Theorem 1: *If $f = O(g)$, then for any constant s, $sf = O(g)$.*

Theorem 2: *If $f_1 = O(g_1)$ and $f_2 = O(g_2)$, then $f_1 + f_2 = O(|g_1| + |g_2|)$.*

    Corollary 2.1: *If for $i = 1, 2, ..., k$, $f_i = O(g_i)$, then $\sum_{i=1}^{k} f_i = O(\sum_{i=1}^{k} |g_i|)$.*

Theorem 3: *If $f_1 = O(g_1)$ and $f_2 = O(g_2)$, then $f_1 + f_2 = O(\max\{|g_1|, |g_2|\})$.*

    Corollary 3.1: *If for $i = 1, 2, ..., k$, $f_i = O(g_i)$, then $\sum_{i=1}^{k} f_i = O(\max_{i=1,...,k} |g_i|)$.*

    Corollary 3.2: *If for $i = 1, 2, ..., k$, $f_i = O(g)$, then $\sum_{i=1}^{k} f_i = O(g)$.*

Theorem 4: *If $f_1 = O(g_1)$ and $f_2 = O(g_2)$, then $f_1 \cdot f_2 = O(g_1 \cdot g_2)$.*

Theorem 5: *If $f_1 = O(g_1)$, $g_2 = O(f_2)$, and g_2 has no zeros. then $f_1 / f_2 = O(g_1 / g_2)$.*

Theorem 6: *If $a \leq b$, then $n^a = O(n^b)$.*

Theorem 7: *If a is any real value, then $a^n = O(n!)$.*

Theorem 8: *If $a < b$, then $n^a = o(n^b)$.*

Theorem 9: If $0 < p < q$, then $p^n = o(q^n)$