Examination 1

CS 336

1. The important issue is the logic you used to arrive at your answer.
2. Use extra paper to determine your solutions then neatly transcribe them onto these sheets.
3. Do not submit the scratch sheets. However, all of the logic necessary to obtain the solution should be on these sheets.
4. Comment on all logical flaws and omissions and enclose the comments in boxes

1. [5] Consider integers in the set {1, 2, 3, …, 1000}. How many are divisible by either 4 or 10?

2. a. [10] Present a combinatorial argument that for all $n \geq 1$:

$$\sum_{k=0}^{n} \binom{n}{k} 2^k = 3^n$$

b. [10] Present a combinatorial argument that for all nonegative integers *p*, *s*, and *n* satisfying $p + s \leq n$

$$\binom{n}{p}\binom{n-p}{s} = \binom{n}{p+s}\binom{p+s}{p}$$

(Hint: Consider choosing two subsets.)

3. [10] For $n \geq 1$, Let $A = \{1, 2, \ldots, 2n\}$. How many subsets of A contain exactly k_1 even numbers and k_2 odd numbers?

4. [10] For $n \geq 1$, how many ordered triples (n_1, n_2, n_3) of non-negative numbers satisfy $n_1 + n_2 + n_3 = n$? (Hint: think about putting balls into bins.)

5. [10] Given a finite event space E (in which all events are equally likely) and subsets A and B of E, show that

$$\Pr(A \cup B) \geq \Pr(A) + \Pr(B) - 1.$$

6. [10] Consider a 52 card deck of cards from which the ace of spades is removed resulting in a 51 card deck. Further, consider two distinct cards drawn from the 51 card deck and assume all such unordered draws are equally likely. Lastly consider the probability of the event that both cards are hearts. Is it more likely that both cards are hearts if it is given that both cards are face cards (i.e., Kings, Queens, or Jacks)?

7. [10] Let A be a set of cardinality *p*. Consider ordered strings of length *m* using the elements of A. How many such strings have the m^{th} component a repetition of one of the preceding *m*-1? (Hint: Think about the complement and think about selecting the m^{th} component first.)