Examination 1

CS 336

1. **The important issue is the logic you used to arrive at your answer.**
2. **Use extra paper to determine your solutions then neatly transcribe them onto these sheets.**
3. **Do not submit the scratch sheets. However, all of the logic necessary to obtain the solution should be on these sheets.**
4. **Comment on all logical flaws and omissions and enclose the**
   comments in boxes

1. [5] Given sets A and B, each of cardinality $n \geq 1$, how many functions map A in a one-to-one fashion onto B ?

2 a. [5] Given the set of r symbols $\{a_1, a_2, \ldots, a_r\}$, how many different strings of length $n \geq 1$ exist?

b. [10] Given the set of r symbols $\{a_1, a_2, \ldots, a_r\}$, how many different strings of length $n \geq 2$ exist that contain at least one a_1 and at least one a_2? (Assume $r \geq 2$.)

3. [10] Present a combinatorial argument that for all positive integers n :

$$3^n = \sum_{k=0}^{n} \binom{n}{k} 2^k .$$

b. [10] Present a combinatorial argument that for all integers $n \geq 3$:

$$\binom{3n}{3} = 3\binom{n}{3} + 3 \cdot 2n \binom{n}{2} + n^3$$

(**Hint:** Consider three pairwise disjoint sets of cardinality n.)

4. [10] A multiset is similar to a set in that order is irrelevant but multiple copies of elements are allowed. For example, the **sets** $\{1,2,3\}$ and $\{1,1,1,2,2,3\}$ are identical and each has cardinality three but the **multisets** $\{1,2,3\}$ and $\{1,1,1,2,2,3\}$ are different and the first has cardinality three but the second has cardinality six. How many multisets of cardinality n are there that employ elements from $a_1, a_2, \ldots, a_r$?

5 a. [5] a. How many strings are there of length $k \geq 1$ using elements from the set $\{1, 2, \ldots, n\}$? (Repetition is allowed.)

b. [10] Now assume each of the different strings in part a. is equally likely. What is the probability that the minimum of the *k* numbers is less than or equal to *r*, where $1 \leq r \leq n$?

6. a. [5] a. How many permutations of *a, b, c, d,* and *e* have both *a* to the left of *b* and *b* to the left of *c*?

b. [10] Assume all such permutations are equally likely, what is the probability that the permutation begins with *a* given that it has both *a* to the left of *b* and *b* to the left of *c*?