Name ______________________________

Examination 1

CS 336H

1. The important issue is the logic you used to arrive at your answer.
2. Use extra paper to determine your solutions then neatly transcribe them onto these sheets.
3. Do not submit the scratch sheets. However, all of the logic necessary to obtain the solution should be on these sheets.
4. Comment on all logical flaws and omissions and enclose the
$\boxed{\textbf{comments in boxes}}$

1. [5] Given m a's, n b's, and p c's, how many distinct sequences are there that employ each of the $m+n+p$ symbols? (Note: All a's are identical as are all b's and all c's)

2 a. [5] How many permutations of *a, b, c, d, e*, and *f* have *b* to the left of *c* and *e* to the left of *f*?

[5] b. How many permutations of *a, b, c, d, e*, and *f* have *b* to the left of *c* and *d*?

3.a [10] Present a combinatorial argument that for all positive integers x and y

$$\sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k} = (x+y)^n .$$

(Hint: Consider sequences drawn from the union of distinct sets A and B of cardinalities x and y, respectively.)

b [10] Present a combinatorial a combinatorial argument that for all positive integers $1 \le k \le m \le r$:

$$\binom{r}{m}\binom{m}{k} = \binom{r}{k}\binom{r-k}{m-k}.$$

4. [10] Consider 5-tuples of the form $< r_1, r_2, r_3, r_4, r_5 >$, where the $r_i \ge 1$. How many such 5-tuples are there satisfying

$$r_1 + r_2 + r_3 + r_4 + r_5 = 20?$$

5. [10] For $n \ge 5$, consider strings of length n using lower case roman letters (i.e., $\{a,b,c,\ldots,x,y,z\}$). Assuming all such strings are equally likely what is the probability that the string occurs in non-decreasing order (i.e. all *a*'s precede all *b*'s, all *b*'s precede all *c*'s ,… etc.)?

6. a. [10] For $n \ge 6$, consider strings of length n using elements of $\{a,b,c,d,e\}$. Assume all such strings are equally likely. What is the probability that a string has three or more *a*'s?

b. [5] What is the probability that such a string has exactly three *b*'s given that it has exactly three *a*'s?

7. [10] Consider strings of length $n \geq 2$ containing exactly k 1's and $n-k$ 0's and having no adjacent 1's (i.e., there is at least one 0 between any 1's). Assuming $k \geq 1$ and $n \geq 2k$, how many such strings are there?