Name _____

Examination 1

CS 336

1. The important issue is the logic you used to arrive at your answer.
2. Use extra paper to determine your solutions then neatly transcribe them onto these sheets.
3. Do not submit the scratch sheets. However, all of the logic necessary to obtain the solution should be on these sheets.
4. Comment on all logical flaws and omissions and enclose the comments in boxes

1. [5] For $n \geq 3$, how many diagonals does a convex polygon with *n* extreme points have? (Consider a convex polygon given by extreme points $< P_1, P_2, ..., P_n >$ in counterclockwise order A "diagonal" is a line segment connecting two **non-adjacent** extreme points.)

2. a. [10] Present a combinatorial argument that for all $n \geq 1$:

$$(2n-1)\cdot(2n-3)\cdots 3\cdot 1 = \frac{(2n)!}{n!2^n}$$

b. [10] Present a combinatorial argument that for all nonegative integers *k* and *n* satisfying $k \leq n-2$

$$\binom{n+2}{k} = \binom{n}{k} + 2\binom{n}{k-1} + \binom{n}{k-2}$$

3. [15] How many partitions are there of a set of 45 elements into a subset of cardinality 3, six subsets of cardinality 4, and three subsets of cardinality 6?

4. [15] For $n \geq 1$, what is the value of $\sum_{i_1=0}^{n} (\sum_{i_2=0}^{n-i_1} (\sum_{i_3=0}^{n-i_1-i_2} 1))$? Present a combinatorial argument: determine the value of the expression, then defend it by establishing a model and counting it. (Hint: Define $i_4 = n - i_1 - i_2 - i_3$ and then think about putting balls into bins.)

5. [10] Consider strings of four *a*s and four *b*s. Assume all such strings are equally likely. What is the probability that two or more *a*s precede all of the *b*s.

6. [10] Consider rolls of a pair of six-sided dice assuming all possible order pairs of outcomes are equally likely. What is the probability that the sum of values shown on the dice is 8 given that either of the dice shows a 2?