Final Examination

CS 336

1. **The important issue is the logic you used to arrive at your answer.**
2. **Use extra paper to determine your solutions then neatly transcribe them onto these sheets.**
3. **Do not submit the scratch sheets. However, all of the logic necessary to obtain the solution should be on these sheets.**
4. **Comment on all logical flaws and omissions and enclose the** **comments in boxes**

1. a. [5] How many strings of length $n \geq 0$ using characters *a*, *b* or *c* with possible repetition, have exactly n_a *a*s and n_b *b*s (where $n_a + n_b \leq n$)?

b. [10] How many strings of length $n \geq 0$ using characters *a*, *b* or *c* with possible repetition, have either exactly n_a *a*s or exactly n_b *b*s or both (where $n_a + n_b \leq n$)?

2. [10] For $n \geq 1$, how many four-tuples $\langle i, j, k, l \rangle$ of non-negative values $i, j, k,$ and l satisfy $i + j + k + l \leq n$? (Hint: First consider the situation $i + j + k + l = n$ and then think about $m = n - (i + j + k + l)$.)

3. a. [10] Using a combinatorial argument, prove that for $n \geq 1$ and $m \geq 2$:

$$\sum_{k=0}^{n} \binom{n}{k} (m-1)^k = m^n$$

b. [10] Using a combinatorial argument, prove that for $n \geq k \geq 0$:

$$\binom{n}{k} k!(n-k)! = n!$$

4. a. [10] For $3 \leq m \leq n$, what is the probability that a string of length m selected without repetition from $\{1, 2, \ldots, n\}$ contains the substring $\langle 1, 2, 3 \rangle$? (You may assume all strings of length m selected without repetition from $\{1, 2, \ldots, n\}$ are equally probable.)

5. [10] Using definition 2' (and no cardinality theorems) show that the set of reciprocals of positive integers (i.e., $\{1/p \mid p \in \mathbb{Z} \wedge p > 0\}$) is infinite.

6. a. [10] Let $A = \{a,b,c,\ldots,z,A,B,C,\ldots,Z\}$ and and let B be the set of finite strings from A, that is $B = \{\langle \alpha_1,\alpha_2,\ldots\alpha_n \rangle | n \in \mathbb{N} \wedge \alpha_i \in A \; for \, i = 1,2,\ldots,n\}$. Is the set B finite, countably infinite, or uncountably infinite? Prove your claim.

b. [5] Prove that the set of finite sets of real values from $[0,1]$ $C = \{\{x_1,x_2,\ldots,x_n\} \mid n \in \mathbb{N} \wedge x_i \in [0,1] for \, i = 1,2,\ldots,n\}$ is uncountably infinite.

7. [10] Prove that if $f_1 = O(g_1)$ and $f_2 = o(g_2)$, then $f_1 f_2 = o(g_1 g_2)$.

8. [10] . For a fixed value of k, define $f(n) = \binom{n}{k}$. Prove that $f(n) = O(n^k)$

9. [10] Assuming x and y are integer variables, prove correct with respect to precondition "$x \ge 0$ and y is defined" and postcondition "$x + y \ne 11$":

```
if y > 2 then
        x := y+6
        if x < 10 then
                y : = 1
        endif
else
        x := y+4
endif
```

10. a. [10] Prove the following code is partially correct with respect to precondition "$n \ge 0$" and postcondition "$s = \sum_{i=1}^{n} a_i b_i$" (assume k and s are integer variables and a and b are integer arrays of length at least n.):

```
k := 1
s := 0
while k≤n do
        s := s + (a[k]*b[k])
        k := k+1
endwhile
```

Be explicit about your loop invariant.

...b. [5] Prove that the loop terminates.

11. a. [10] Determine the weakest precondition with respect to the postcondition "$w > 0$" for the following (assume w, z, y, and x are integer variables and that y and z are defined):

```
x := y
y := x
x := z
y := x
w := x+y+z
```

b. [5] For the same piece of code, determine the weakest precondition with respect to the postcondition "$wy = 12$"

12. [10] Determine the weakest precondition with respect to the postcondition "$y = 1$" for the following code (assume *z*, *y*, and *x* are integer variables and that x and z are defined):

```
if x<3 then
        y := z
        if y<z then
                y := 2*y
        endif
else
        y := z-y
endif
```