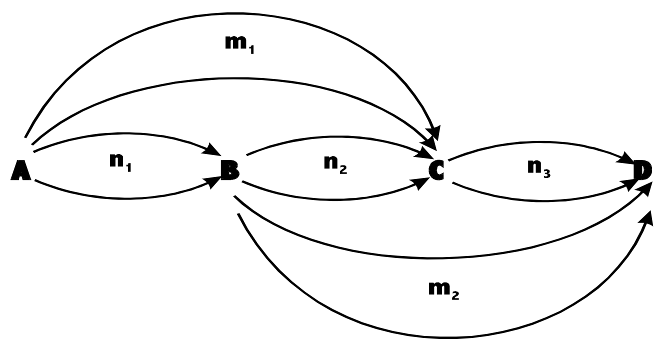
Final Examination
CS 336

1. The important issue is the logic you used to arrive at your answer.
2. Use extra paper to determine your solutions then neatly transcribe them onto these sheets.
3. Do not submit the scratch sheets. However, all of the logic necessary to obtain the solution should be on these sheets.
4. Comment on all logical flaws and omissions and enclose the
comments in boxes

1. a. [5] For $n \geq 1$, how many Boolean (i.e. true- or false-valued) functions exist for n Boolean variables?



2. [10] For $n \geq 1$, how many five-tuples $\langle i, j, k, l, m \rangle$ of non-negative values $i, j, k,$ and l satisfy $i + j + k + l + m \leq n$? (Hint: First consider the situation $i + j + k + l + m = n$ and then think about $p = n - (i + j + k + l + m)$.)

3. a. [10] Using a combinatorial argument, prove that for $n \geq 1$ and $m \geq 2$:

$$\sum_{k=0}^{n} \binom{n}{k} (m-1)^k = m^n$$

b. [10] Using a combinatorial argument, prove that for $n \geq k \geq 0$:

$$\binom{n}{k} k!(n-k)! = n!$$

4. a. [10] For $n \geq 5$, what is the probability that a string of n zeros and ones has exactly 5 ones. (You may assume all strings of n zeros and ones are equally probable.)

b. [5] For $n \geq 5$, what is the probability that a string of n zeros and ones has exactly 5 ones given that it has at least 4 ones. (You may assume all strings of n zeros and ones are equally probable.)

5. [15] Prove: If A is a nonempty set, $\mathcal{P}(A)$, the power set of A, is not countably infinite.

6. a. [10] Prove this corollary to Theorem 6:
Given a countably infinite collection of finite sets $\{A_i\}_{i\in\mathbb{N}}$ satisfying $A_0 \neq \varnothing$ and for $i \geq 1$,

$$A_i \not\subset \bigcup_{j=0}^{i-1} A_j$$

the union $\bigcup_{i\in\mathbb{N}} A_i$ is countably infinite. (In other words, if each set contains at least one element not contained in its predecessors, the union cannot be finite.)

7. [10] Prove that if $f, g,$ and h are real-valued functions defined on the natural numbers, then $f = o(g)$ and $g = O(h)$ imply $f = o(h)$.

8. [10] . Prove that if $0 < a < b$, then $n^b \neq O\,(n^a)$

9. [10] Assuming x and y are integer variables, prove correct with respect to precondition " y is defined" and postcondition "$x \neq y$":

```
if y > 3 then
        x := y+6
        if x > 11 then
                y : = 11
        endif
else
        x := y-2
        y := y-1
endif
```

10. [10] Prove the following code is partially correct with respect to precondition "true" and postcondition "$x = 1$" (assume x is an integer variable.):

```
        x := 0
        while x = 0 do
                x := 1
        endwhile
```

Be explicit about your loop invariant: I =

11. a. [10] Prove the following code is partially correct with respect to precondition "$n \geq 1$" and postcondition "$(k/2 < n) \wedge (k \geq n) \wedge (\exists j \geq 0 \ni k = 2^j)$" (assume k and n are integer variables.):

```
k := 1
while k < n  do
        k := 2*k
endwhile
```

Be explicit about your loop invariant: I =

b. [5] Prove that the loop terminates.

12. [10] Assuming max, a, b, and c are integer variable and that a, b, and c are defined, determine the weakest precondition with respect to the postcondition

"$(\min = a \vee \min = b \vee \min = c) \wedge (\min \leq a) \wedge (\min \leq b) \wedge (\min \leq c)$":

```
if b < a then
        {if b < c then
                min : = b
        else
                min := c}
else
        {if c < a then
                min : = c}
```

13. a. [10] Determine the weakest precondition with respect to the postcondition "$z = 2$" for the following (assume z, y, and x are integer variables). Simplify your answer so that there are NO logical operators.

```
x := 3
z := 2*x-y
if y>0 then
        z := z-2
else
        z := -z
endif
```

b. [5] Determine the weakest precondition with respect to the postcondition "$(x = y) \wedge (y = x')$" for the following (assume y, and x are integer variables and are defined):

```
x = y
```