Name ________________________________

Final Examination

CS 336

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1. The important issue is the logic you used to arrive at your answer.
2. Use extra paper to determine your solutions then neatly transcribe them onto these sheets.
3. Do not submit the scratch sheets. However, all of the logic necessary to obtain the solution should be on these sheets.
4. Comment on all logical flaws and omissions and enclose the
comments in boxes

1. [10] How many five digit decimal numbers (without leading zeros) include at least one digit of 3 or 5?

2. [10] Suppose 13 card hands are to be drawn from a regular 52-card deck except that the numbers on the cards are ignored and only the suits matter. Thus, there are 14 spades, 14 hearts etc. . Also, the order of the cards is irrelevant. How many different hands exist?

3. a. [10] Using a combinatorial argument, prove that for $n \geq 1$ and $k \geq 1$:

$$n^k - n^{k-1} = (n-1)n^{k-1}$$

b. [10] Using a combinatorial argument, prove that for $m \geq n \geq p \geq 0$:

$$\binom{m}{n}\binom{n}{p} = \binom{m}{p}\binom{m-p}{n-p}$$

4. a. [5] Consider all distinctly appearing arrangements of the letters of TALLAHASSEE equally likely. What is the probability that such an arrangement spells TALLAHASSEE or EESSA-HALLAT?

b. [5] What is the probability that such an arrangement spells TALLAHASSEE or EESSAHALLAT given that the two ls appear together?

5. [15] Prove: If A and B are countably infinite so is $A \times B$.

6. [10] You are given that if sets A and B are finite, then so is $A \cup B$. Using induction, prove for $k \geq 1$, if $A_1, A_2, , \ldots, A_k$ are finite, then so is $\bigcup_{i=1}^{k} A_i$.

7. [15] Over the set of real-valued functions defined on the natural numbers, define the relation Θ by $f \Theta g$ if and only if $f = O(g)$ and $g = O(f)$. Prove Θ is an equivalence relation.

8. [10] . Prove that if $1 < a$, then $n + \frac{1}{n} = o(n^a)$. (Hint: First bound $n + \frac{1}{n}$ by something that is also $o(n^a)$.)

9. [10]. Prove the code below is correct with respect to precondition "$(x = a) \wedge (y = b)$" and post-condition "$(b > 2a^8) \vee (1 \le y)$". Assume x, and y are integer variables. (Hint: To simplify the process, keep the actual postcondition in mind – it may be much weaker than the strongest post-condition.)

```
x := x*x
x := x*x
x := x*x
if y > 3*x then
        y := 0
        if x > 100 then
                x : = 15*x-1
        else
                x := -x
        endif
else
        y := 1
        x := 2*x
endif
```

10. [10] The code below purports to compute the quotient and remainder of two given positive integers. Prove the code is partially correct with respect to precondition "$(x > 0) \wedge (y > 0)$" and postcondition "$(y = d \cdot x + r) \vee (0 \le r < y)$" (assume x, and y are integer variables.):

```
d := 0
r := x
while r ≥ y do
        r := r-y
        d : d+1
endwhile
```

Be explicit about your loop invariant: I =$(y = d \cdot x + r) \wedge (0 \le r)$

b. [5] Prove that the loop terminates.

11. [10] Assuming x, y, and z are integer variables and are defined, determine the weakest precondition with respect to the postcondition

$$“(x > 0) \wedge (z > y)”:$$

```
if ((x > y) ∨ (y > z)) then
        x: = x+z
        y: = y-x
else
        z := y
        x := z-x
endif
```

State your answer using only relational operators, $\wedge$, and $\vee$.

12. a.[2] Translate the assertion “$m = \max\{x, y\}$” into an equivalent form using only relational and logical operators.

b. [10] Determine the weakest precondition with respect to the postcondition “$m = \max\{x, y\}$” for the following (assume y and x are integer variables and are defined). Substitute your answer to part a for “$m = \max\{x, y\}$”.

```
if ((x > y) ∧ (x > 0)) then
        m := x
else
        m := y
endif
```

. c.[3] Is the code correct with respect to the precondition “$x^2 < 5$” (i.e. does “$x^2 < 5$” imply your weakest precondition)?