Name ________________________________

Final Examination

CS 336

1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
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1. The important issue is the logic you used to arrive at your answer.
2. Use extra paper to determine your solutions then neatly transcribe them onto these sheets.
3. Do not submit the scratch sheets. However, all of the logic necessary to obtain the solution should be on these sheets.
4. Comment on all logical flaws and omissions and enclose the
comments in boxes

1. [10] How many five digit decimal numbers (allowing leading zeros) either begin with 3, end with 5, or contain at least one 7 someplace? (**Do not waste time simplifying**.)

2. [10] Consider arrays of the form $\langle r_1, r_2, r_3, r_4 \rangle$, where $r_1 \geq 2, r_2 \geq 3, r_3 \geq 4,$ and $r_4 \geq 5$. How many such arrays are there satisfying

$$r_1 + r_2 + r_3 + r_4 = 25.$$

(Hint: Consider the excesses.)

3. a. [10] Using a combinatorial argument, prove that for $n \geq 2$ and $m \geq 2$:

$$\binom{n+m}{2} = n \cdot m + \binom{n}{2} + \binom{m}{2}$$

b. [10] Using a combinatorial argument, prove that for $n \geq 1$:

$$\sum_{k=1}^{n} k \binom{n}{k} = n2^{n-1}$$

(Hint: Let A be a set of cardinality n. Consider pairs $\langle a, B \rangle$ where $a \in A \sim B$ and $B \subseteq A \sim \{a\}$.)

4. a. [5] Three dice are rolled. Consider all ordered outcomes equally likely. What is the probability that at least one die shows a 4 given that the sum of the rolls is 12?

b. [5] Three dice are rolled. Consider all ordered outcomes equally likely. Is the event that at least one die shows a 4 statistically independent of the event that the sum of the rolls is 12?

5. [10] Prove: The unit circle $C = \{x + iy : x^2 + y^2 = 1\}$ in the complex plane is uncountably infinite.

6. [10] Let A be a nonempty set. Prove that $\mathcal{P}(A)$, the power set of A, cannot be put into one-to-one correspondence with A (i.e., there exists no function $f : A \xrightarrow[onto]{1-1} \mathcal{P}(A)$). (Hint: What about elements $a \in A$ satisfying $a \notin f(a)$?)

7. [10] Given $f : \square \to \square$ and $g : \square \to \square$ prove that if $f = O(g)$ then $f^2 = O(g^2)$ (where $f^2(n) = (f(n))^2$).

8. [10] . Prove that for any $f : \square \to \square$, the function $g : \square \to \square$ defined as

$$g(n) = \begin{cases} 1 & n = 0 \\ \frac{1}{n} f(n) & n \geq 1 \end{cases} \quad \text{satisfies } g = o(f).$$

9. [10] Assuming x and y are integer variables, prove correct with respect to precondition "y is defined" and postcondition "$x \geq 1$":

```
if y > 0 then
        x := y+6
        if x > 11 then
                x : = x-10
        endif
else
        x := 4-y
        y := y-1
        if y = -3 then
                x : = x-3
        endif
endif
```

10. [10] Consider a function $parity : \square \to \{0,1\}$ defined by $parity(n) = \begin{cases} 0 & \textit{if n is even} \\ 1 & \textit{if n is odd} \end{cases}$. Prove the following code is partially correct with respect to precondition "$n \geq 0$" and postcondition "$p = parity(n)$". (Assume p and i are integer variables.)

```
p := 0
i := 1
while i ≤= n do
        p := 1-p
        i := i+1
endwhile
```

Be explicit about your loop invariant: I $= ((p = parity(i-1)) \wedge (i \leq n+1))$

(Hint: You may want to prove a lemma: $\forall n \in \square$, $parity(n) = 1 - parity(n-1)$.)

11. [10] Prove that the code below terminates. (Assume s and i are integer variables.):

```
s := 0
i := 1
while i ≤= 100000 do
    s := s+i
    i := 4*i+2
endwhile
```

12. [10] Determine the weakest precondition with respect to the postcondition "$S = 0$" for the following (assume S, y, and x are integer variables and y and x are defined

```
if x≠0 then
    x := y
    S := x-y
else
    S := y+x
endif
```

13. [10] Determine the weakest precondition with respect to the postcondition "$x \neq y$" for the following (assume y and x are integer variables and x is defined). Simplify your answer so that there are **NO** logical operators.

```
if x ≥ 3 then
    y:= 2
else
    if x = 2 then
        y := 6
    else
        y := x+1
    endif
endif
```