CS 336

1. The important issue is the logic you used to arrive at your answer.
2. Use extra paper to determine your solutions then neatly transcribe them onto these sheets.
3. Do not submit the scratch sheets. However, all of the logic necessary to obtain the solution should be on these sheets.
4. Comment on all logical flaws and omissions and enclose the
$\boxed{\textbf{comments in boxes}}$

1. [10] Let A and B be non-empty sets with cardinalities m and n, respectively. How many functions from A to B are **not** one-to-one?

2. Consider choosing k objects from a set $\{a_1, a_2, \ldots, a_n\}$ of n objects with order unimportant but repetition allowed.

a. **[5]** How many such selections have exactly one a_1 ?

b. **[5]** How many such selections have at least a_1 ?

3. a. [10] Using a combinatorial argument, prove that for $n \geq 1$:

$$3^n = \sum_{k=0}^{n} \binom{n}{k} 2^k$$

b. [10] Using a combinatorial argument, prove that for $m, n, p \geq 1$:

$$\binom{m+n+p}{m}\binom{n+p}{n} = \binom{m+n+p}{p}\binom{m+n}{n}.$$

4. a. [5] For $n \geq k \geq 1$, suppose k items are selected from $\{a_1, a_2, \ldots, a_n\}$ without repetition and **with no order imposed**. Consider all such selections equally likely. What is the probability that a selection has exactly two of $\{a_1, a_2, a_3\}$?

b. [5] For $n \geq k \geq 1$, suppose k items are selected from $\{a_1, a_2, \ldots, a_n\}$ without repetition and **with order imposed**. Consider all such selections equally likely. What is the probability that a selection has exactly two of $\{a_1, a_2, a_3\}$?

5. [10] Using Definition 2', prove that the set

$$P = \{\textit{infinitely long strings of } 0s \textit{ and } 1s \textit{ with exactly two } 1s\}$$

is infinite.

6. [10] Let A be a countably infinite set , B be an uncountably infinite set nonempty set and $C = A \times B$. Is the C finite, countably infinite, or uncountably infinite? Prove your assertion.

7. [10] Given that for $\alpha > 0, (1+\alpha)^n > 1 + n\alpha,$ prove that $n \cdot 2^n = O(3^n)$.

8. [10] Prove that if $f_1 = O(g_1)$ and $f_2 = o(g_2)$, then $f_1 f_2 = o(g_1 g_2)$.

9. [10] Assuming x and y are integer variables, prove correct with respect to precondition "y is defined" and postcondition "$x > 0$":

```
if y > 0 then
        x := 2*y
        if x > 5 then
                x : = x-4
        endif
else
        x := 4-y
        y := y-1
        if y = -3 then
                x : = x-3
        endif
endif
```

10. [10] Prove the following code is partially correct with respect to precondition "*true*" and postcondition "*k is even*" . (Assume k, n, and i are integer variables and that n and i are defined at input.)

```
k := 1234
while i ≤= n do
        if i ≥ 7 then
                k : = k-12
        else
                k := 4*k-6
        endif
        i := i+5
endwhile
```

Be explicit about your loop invariant: I = ________________________________

11. [10] Prove that the code below terminates. (Assume s and i are integer variables.):

```
s := 0
i := 1
while i ≤= 100000 do
        s := s+i
        i := 4*i+2
endwhile
```

12. [10] Determine the weakest precondition with respect to the postcondition "$S = 0$" for the following (assume S, y, and x are integer variables and y and x are defined

```
if x≠0 then
        x := y
        S := x-y
else
        S := y+x
endif
```

13. [10] Determine the weakest precondition with respect to the postcondition "$x \neq y$" for the following (assume y and x are integer variables and x is defined). Simplify your answer so that there are **NO** logical operators.

```
if x ≥ 3 then
        y:= 2
else
        if x = 2 then
                y := 6
        else
                y := x+1
        endif
endif
```