English to Predicate Calculus Translation Example Solutions

Let

S be a set of students,

G be a set of exam grades, and

E be a set of exam numbers.

For $s \in S, g \in G$, and $e \in E$, define $Gr(s, g, e)$ to be true if and only if student s made grade g on exam number e.

For $g_1, g_2 \in G$, define $H(g_1, g_2)$ to be true if and only if grade g_1 is higher than grade g_2.

Express these English sentences in the notation of Predicate Calculus:

1. Someone got an A on Exam 1.

$$(\exists s \in S)\ Gr(s, A, 1)$$

2. No one failed (i.e., got grade F) both exams 1 and 2.

$$\sim((\exists s \in S)\ (Gr(s, F, 1) \wedge Gr(s, F, 2)))$$

3. Bill got the strictly highest grade on Exam 1.

$$(\exists g_1 \in G)\ (Gr(Bill, g_1, 1) \wedge (\forall s \in S)(\forall g_2 \in G)(((s \neq Bill) \wedge Gr(s, g_2, 1)) \Rightarrow H(g_1, g_2)))$$

4. Exactly two students got As on Exam 1.

$$(\exists s_1 \in S)(\exists s_2 \in S)((s_1 \neq s_2) \wedge Gr(s_1, A, 1) \wedge Gr(s_2, A, 1)$$
$$\wedge (\forall s_3 \in S)((s_3 \neq s_1) \wedge (s_3 \neq s_1)) \Rightarrow \sim Gr(s_3, A, 1)))$$

5. At least two students got Bs on Exam 2.

$$(\exists s_1 \in S)(\exists s_2 \in S)((s_1 \neq s_2) \wedge Gr(s_1, B, 2) \wedge Gr(s_2, B, 2))$$

6. At most two students got Cs on Exam 3.

$$\sim((\exists s_1 \in S)(\exists s_2 \in S)(\exists s_3 \in S)((s_1 \neq s_2) \wedge (s_1 \neq s_3) \wedge (s_2 \neq s_3) \wedge$$
$$Gr(s_1, C, 3) \wedge Gr(s_2, C, 3) \wedge Gr(s_3, C, 3))$$

7. All the students who got As on Exam 1 got Bs on Exam 2 and vice versa (i.e., all the students who got As on Exam 2 got Bs on Exam 1).

$$(\forall s \in S)\ ((Gr(s, A, 1) \Rightarrow Gr(s, B, 2)) \wedge (Gr(s, A, 2) \Rightarrow Gr(s, B, 1)))$$

8. If everyone got an A on Exam 1 then only one student got an A on Exam 2.

$((\forall s \in S)\, Gr(s,\, A, 1)) \Rightarrow$
$\quad (\exists s_1 \in S)\, (Gr(s_1,\, A, 1) \wedge (\forall s_2 \in S)\, ((s1 \neq s2) \Rightarrow\, \sim Gr(s_2,\, A, 1)))$

9. Even if everyone got an A on Exam 1 then only one student got an A on Exam 2.

$(\exists s_1 \in S)\, (Gr(s_1,\, A, 1) \wedge (\forall s_2 \in S)\, ((s1 \neq s2) \Rightarrow\, \sim Gr(s_2,\, A, 1)))$