Some Examples of Predicate Usage

1. Let's do a few examples without quantifiers to get started. Suppose there some predicates. The domain for the predicates is the set of people.

mechanic *x*	*x* is a mechanic.
likes *xy*	*x* likes *y*.
introduced *xyz*	*x* introduced *y* to *z*.

a. Then "Cathy and Bob are mechanics" is no more than:

$$\text{mechanic Cathy} \wedge \text{mechanic Bob}$$

b. "Bob likes Cathy" is:

likes Bob Cathy

c. "Bob likes himself" is:

likes Bob Bob

d. "Cathy likes either Bob or Alex" is:

$$\text{likes Cathy Bob} \vee \text{likes Cathy Alex}$$

e. "Cathy introduced herself to Bob but not Alex" is:

$$\text{introduced Cathy Cathy Bob} \wedge \sim\text{introduced Cathy Cathy Alex}$$

Notice that logically "and" and "but" have the very same meaning.

2. Now we introduce quantifiers ("introduced Cline quantifiers you", using the notation from problem 1). Here are some predicates and again the domain for all the predicates is the set of people.

E *x*	*x* is an embezzler.
W *x*	*x* is wicked.
R *x*	*x* is in this room.
L *x*	*x* is leaving.

As you consider the use of quantifiers, you should be constantly asking yourself "Am I saying something about ***some*** or about ***all*** ?".

a. "All embezzlers are wicked" may be written:

$$\forall x(Ex \Rightarrow Wx)$$

b. "Not all embezzlers are wicked" is precisely the negation of the previous sentence so it may be written:

$$\sim \forall x(Ex \Rightarrow Wx)$$

But we could also say that this is equivalent to saying that "There exists at least one person who is an embezzler but who is not wicked". This equivalent form is written:

$$\exists x(Ex \wedge \sim Wx)$$

Soon, we will introduce a rewrite rule called "Quantifier Excahange" that formalizes the identity of the two, but for right now, we'll say we know these are equivalent.

c. Thus, the statement "Some embezzlers are not wicked." is the same thing so can be written as

$$\sim \forall x(Ex \Rightarrow Wx)$$

or

$$\exists x(Ex \wedge \sim Wx).$$

d. "If Jack is an embezzler, then Jack is wicked" is a weaker version of 2a above. Above the statement about embezzling and wickedness holds for everyone; here we are just claiming it for one individual: Jack. So we get

$$E\text{ Jack} \Rightarrow W\text{ Jack}.$$

e. "Nobody in the room is leaving" can be stated using either existential or universal quantification. If you see it as the negation of an existential then you say:

$$\sim \exists x(Rx \wedge Lx).$$

However, if you see it as a universal with a "guarded" negation then you have the logically equivalent:

$$\forall x(Rx \Rightarrow \sim Lx)$$

By "guarded" statement, I mean there is some sort of conditional. To say "Today is Tuesday" is non-conditional. However, if I modify that to "If my memory is correct, today is Tuesday" then I have *guarded* the "Today is Tuesday" statement with the premise "My memory is correct". Students often find such translation from English a bit tricky in the beginning – especially when there are negations floating around.

f. "Someone in the room is leaving and someone isn't". is the conjunction of two existentials:

$$\exists x(Rx \wedge Lx) \wedge \exists y(Ry \wedge \sim Ly).$$

This alternative is also perfectly correct:

$$\exists x(Rx \wedge Lx) \wedge \exists x(Rx \wedge \sim Lx).$$

The "scope" of the first quantifier is just $(Rx \wedge Lx)$ so the symbol x maybe reused. (Think of dummy variables within procedures. More on scope later.)

3. Here are a few more. They may be slightly more tricky simply because when no quantification is explicitly stated in English, the universal quantification is "understood" . Thus when I say "Martians are green" I mean "**ALL** Martians are green". The predicates used are

F x	x is a frog.
G x	x is green.

and the set of discourse is the set of things.

a. “Frogs are green” is:

$$\forall x(Fx \Rightarrow Gx).$$

(Notice we are not claiming everything is green, we guard the claim by applying it only to frogs.)

b. “Some frogs are not green” is the negation of a. We could say:

$$\sim \forall x(Fx \Rightarrow Gx)$$

or

$$\exists x(Fx \wedge \sim Gx).$$

c. “Frogs are not green” is a much stronger statement than b, however. It says “**NO** frogs are green”. So if we write it with a universal we have:

$$\forall x(Fx \Rightarrow \sim Gx)$$

or if we write it equivalently with an existential:

$$\sim \exists x(Fx \wedge Gx).$$

d. “Not everything is a frog” is

$$\sim \forall x(Fx).$$

You may be tempted to say this is the same as

$$\exists x(\sim Fx)$$

And that is correct as long as the set of “things” is non-empty. (More on this later, too.)

e. “Only frogs are green” is a conditional. That is signaled by the “only”. We have:

$$\forall x(Gx \Rightarrow Fx).$$

Notice the order of G and F. Make sure you understand that we are saying greenness implies frogness and not the other way around. Of course, since you could think of it meaning “If you are not a frog, you are not green”, you could have said it equivalently as:

$$\forall x(\sim Fx \Rightarrow \sim Gx).$$