Weak and Strong Assertions

"p is stronger than q" is another way of saying "p implies q". Thus the statement that "x is a cow" is stronger than the statement "x is an animal".

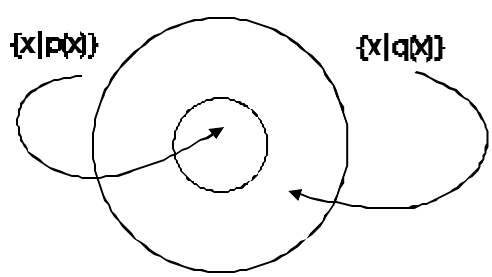
"p is weaker than q" is another way of saying "q implies p". Thus the statement that "x is a animal" is weaker than the statement "x is an cow".

Any assertion is strengthened by forming a conjunction with another assertion. Thus "x is a cow and x is 3 years old" is stronger that "x is a cow". This is just another way of saying $(p \wedge q) \Rightarrow p$ for any *p* and *q*.

Any assertion is weakened by forming a disjunction with another assertion. Thus "x is a cow or x is 3 years old" is weaker that "x is a cow". This is just another way of saying $p \Rightarrow (p \vee q)$ for any *p* and *q*.

Now imagine starting with an assertion and conjuncting it with everything in sight: p and q and w and r and ... . Notice this assertion is getting stronger and stronger as you add assertions. Is there a "limit"? The answer is yes and that limit is just FALSE. We call FALSE the strongest possible assertion. This is totally consistent since we know that FALSE implies q for any q. The statement FALSE can be written other ways such as "x is positive and x is not positive" or "x has children and x has no children". They are all the strongest possible statements since if there were to exist a stronger assertion p then, by definition, we would have "p implies FALSE". We know this holds only if p = FALSE. Starting with the hypothesis FALSE we can conclude anything.

Go the other way. Imagine starting with an assertion and disjuncting it with everything in sight: p or q or w or r or ... . Notice this assertion is getting weaker and weaker as you or it with assertions. Is there a "limit"? The answer is yes and that limit is just TRUE. We call TRUE the weakest possible assertion. This is totally consistent since we know that q implies TRUE for any q. The statement TRUE can be written other ways such as "x is positive or x is not positive" or "x has children or x has no children". They are all the weakest possible statements since if there were to exist a weaker assertion p then, by definition, we would have "TRUE implies p". We know this holds only if p = TRUE. Starting with the hypothesis TRUE is equivalent to starting with nothing.



Now think about sets of the form P = {x | p(x)} where p is an assertion with variable x. Suppose p(x) is stronger than q(x). What about the associated set Q = {x | q(x)} relative to P = {x | p(x)}? Do you see that P must be a subset of Q? (Do some examples using p(x)= "x is a cow" and q(x) = "x is an animal" if it helps.) Notice that the set {x | FALSE(x)} is the empty set. If you want a way to remember this, think "stronger means smaller" in terms of sets.

Turning to the concept of "weaker", we see that if p(x) is weaker than q(x) then P is a superset of Q. Remember this as "weaker means wider" in terms of sets.