

1. Given floating point numbers a, b , and c such that both $a + b$ and $fl(a + b) + c$ are in range then there exist real numbers \bar{a}, \bar{b} , and \bar{c} so that $fl((a + b) + c) = (\bar{a} + \bar{b}) + \bar{c}$, where for some $\mathbf{e}_a, \mathbf{e}_b$, and \mathbf{e}_c satisfying $|\epsilon_a| \leq 2\epsilon_0 + O(\epsilon_0^2)$, $|\epsilon_b| \leq 2\epsilon_0 + O(\epsilon_0^2)$, and $|\epsilon_c| \leq \epsilon_0$, we have $\bar{a} = a(1 + \epsilon_a)$, $\bar{b} = b(1 + \epsilon_b)$, and $\bar{c} = c(1 + \epsilon_c)$.

2. Given floating point numbers a, b, c , and d such that $a \cdot b, c / d$ and $fl(a \cdot b) + fl(c / d)$ are in range then there exist real numbers $\bar{a}, \bar{b}, \bar{c}$ and \bar{d} so that $fl(a \cdot b + c / d) = \bar{a} \cdot \bar{b} + \bar{c} / \bar{d}$, where for some $\mathbf{e}_a, \mathbf{e}_b, \mathbf{e}_c$, and \mathbf{e}_d satisfying $|\epsilon_a| \leq 2\epsilon_0 + O(\epsilon_0^2)$, $|\epsilon_b| \leq 2\epsilon_0 + O(\epsilon_0^2)$, $|\epsilon_c| \leq 2\epsilon_0 + O(\epsilon_0^2)$, and $|\epsilon_d| \leq 2\epsilon_0 + O(\epsilon_0^2)$, we have $\bar{a} = a(1 + \epsilon_a)$, $\bar{b} = b(1 + \epsilon_b)$, $\bar{c} = c(1 + \epsilon_c)$, and $\bar{d} = d(1 + \epsilon_d)$.