Gaussian Elimination Algorithm
with Partial Pivoting and Elimination Separated from Solving, and Column Operation

Forward Elimination Applied to Matrix

for *k* = 1:*n* .....*The outer loop - this eliminates variable k*
  choose ip_k such that $|A_{ip_k,k}| = \max\{|A_{i,k}| : i \geq k\}$ .....*Find the largest of the candidate pivots*
  if $A_{ip_k,k}$ = 0.....*if the largest is zero, no possible pivot*
    warning ('Pivot in Gaussian Elimination is zero').....and maybe get out of here
  end
  swap $A_{k,k}$ **with** $A_{ip_k,k}$ .....swap just the pivot element
  for *j* = *k*+1:*n* .....*get all of the multipliers*
    $A_{i,k} = A_{i,k} / A_{k,k}$
  end
  for *j* = *k*+1:*n* .....*loop on the columns*
    swap $A_{k,j}$ **with** $A_{ip_k,j}$ $A_{ip_k,k}$ .....swap just the elements in the jth column
    for i = *k*+1:*n*.....*loop on the rows – innermost loop*
      $A_{i,j} = A_{i,j} - A_{i,k}A_{k,j}$ .....update the i,j element
    end
  end
end

This results in the upper triangle of the eliminated system in the upper triangle of A, the multipliers in the strict lower triangle of A, and the swapping information in the ip array.

Solving ..... *notice no appearance of b until now*
for *k* = 1:*n* .....*here is where we apply the swapping and elimination to b*
  swap b_k **with** b_{ip_k} ..... *here is where we need to remember the swapping information*
  for *i* = *k*+1:*n* .....*loop on the rows*
    $b_i = b_i - A_{i,k}\, b_k$ .... compare the k-loop to the one above
  end
end

x = **b**
for *j* = *n*:-1:1.....*this code switches the I and j loops from before*
  $x_j = x_j / A_{j,j}$
  for *i*= 1:*j*-1
    $x_i = x_i - A_{i,j}x_j$
  end
end

and the output is the solution x.