M 340L – CS
Homework Set 2

Find the general solutions of the systems whose augmented matrices are given in Problems 1 - 4.

1.
$$\begin{bmatrix} 1 & -3 & 0 & -5 \\ -3 & 7 & 0 & 9 \end{bmatrix}$$

2.
$$\begin{bmatrix} 1 & -2 & -1 & 4 \\ -2 & 4 & -5 & 6 \end{bmatrix}$$

3.
$$\begin{bmatrix} 3 & -2 & 4 & 0 \\ 9 & -6 & 12 & 0 \\ 6 & -4 & 8 & 0 \end{bmatrix}$$

4.
$$\begin{bmatrix} 1 & 0 & 5 & 0 & -8 & 3 \\ 0 & 1 & 4 & -1 & 0 & 6 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

5. Key statements from Lay, Section 1.2 are either quoted directly, restated slightly (but still true), or altered in some way that makes them false in some cases. Mark each statement True or False, and justify your answer. (If true, give the approximate location where a similar statement appears, or refer to a definition or theorem. If false, cite an example that shows the statement is not true in all cases or give the location of a statement that has been quoted or used incorrectly.)

a. In some cases, a matrix may be row reduced to more than one matrix in reduced echelon form, using different sequences of row operations.

b. The row reduction algorithm applies only to augmented matrices for a linear system.

c. A basic variable in a linear system is a variable that corresponds to a pivot column in the coefficient matrix.

d. Finding a parametric description of the solution set of a linear system is the same as solving the system.

e. If one row in an echelon form of an augmented matrix is [0 0 0 5 0] , then the associated linear system is inconsistent.

6. Write a system of equations that is equivalent to the vector equation:

$$x_1\begin{bmatrix}3\\-2\end{bmatrix}+x_2\begin{bmatrix}7\\3\end{bmatrix}+x_3\begin{bmatrix}-2\\1\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}.$$

7. Write a vector equation that is equivalent to system of equations:

$$\begin{aligned}3x_1-2x_2+4x_3&=3\\-2x_1-7x_2+5x_3&=1\\5x_1+4x_2-3x_3&=2\end{aligned}$$

8. Let $A=\begin{bmatrix}2&0&6\\-1&8&5\\1&-2&1\end{bmatrix}$, let $\boldsymbol{b}=\begin{bmatrix}10\\3\\7\end{bmatrix}$, and let and let W be the set of all linear combinations of the columns of A.

a. Determine if b is in W?

b. Show that the second column of A is in W.

9. Compute the product:

$$\begin{bmatrix} 1 & 3 & -4 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

10. Write the matrix equation as a vector equation:

$$\begin{bmatrix} 2 & -3 \\ 3 & 2 \\ 8 & -5 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} -3 \\ 5 \end{bmatrix} = \begin{bmatrix} -21 \\ 1 \\ -49 \\ 11 \end{bmatrix}$$

11. Write the vector equation as a matrix equation:

$$z_1 \begin{bmatrix} 2 \\ -4 \end{bmatrix} + z_2 \begin{bmatrix} -1 \\ 5 \end{bmatrix} + z_3 \begin{bmatrix} -4 \\ 3 \end{bmatrix} + z_4 \begin{bmatrix} 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \end{bmatrix}$$

12. Given A and b, write the augmented matrix for the linear system that corresponds to the matrix equation. $Ax = b$. Then solve the system and write the solution as a vector.

$$A = \begin{bmatrix} 1 & 2 & -1 \\ -3 & -4 & 2 \\ 5 & 2 & 3 \end{bmatrix}, b = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}.$$

13. Key statements from Lay, Section 1.4 are either quoted directly, restated slightly (but still true), or altered in some way that makes them false in some cases. Mark each statement True or False, and justify your answer. (If true, give the approximate location where a similar statement appears, or refer to a definition or theorem. If false, give the location of a statement that has been quoted or used incorrectly, or cite an example that shows the statement is not true in all cases.)

a. If the equation $Ax = b$ is consistent, then b is in the set spanned by the columns of A.

b. The solution set of a linear system whose augmented matrix is $\begin{bmatrix} \boldsymbol{a}_1 & \boldsymbol{a}_2 & \boldsymbol{a}_3 & \boldsymbol{b} \end{bmatrix}$ is the same as the solution set of $Ax = b$, if $A = \begin{bmatrix} \boldsymbol{a}_1 & \boldsymbol{a}_2 & \boldsymbol{a}_3 \end{bmatrix}$.