M 340L – CS
Homework Set 4

1. Mark T(rue) or F(alse) for each of the following statements:

____a. The column space of A is the set of all vectors that can be written as Ax for some x.

____b. Elementary row operations on an augmented matrix can change the solution set of the associated linear system.

____c. If b is in the set spanned by the columns of A then the equation $Ax = b$ is consistent.

____d. For an $n \times n$ system of linear equations, the Gaussian Elimination Algorithm with Partial Pivoting and Elimination Separated from Solving uses approximately $n^3/3$ floating point multiplications and $2n^3/3$ floating point additions/subtractions.

____e. the Gaussian Elimination Algorithm with Partial Pivoting has multipliers no larger than one in absolute value.

____f. A homogeneous equation is always consistent.

____g. The homogeneous equation $Ax = 0$ has the trivial solution if and only if the equation has at least one free variable.

____h. If x is a nontrivial solution of $Ax = 0$, then every entry in x is nonzero.

____i. The effect of adding p to a vector is to move the vector in a direction parallel to p.

____j. The equation $Ax = b$ is homogeneous if the zero vector is a solution.

2. a Find the general solutions of the systems $Ax = 0$ whose matrix is:

$$\begin{bmatrix} 1 & -2 & 3 & -6 & 5 & 0 \\ 0 & 0 & 0 & 1 & 4 & -6 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

b. Express this solution in parametric vector form.

3. Show that if $Ax = b$ and $Ay = 0$, then for any scalar α, $A(x + \alpha y) = b$. (Remember a proof **begins** with the hypothesis and **ends** with the conclusion.)

4. Suppose you are to solve m different linear systems of n equations in n unknowns. All of the equations have the same matrix; however, they just differ in right hand sides. Estimate how many multiplications are required.

5. Use the Gaussian Elimination with Partial Pivoting and Solution algorithm to solve

$$
\begin{aligned}
3x_1 + 5x_2 - 2x_3 &= -16 \\
-3x_1 \qquad\quad - x_3 &= -5 \\
6x_1 + 2x_2 + 4x_3 &= 8
\end{aligned}
$$

Show what occupies storage in the A matrix and the ip array initially and after each major step of elimination.

A ip

6. Fill in the **five** blanks in the code for Gaussian Elimination with Partial Pivoting and Solution

for k = 1:n

  choose ip_k such that $|A_{ip_k,k}| = \max\{|A_{i,k}| : i \geq k\}$

  if $A_{ip_k,k}$ = 0

    warning ('Pivot in Gaussian Elimination is zero')

  end

  swap $A_{k,k}, \ldots, A_{k,n}$ with $A_{ip_k,k}, \ldots, A_{ip_k,n}$

  for i = _____:n

    $A_{i,k}$ = _________

    for j = k+1:n

      $A_{i,j} =$ ____________

    end

  end

end

for k = 1:n

  swap b_k with ______

  for i = k+1:n

    $b_i = b_i - A_{i,k}\, b_k$

  end

end

x = b

for i = n:-1:1

  for j = i+1:n

    $x_i = x_i - A_{i,j} x_j$

  end

  $x_i =$ ________

end