CS 341 Homework 7
Review of Equivalence Relations

1. Assume a finite domain that includes just the specific cities mentioned here. Let R = the reflexive, symmetric, transitive closure of:

(Austin, Dallas), (Dallas, Houston), (Dallas, Amarillo), (Austin, San Marcos),
(Philadelphia, Pittsburgh), (Philadelphia, Paoli), (Paoli, Scranton),
(San Francisco, Los Angeles), (Los Angeles, Long Beach), (Long Beach, Carmel)

(a) Draw R as a graph.
(b) List the elements of the partition defined by R on its domain.

2. Let R be a relation on the set of positive integers. Define R as follows:

{(a, b) : (a mod 2) = (b mod 2)} In other words, R(a, b) iff a and b have the same remainder when divided by 2.

(a) Consider the following example integers: 1, 2, 3, 4, 5, 6. Draw the subset of R involving just these values as a graph.
(b) How many elements are there in the partition that R defines on the positive integers?
(c) List the elements of that partition and show some example elements.

3. Consider the language L, over the alphabet $\Sigma = \{a, b\}$, defined by the regular expression

$a^*(b \cup \varepsilon)\, a^*$

Let R be a relation on Σ^*, defined as follows:

R(x, y) iff both x and y are in L or neither x nor y is in L. In other words, R(x,y) if x and y have identical status with respect to L.

(a) Consider the following example elements of Σ^*: ε, b, aa, bb, aabaaa, bab, bbaabb. Draw the subset of R involving just these values as a graph.
(b) How many elements are there in the partition that R defines on Σ^*?
(c) List the elements of that partition and show some example elements.

Solutions

1. **(b)** [cities in Texas], [cities in Pennsylvania], [cities in California]

2. **(b)** Two
(c) [even integers] Examples: 2, 4, 6, 106
[odd integers] Examples: 1, 3, 5, 17, 11679

3. **(a)** (Hint: L is the language of strings with no more than one b.)
(b) Two
(c) [strings in L] Examples: ε, aa, b, aabaaa
[strings not in L] Examples: bb, bbaabb, bab