CS 341 Homework 19
Turing Machine Extensions

1. Consider the language $L = \{ww^R\}$.
(a) Describe a one tape Turing machine to accept L.

(b) Describe a two tape Turing machine to accept L.

(c) How much more efficient is the two tape machine?

2. Give (in abbreviated notation) a nondeterministic Turing machine that accepts the language
$L = \{ww^Ruu^R : w, u \in \{a, b\}^*\}$

Solutions

(1) (a) The one tape machine needs to bounce back and forth between the beginning of the input string and the end, marking off matching symbols.

(b) The two tape machine works as follows: If the input is ε, accept. If not, copy the input to the second tape and record in the state that you have processed an even number of characters so far. Now, start the first tape at the left end and the second tape at the right end . Check that the symbols on the two tapes are the same. If not, reject. If so, move the first tape head to the right and the second tape head to the left. Also record that you have processed an odd number and continue, each time using the state to keep track of whether you've seen an even or odd number of characters so far. When you reach the end of the input tape, accept if you've seen an even number of characters. Reject if you've seen an odd number. (The even/odd counter is necessary to make sure that you reject strings such as aba.)

(c) The one tape machine takes time proportional to the square of the length of the input, since for an input of length n it will make n passes over the input, each of which takes on average n/2 steps. The two tape machine takes time that's linear in n. It takes n steps to copy, then another n steps to compare.

2. The idea is just to use nondeterminism to guess the location of the boundary between the w and u regions. Each path will choose a spot, shift the u region to the right, and insert a boundary marker #. Once this is done, the machine simply checks each region for ww^R. If we get a string in L, one of the guessed paths will work.