M 340L – CS
Homework Set 3

1. Determine if the system has a nontrivial solution. (One SAXPY operation – and no other row operations – is sufficient to determine the answer.)

$$\begin{aligned} 5x_1 - 3x_2 + 2x_3 &= 0 \\ -3x_1 - 4x_2 + 2x_3 &= 0 \end{aligned}$$

$$\begin{bmatrix} 5 & -3 & 2 & 0 \\ -3 & -4 & 2 & 0 \end{bmatrix} \to \begin{bmatrix} 5 & -3 & 2 & 0 \\ 0 & -29/5 & 16/5 & 0 \end{bmatrix}.$$ x_3 is free so there is a nontrivial solution.

2. Write the solution set of the given homogeneous system in parametric vector form. (Follow the method of Examples 1 and 2 in Lay 1.5)

$$\begin{aligned} x_1 + 2x_2 - 3x_3 &= 0 \\ 2x_1 + x_2 - 3x_3 &= 0 \\ -x_1 + x_2 &= 0 \end{aligned}$$

$$\begin{bmatrix} 1 & 2 & -3 & 0 \\ 2 & 1 & -3 & 0 \\ -1 & 1 & 0 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & -3 & 0 \\ 0 & -3 & 3 & 0 \\ 0 & 3 & -3 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & -3 & 0 \\ 0 & -3 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & -3 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\to \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

x_3 is free, $x_2 = x_3, x_1 = x_3$. In parametric vector form this is $x = x_3 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

In problems 3 and 4, describe all solutions of $Ax = 0$ in parametric vector form, where A is row equivalent to the given matrix.

3.

$$\begin{bmatrix} 1 & -3 & -8 & 5 \\ 0 & 1 & 2 & -4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -3 & -8 & 5 \\ 0 & 1 & 2 & -4 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & -2 & -7 \\ 0 & 1 & 2 & -4 \end{bmatrix}.$$ x_4 is free and x_3 is free, $x_2 = -2x_3 + 4x_4$,

$x_1 = 2x_3 + 7x_4$. In parametric vector form this is $x = x_3 \begin{bmatrix} 2 \\ -2 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 7 \\ 4 \\ 0 \\ 1 \end{bmatrix}$.

4.

$$\begin{bmatrix} 1 & -2 & 3 & -6 & 5 & 0 \\ 0 & 0 & 0 & 1 & 4 & -6 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 3 & -6 & 5 & 0 \\ 0 & 0 & 0 & 1 & 4 & -6 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & -2 & 3 & 0 & 29 & -36 \\ 0 & 0 & 0 & 1 & 4 & -6 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & -2 & 3 & 0 & 29 & 0 \\ 0 & 0 & 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

$x_6 = 0, x_5$ is free, $x_4 = -4x_5, x_3$ is free, x_2 is free, and $x_1 = 2x_2 - 3x_3 - 29x_5$. In parametric vector form this is $x = x_2 \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -3 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} -29 \\ 0 \\ 0 \\ -4 \\ 1 \\ 0 \end{bmatrix}$.

5. Mark each statement True or False then justify your answer.

a. A homogeneous system of equations can be inconsistent.

False. Since the zero vector is always a solution, a homogeneous system of equations can never be inconsistent.

b. If x is a nontrivial solution of $Ax = 0$, then every entry in x is nonzero.

False. A nontrivial solution need only have at least one non-zero component.

c. The effect of adding p to a vector is to move the vector in a direction parallel to p.

True. The vertices $0, p, x+p, x$ form a parallelogram so the effect of adding p to x to get $x+p$ is a line parallel to the line from zero to p.

d. The equation $Ax = b$ is homogeneous if the zero vector is a solution.

True. Since $A0 = b = 0$, the equation $Ax = b$ is also $Ax = 0$ and thus is homogeneous.

6. If $b \neq 0$, can the solution set of $Ax = b$ be a plane through the origin? Explain.

No, if $b \neq 0$, the solution set of $Ax = b$ cannot be a plane through the origin. If it were then $A0 = 0 = b \neq 0$, which is a contradiction.

7. Construct a 3×3 nonzero matrix A such that the vector $\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$ is a solution of $Ax = 0$.

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

8. Given $A = \begin{bmatrix} 3 & -2 \\ -6 & 4 \\ 12 & -8 \end{bmatrix}$, find one nontrivial solution of $Ax = 0$ by inspection.

Since the twice the first column plus three times the third column is zero, $x = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$