Theorem: If $n \geq 2$, then $n^3 > 2n+3$.

Proof 1:

We proceed by induction. The base case is $n=2$, and for that we have $n^3 = 8 > 7 = 2n+3$. We now assume $n^3 > 2n+3$, and consider the result for $n+1$. We have:

$$
\begin{aligned}
(n+1)^3 &= n^3 + 3n^2 + 3n + 1 \\
&\geq n^3 + 3n + 1 \\
&\geq n^3 + 7 \quad (\text{since } n \geq 2) \\
&> 2n + 3 + 7 \quad (\text{by the inductive hypothesis}) \\
&> 2n + 2 + 3 \\
&> 2(n+1) + 3,
\end{aligned}
$$

which concludes the proof.

Notes:

1. Some people may object to all of the uses of $\geq$ when I could have used $>$. Why did I do something strange like that? First ask yourself: "Did he do something **incorrect**?". To that you must answer that what I did is correct. You may be telling yourself "... but you could have made a stronger statement by using $>$.". I totally agree, but why should I? I want to produce a proof that is as easy to believe as possible. Please agree that proving a weaker claim with $\geq$ is always easier than the stronger claim with $>$. If I have a string of inequalities with $\geq$ and at least one use of $>$, then I get the strict inequality overall. If > actually is obvious (and I admit some of these are) use it but realize that $\geq$ works as long as there is at least one > in the chain. (I was trying to make a point here.)
2. Now look at the last three lines of the chain of inequalities. How did I have the foresight to see that 2+3 stuff and all? **I didn't**. I did some scratch paper work to see what I would need on those lines **knowing** that the final line had to be $> 2(n+1)+3$. As always, that scratch paper work is not part of the proof.
3. There are non-inductive proofs of this result. Here is one that uses the Fundamental Theorem of Calculus.

Theorem: If $n \geq 2$, then $n^3 > 2n+3$.

Proof 2:

The result is equivalent to "If $n \geq 2$, then $n^3 - 2n - 3 > 0$.". Define $f(n) = n^3 - 2n - 3$, and notice that $f(2) = 2^3 - 2 \cdot 2 - 3 = 1 > 0$. Also, notice that

$$f'(t) = 3t^2 - 2 \geq 3 \cdot 4 - 2 = 10 > 0, \text{ for } n \geq 2.$$

Thus, $\int_2^n f'(t)dt \geq 0,$ since it is the integral of a positive function. From the Fundamental Theorem of Calculus, we have

$$f(n) = f(2) + \int_2^n f'(t)dt \geq f(2) = 1 > 0.$$

The proof I did on the board of the next one was fine **unless** someone wants a Querium style proof. Here is the Querium style proof:

Theorem: For any sets $A, B, \text{and } C,$ if $A \subseteq B,$ then $C - B \subseteq C - A.$

Proof:

1.	$x \in C - B$		
2.	$x \in C \wedge \neg(x \in B)$	Definition of set difference applied to	[1]
3.	$x \in C$	Simplification	[2]
4.	$\neg(x \in B)$	Simplification	[2]
5.	$A \subseteq B$	Premise	
6.	$x \in A \rightarrow x \in B$	Definition of subset applied to	[5]
7.	$\neg(x \in A)$	Modus Tolens	[4,6]
8.	$x \in C \wedge \neg(x \in A)$	Conjunction	[3,7]
9.	$x \in C - A$	Definition of set difference applied to	[8]