The next time we meet, I will discuss the role of probability on the drawing of conclusions. In particular, I'll mention situations where actions that are not random may be disguised as random events and thus mislead us. It will be important not just to discuss the "correct" solutions to these problems but also how you determined your solutions. (Let me add that, based upon previous experience, everyone who believed the problems were simple and could be solved immediately without simulation or experimentation was wrong. Remember, the idea is that humans do bad jobs of these things.) For the purpose of beginning the discussion, please consider this variant of the common shell game:

Shell Game Problem

A pea is placed under one of three shells and the shells are then manipulated in such a fashion that all of the three appear to be equally likely to contain the pea. Nevertheless, you win a prize if you guess the correct shell, so you make a guess. The person running the game does know the correct shell, however, and uncovers one of the shells that you did not choose and that is empty. Thus, what remain are two shells: one you chose and one you did not choose. Furthermore, since the uncovered shell did not contain the pea, one of the two remaining shells does contain it. You are offered the opportunity to change your selection to the other shell. Should you?

You may answer yes, no, or it makes no difference. In any case, think carefully and defend your choice.

Number Guessing Problem

You are asked to guess a number between 1 and 100. You announce your selection (call it a) and the person running the game tells you that the correct number is not some b that is not equal to a. For example, you guess 12 and you are told that 17 is incorrect. You are given the option to change your mind and choose another number. Should you change? If so, to what? If the original game were a fair one, you would expect that if you paid $1 to play, you should get $100 if you win. However, what about the modified game: how much should you win for it given an entry fee of $1?

Card Guessing Problem

Someone shows you three special cards. One is red on both sides, one is black on both sides, and one is red on one side but black on the other. The person puts the cards in a bag, mixes them up, and draws one in such a manner that only one side shows to either of you (i.e. neither she nor you know what's on the other side). The side that shows is red and the person says: "Since this side is red, this cannot be the card that is black on both sides and thus must be either the card that is red on both sides or the one with two colors. Since the cards have been thoroughly mixed, it is equally likely either of those two. For your $1 then, I will give you $2 if it's the card with two colors but nothing if it's the two-sided red card." Should you play?

Another Card Guessing Problem

Someone tells you she has a bag containing cards with values written on both sides. In fact for each card, the value on one side is exactly twice the value on the other side. However, there are many cards in the bag and the values on distinct cards may be quite different. For example, there may be a $1/$2 card in the bag as well as a $1,000,000/$2,000,000 card - you have no idea of the extremes. She draws a card and shows you one side: it says $10. Neither you nor she knows whether the side with $10 is the low side or the high side of the card. She offers to give you the $10 or the unknown value written on the other side (i.e. either $5 or $20). Should you take the $10 (or do you figure that there is equal chance of the other side having either $5 or $20 in which case the expected value of the other side is $12.50)?