A Proof that all Pool Balls are the Same Color

Let $P(n)$ be the proposition that, within every set of pool balls of cardinality n, all of the balls have the same color. We prove this by induction. Consider first $n=1$. Certainly, within every set of pool balls of cardinality 1, all of the balls have the same color. Now assume the inductive hypothesis and consider sets of cardinality $n+1$. Designate one of the balls as A and remove it temporarily from the set. The remaining set has cardinality n, thus from the inductive hypothesis all of the balls have the same color. Designate two of the balls *of the remaining set* as B and C. Since B and C come from the remaining set, they have the same color. (If it helps, draw a picture of a bunch of balls and label A, B, and C.) Remove B from the set and add A back in. The set again has cardinality n, thus from the inductive hypothesis all of the balls have the same color. Since this set includes C, the color of all the balls must be the same color as C. However, A is also in this set of cardinality n, so the color of A must match that of C. But, as said above, B has the same color as C, so all $n+1$ balls have the same color. We have now proved that $P(n) \rightarrow P(n+1)$.

Take $n = $ the number of pool balls in the universe, and we have that all of the balls in the universe have the same color.