# Lattice-Based SNARGs and Their Application to More Efficient Obfuscation

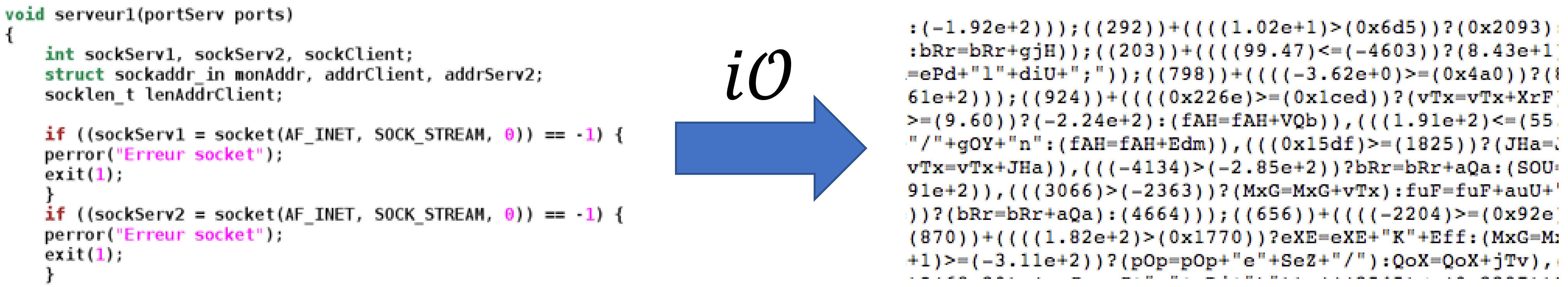
Dan Boneh, Yuval Ishai, Amit Sahai, and David J. Wu

# Program Obfuscation [BGIRSVY01, GGHRSW13]

Indistinguishability obfuscation ($i\mathcal{O}$) has emerged as a “central hub for cryptography” [BGIRSVY01, GGHRSW13]

[GGHRSW13, SW14, BZ14, BST14, GGHR14, GHRW14, BP15, CHNVW15, CLTV15, GP15, GPS16, BPW16 …]

Takes a program as input and “scrambles” it



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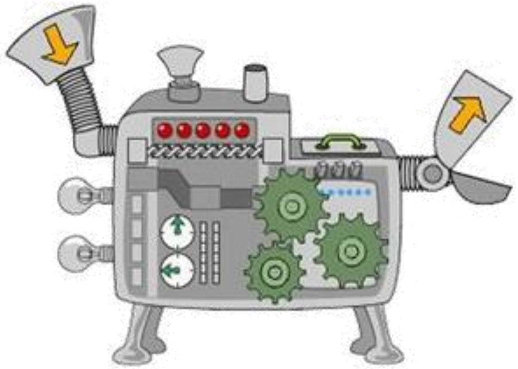
Many applications, yet extremely far from practical


Not just engineering challenges – fundamental theoretical challenges

Polynomial-time, but constant factors are $\geq 2^{100}$

# Our Goal

Obtain an "obfuscation-complete" primitive with an emphasis on concrete efficiency



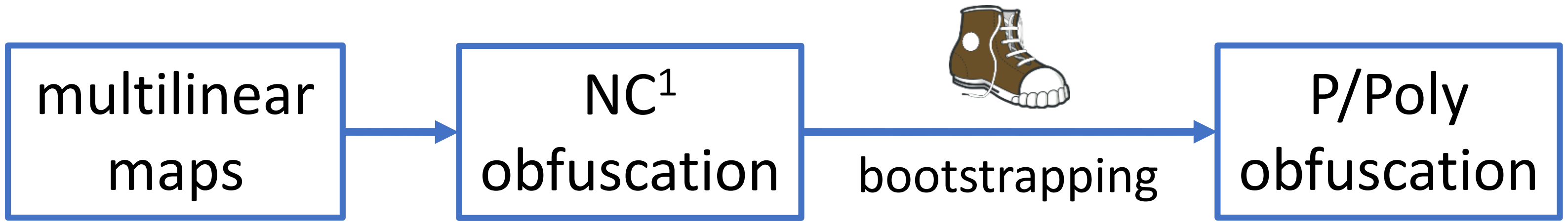
- Functionality whose (ideal) obfuscation can be used to obfuscate arbitrary circuits
- Obfuscated primitive should need to invoked once for function evaluation
- **Our solution:** obfuscate FHE decryption and SNARG verification

**Concurrently:** improve the asymptotic efficiency of SNARGs

# How (Im)Practical is Obfuscation?

Existing constructions rely on multilinear maps [BS04, GGH13, CLT13, GGH15]

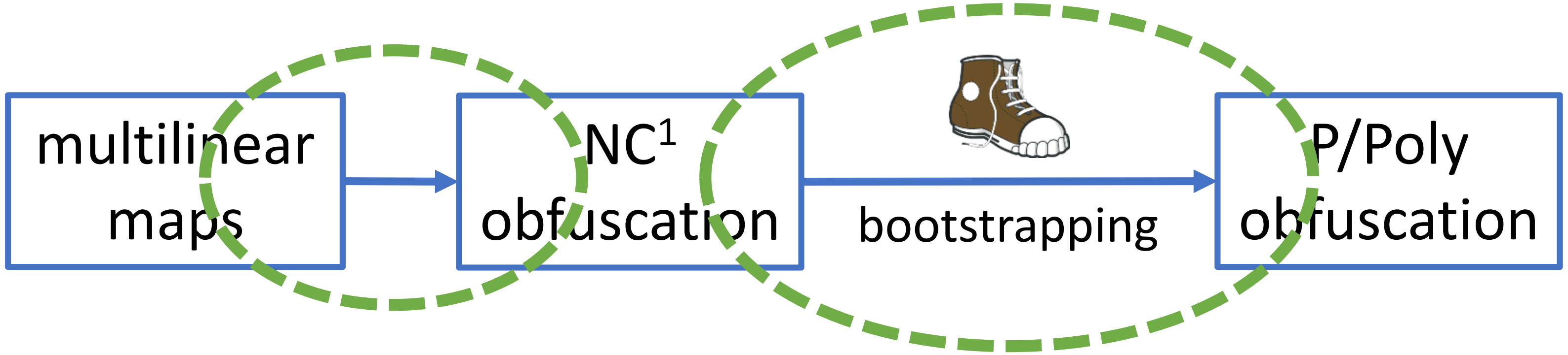
- Bootstrapping: [GGHRSW13, BR14, App14]




  - For AES, requires $\gg 2^{100}$ levels of multinearity and $\gg 2^{100}$ encodings
- Direct obfuscation of circuits: [Zim15, AB15]
  - For AES, already require $\gg 2^{100}$ levels of multilinearity
- Non-Black Box: [Lin16a, LV16, Lin16b, AS17, LT17]
  - Only requires constant-degree multilinear maps (e.g., 3-linear maps [LT17])
  - Multilinear maps are complex, so non-black box use of the multilinear maps will be difficult to implement

# How (Im)Practical is Obfuscation?

Focus of this work will be on candidates that make black-box use of multilinear map



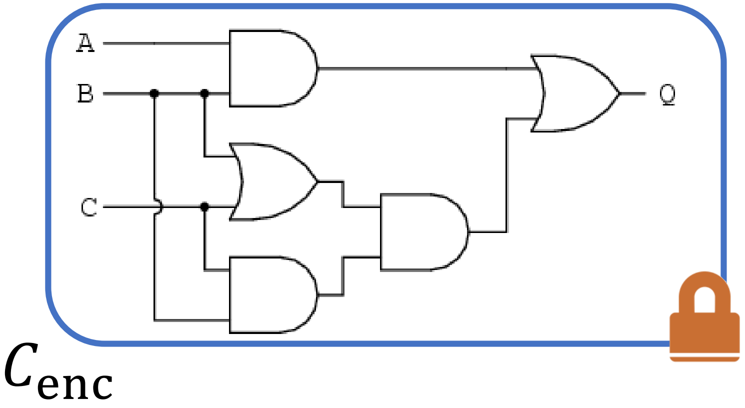

prior works have focused on improving the efficiency of obfuscation for $NC^1$ (branching programs) [AGIS14, BMSZ16]

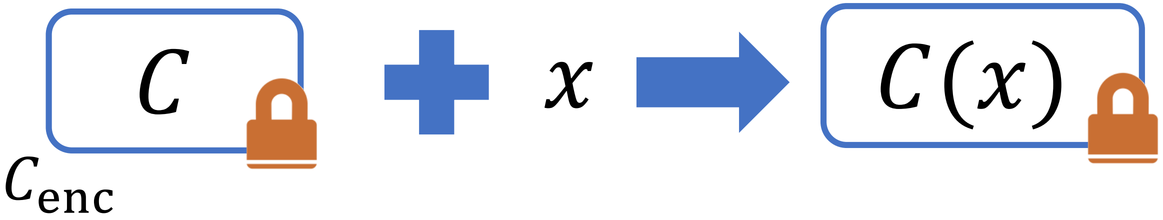
our goal: improve efficiency of **bootstrapping**

for AES, we require $\approx 4000$ levels of multilinearity (compare with $\gg 2^{100}$ from before)

# Bootstrapping Obfuscation [GGHRSW13, BR14]

To obfuscate a circuit $C \in \mathrm{P/Poly}$:




encrypt the circuit $C$ using a public key FHE scheme to obtain encrypted circuit $C_{\mathrm{enc}}$

given $C_{\mathrm{enc}}$, evaluator can homomorphically compute encryption of $C(x)$

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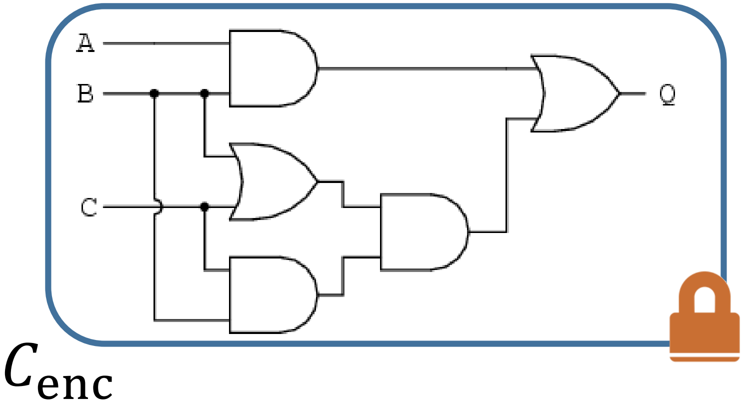
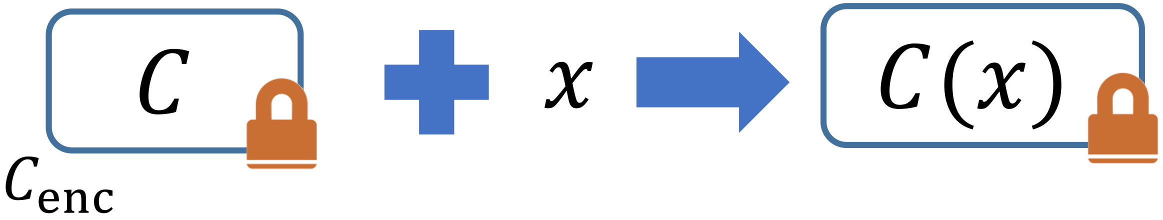



- Provide obfuscated program that decrypts the FHE ciphertext
- Should not decrypt arbitrary FHE ciphertexts, only those that correspond to honest evaluations
- Evaluator includes a proof that evaluation done correctly

# Bootstrapping Obfuscation [GGHRSW13, BR14]

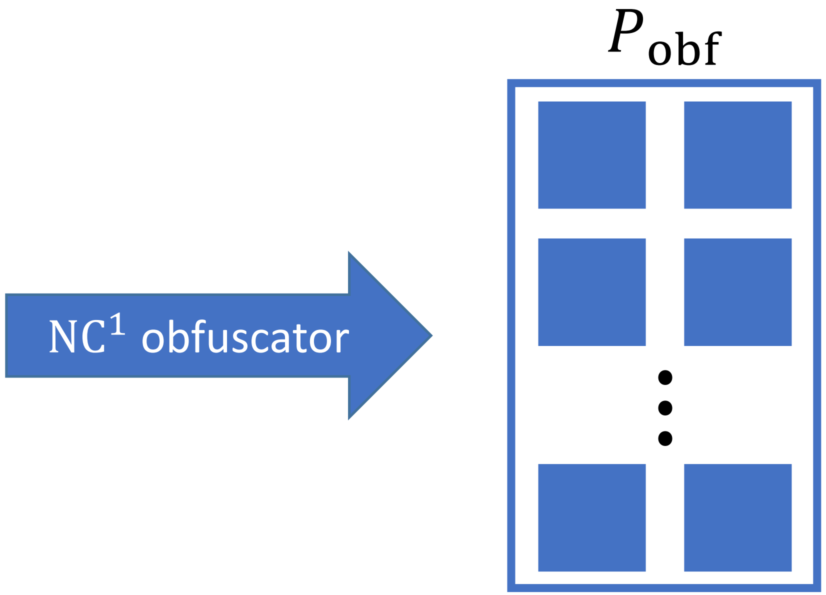
FHE secret key

CRS for proof system

constants: $\mathrm{sk}$ and $\sigma$

on input $(x, \mathrm{ct}, \pi)$:

1. Verify the proof $\pi$ that $\mathrm{ct}$ corresponds to an evaluation of $C_{\mathrm{enc}}$ on $x$
2. If valid, output $\mathrm{FHE.Decrypt}(\mathrm{sk}, \mathrm{ct})$ and 0 otherwise

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For VBB obfuscation, can use a succinct argument (SNARG)

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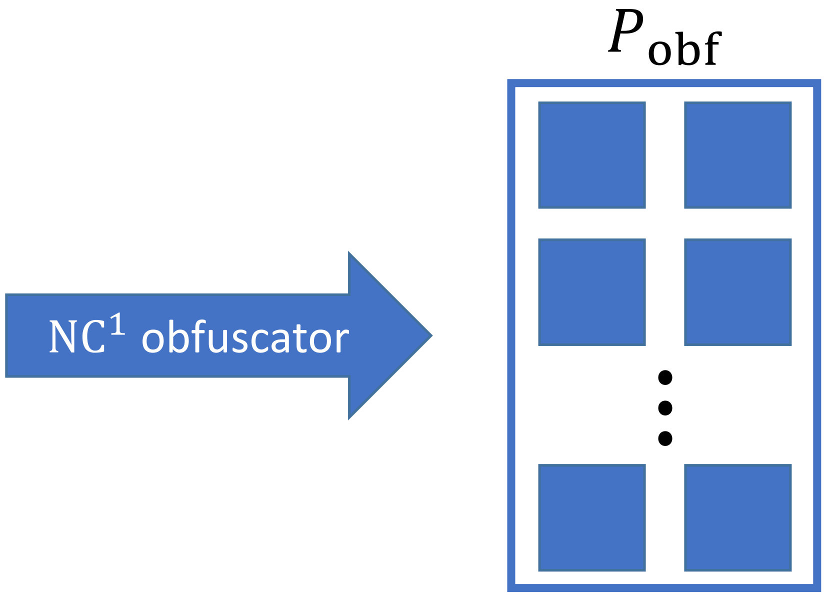


- Obfuscated program does two things: proof verification and FHE decryption
- $\mathrm{NC}^1$ obfuscator works on *branching programs*, so need primitives with short branching programs (e.g., computing an inner products over a small field)

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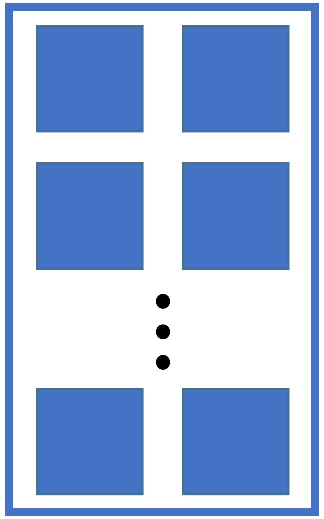
$\mathrm{NC}^1$ obfuscator

$P_{\mathrm{obf}}$



Require primitives that minimize branching-program complexity

- Obfuscated program doe … and FHE decryption
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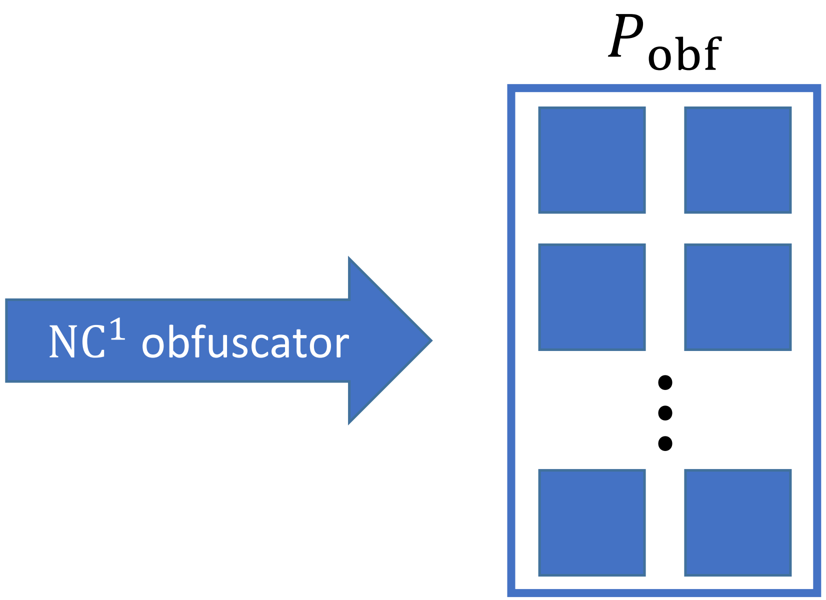


- Obfuscated program does two things: proof verification and FHE decryption
- $\mathrm{NC}^1$ obfuscator works on *branching programs*, so need primitives with short branching programs (e.g., computing an inner products over a small field)
- FHE decryption is (rounded) inner product [BV11, BGV12, Bra12, GSW13, AP14, DM15, …], so just need a SNARG with simple verification

# Branching-Program-Friendly SNARGs

Goal: construct a succinct non-interactive argument (SNARG)
that can be verified by a <u>short</u> branching program

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Succinct non-interactive arguments (SNARG) for NP relation [GW11]

- $\mathrm{Setup}(1^{\lambda}) \rightarrow (\sigma, \tau)$: outputs common reference string $\sigma$ and verification state $\tau$
- $\mathrm{Prove}(\sigma, x, w) \rightarrow \pi$: on input a statement $x$ *a*nd witness $w$, outputs a proof $\pi$
- $\mathrm{Verify}(\tau, x, \pi) \rightarrow 0/1$: on input the verification state $\tau$, the statement $x$, decides if proof $\pi$ is valid

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Succinct non-interactive arguments (SNARG) for $\mathsf{NP}$ relation [GW11]

- Must satisfy usual notions of completeness and computational soundness
- Succinctness: proof size and verifier run-time should be polylogarithmic in the circuit size (for circuit satisfiability)
  - Verifier run-time: $\mathrm{poly}(\lambda + |x| + \log\,|C|)$
  - Proof size: $\mathrm{poly}(\lambda + \log\,|C|)$

# Branching-Program-Friendly SNARGs

Goal: construct a succinct non-interactive argument (SNARG)
that can be ... hort branc...

Verification state $\tau$ must be secret

Allow Setup algorithm to run in time $\mathrm{poly}(\lambda + |C|)$

Main result: new designated-verifier SNARGs in the preprocessing model with the following properties:

# Branching-Program-Friendly SNARGs

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proofs have size $\tilde{O}(\lambda)$

prover complexity is $\tilde{O}(|C|)$

…lt: new designate… the preprocessing model with the
…properties:

- Quasi-optimal succinctness
- Quasi-optimal prover complexity

} <u>first</u> SNARG that is "quasi-optimal"

Asymptotics based on achieving $\mathrm{negl}(\lambda)$ soundness
error against provers of size $2^{\lambda}$

# Branching-Program-Friendly SNARGs

Goal: construct a succinct non-interactive argument (SNARG) that can be verified by a short branching program

Main result: new designated-verifier SNARGs in the preprocessing model with the following properties:

- Quasi-optimal succinctness
- Quasi-optimal prover complexity
- Post-quantum security
- Works over polynomial-size fields

first SNARG that is "quasi-optimal"

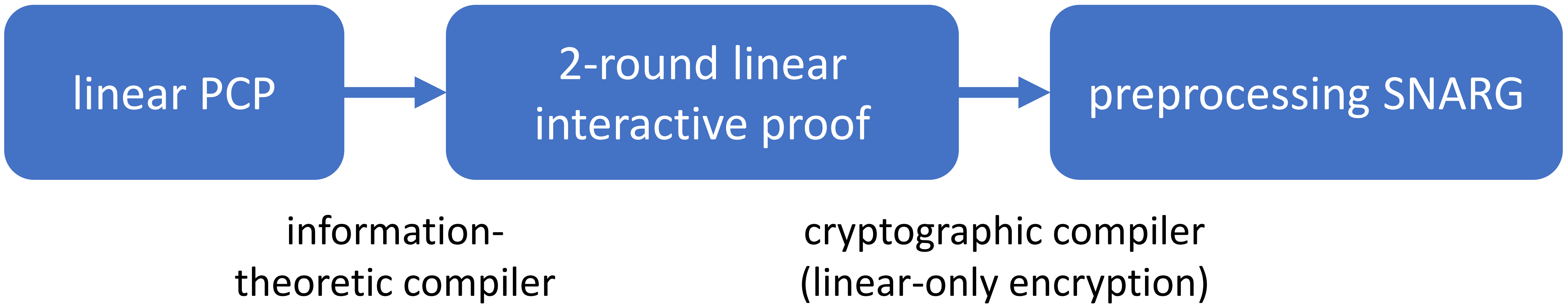
New SNARG candidates are lattice-based

- Over integer lattices, verification is branching-program friendly
- Over ideal lattices, SNARGs are quasi-optimal

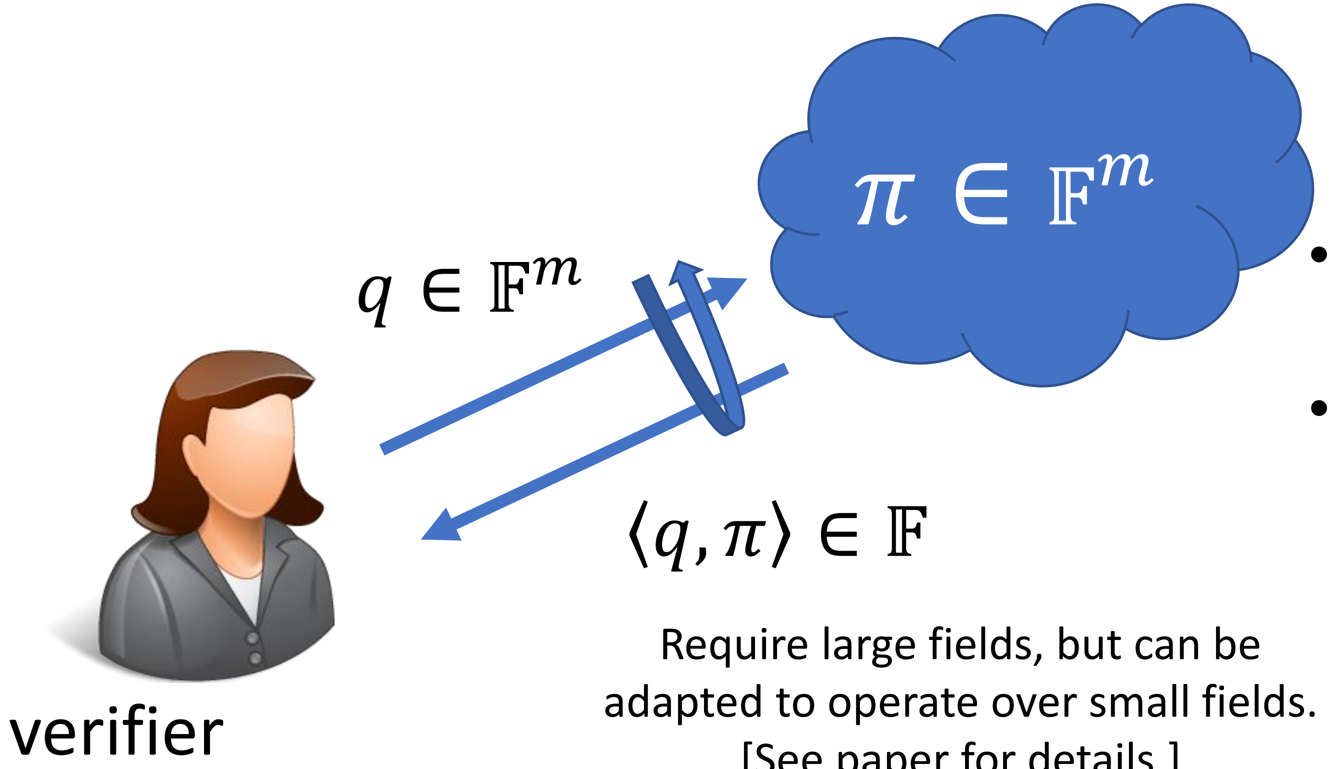
# Branching-Program-Friendly SNARGs

Goal: construct a succinct non-interactive argument (SNARG)
that can be verified by a short branching program

Starting point: preprocessing SNARGs from [BCIOP13]



# Linear PCPs (LPCPs) [IKO07]


Require large fields, but can be adapted to operate over small fields. [See paper for details.]

- Verifier given oracle access to a *linear* function $\pi \in \mathbb{F}^m$
- Several instantiations:
  - 3-query LPCP based on the Walsh-Hadamard code: $m = O(|C|^2)$ [ALMSS92]
  - 3-query LPCP based on quadratic span programs: $m = O(|C|)$ [GGPR13]

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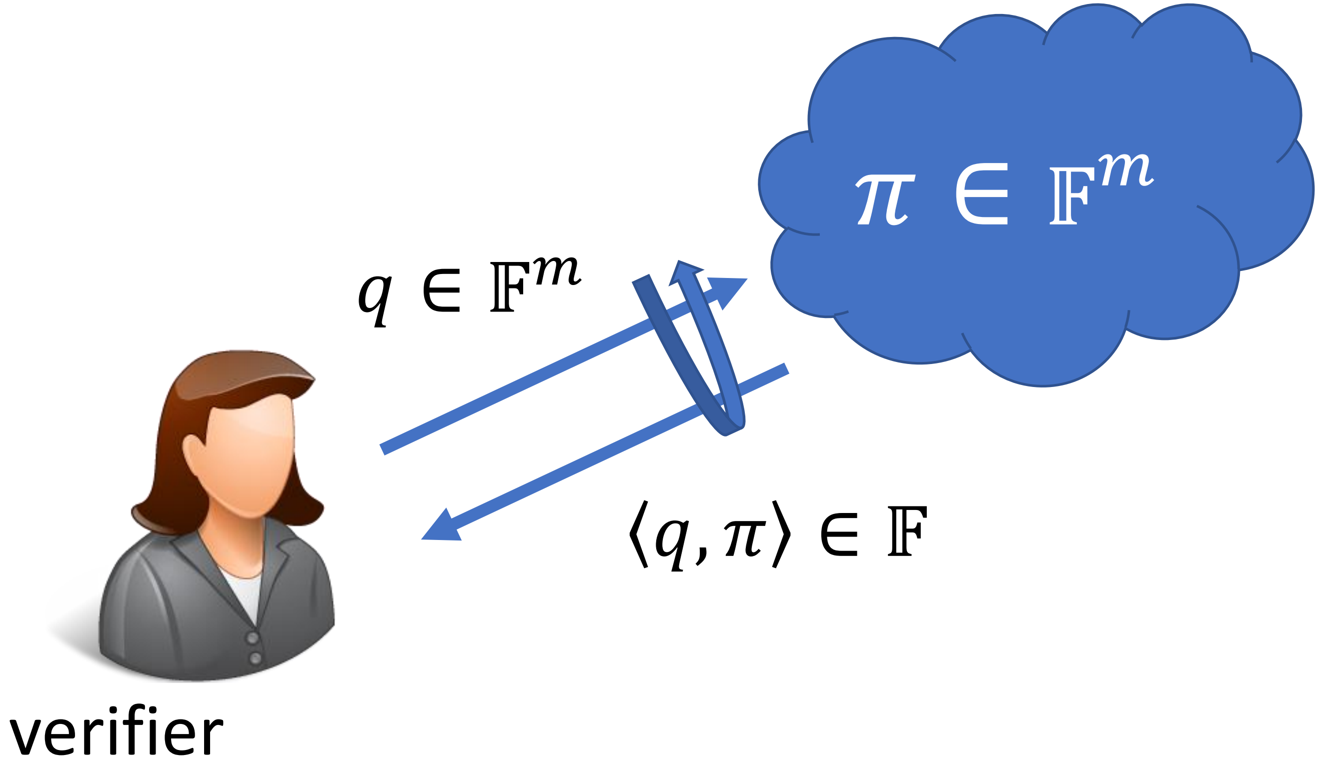
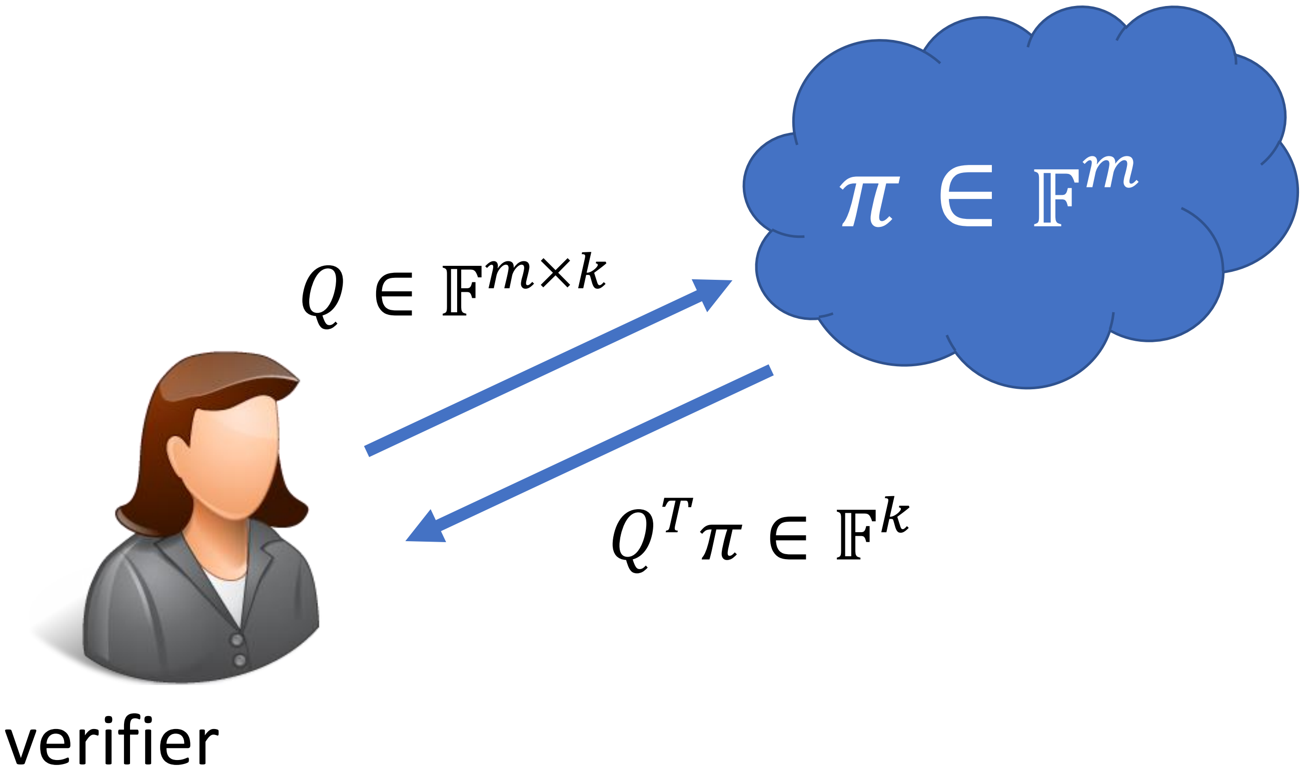



Oftentimes, verifier is *oblivious*: the queries $q$ do not depend on the statement $x$

# Linear PCPs (LPCPs) [IKO07]

Equivalent view (if verifier is oblivious):




$$Q = \begin{bmatrix} q_1 & q_2 & q_3 & \cdots & q_k \end{bmatrix} \in \mathbb{F}^{m \times k}$$

pack all queries into single matrix

# From Linear PCPs to Preprocessing SNARGs [BCIOP13]

*Oblivious* verifier can “commit” to its queries ahead of time

$$Q = \underbrace{\begin{bmatrix} q_1 & q_2 & q_3 & \cdots & q_k \end{bmatrix}}_{\text{part of the CRS}}$$

Honest prover takes $(x, w)$ and constructs linear PCP $\pi \in \mathbb{F}^m$ and computes $Q^{\mathrm{T}}\pi$

Two problems:

- Malicious prover can choose $\pi$ based on queries
- Malicious prover can apply different $\pi$ to the different columns of $Q$

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Step 1: Encrypt elements of $Q$ using additively homomorphic encryption scheme

- Prover homomorphically computes $Q^T\pi$
- Verifier decrypts encrypted response vector and performs LPCP verification

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Honest prover takes $(x, w)$ and constructs linear PCP $\pi \in \mathbb{F}^m$ and computes $Q^{\mathrm{T}}\pi$

Step 2: Conjecture that the encryption scheme only supports a limited subset of homomorphic operations (linear-only vector encryption)

# Linear-Only Vector Encryption

$v_1 \in \mathbb{F}^k$

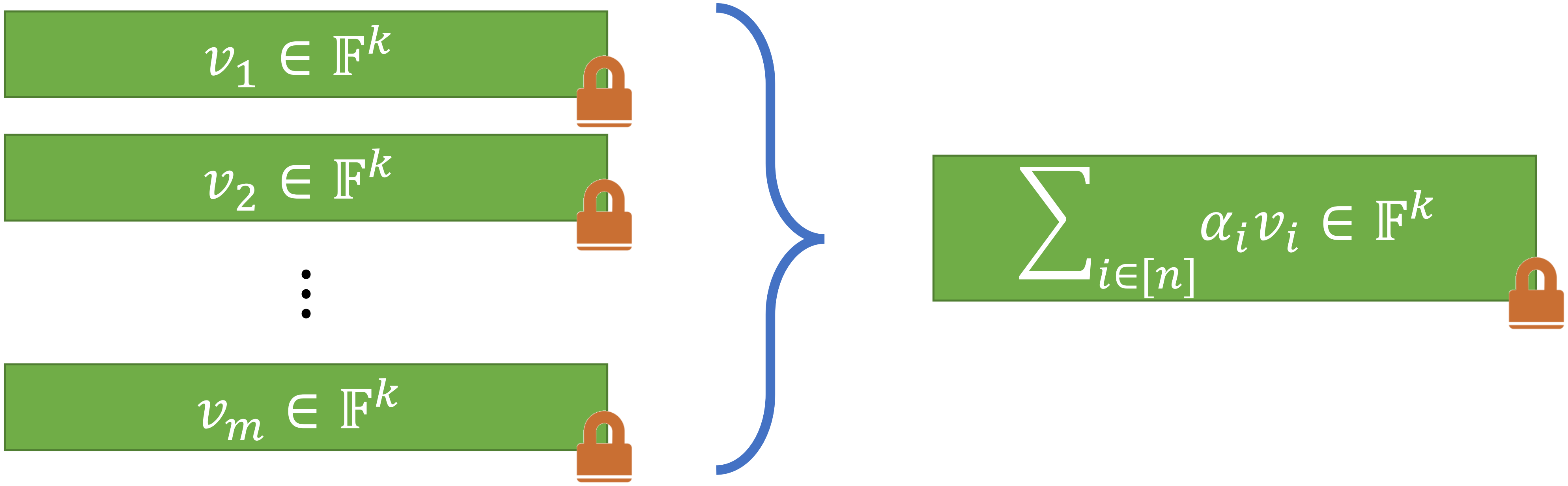
$v_2 \in \mathbb{F}^k$

$\vdots$

$v_m \in \mathbb{F}^k$

plaintext space is a *vector* space

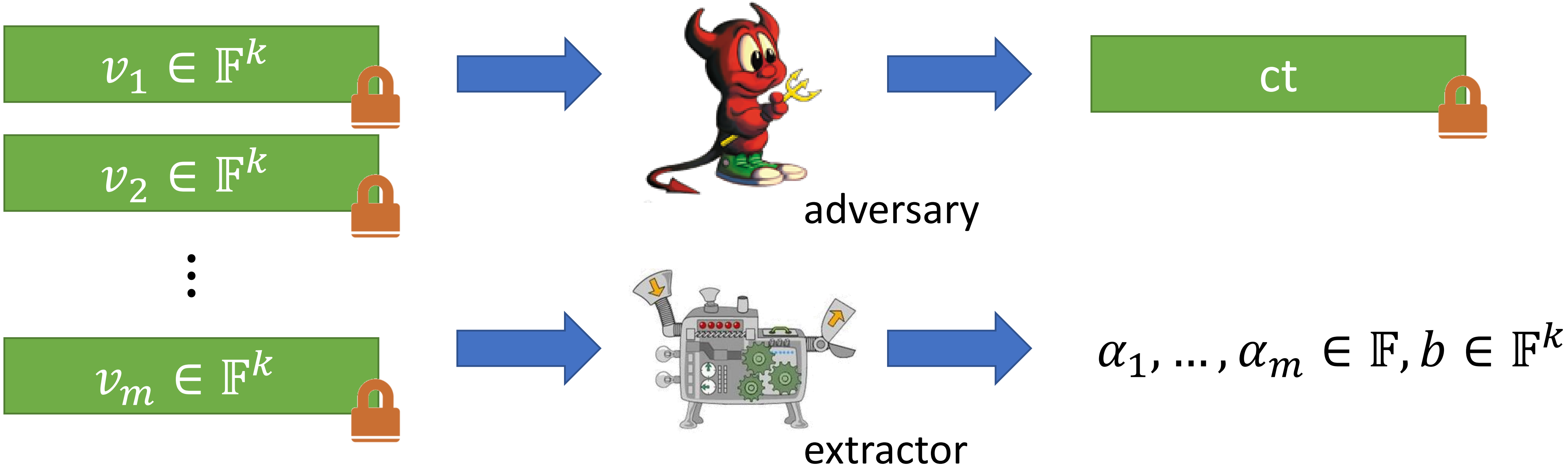
# Linear-Only Vector Encryption




plaintext space is a *vector* space

encryption scheme is semantically-secure and additively homomorphic

# Linear-Only Vector Encryption




For all adversaries, there is an efficient extractor such that if ct is valid, then the extractor is able to produce a vector of coefficients $(\alpha_1, \ldots, \alpha_m) \in \mathbb{F}^m$ and $b \in \mathbb{F}^k$ such that $\text{Decrypt}(\text{sk}, \text{ct}) = \sum_{i \in [n]} \alpha_i v_i + b$

Weaker property also suffices. [See paper for details.]

# Linear-Only Vector Encryption

$v_1 \in \mathbb{F}^k$

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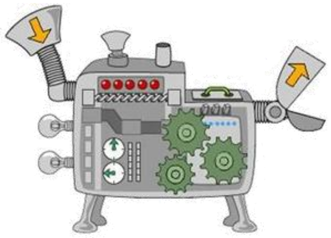
$v_m \in \mathbb{F}^k$

adversary



extr

ct

$\in \mathbb{F}^k$

extractor can "explain" the ciphertexts as an affine function of its inputs

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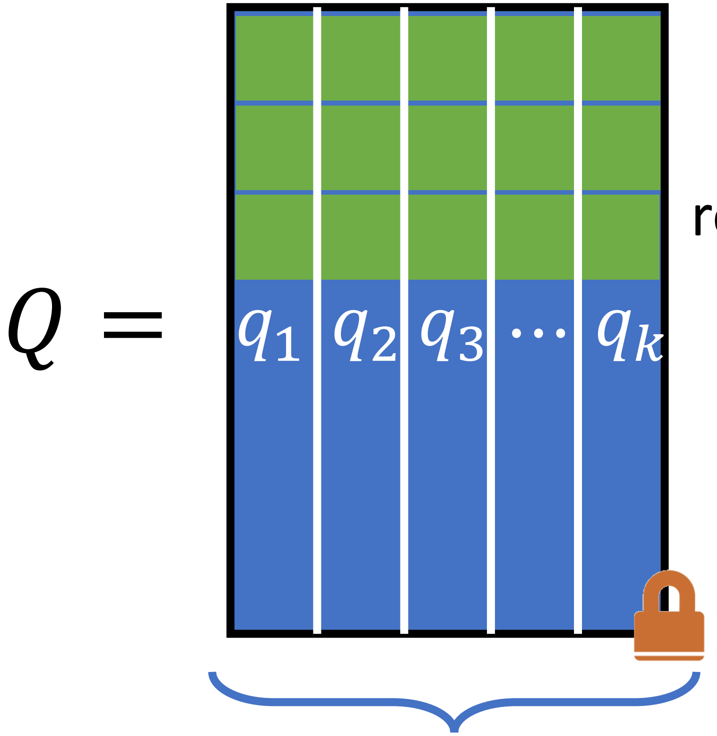



encrypt row by row

part of the CRS

Honest prover takes $(x, w)$ and constructs linear PCP $\pi \in \mathbb{F}^m$ and computes $Q^{\mathrm{T}}\pi$

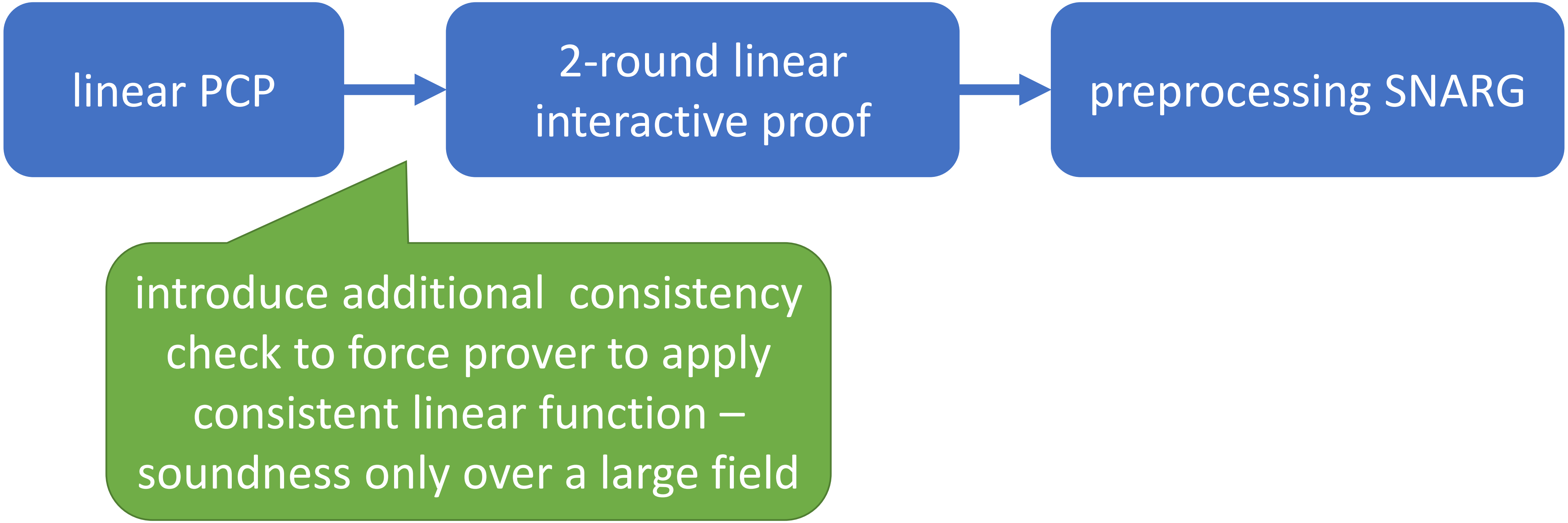
Step 2: Conjecture that the encryption scheme only supports a limited subset of homomorphic operations (linear-only vector encryption)

Linear-only vector encryption $\Rightarrow$ all prover strategies can be explained by $(\pi, b)$ as $Q^T\pi + b$

[See paper for full details.]

# Comparison with [BCIOP13]

Preprocessing SNARGs from [BCIOP13]:



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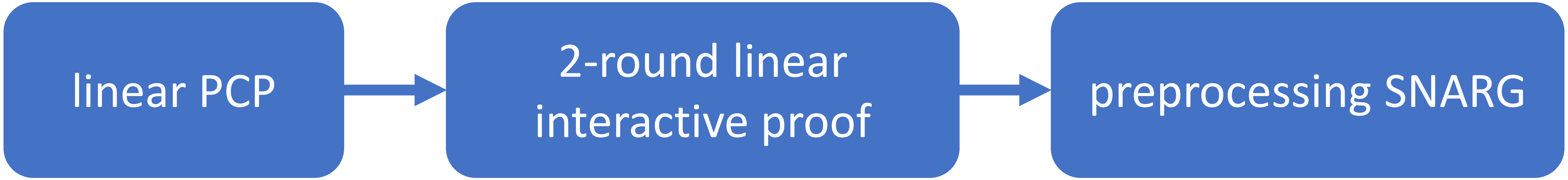
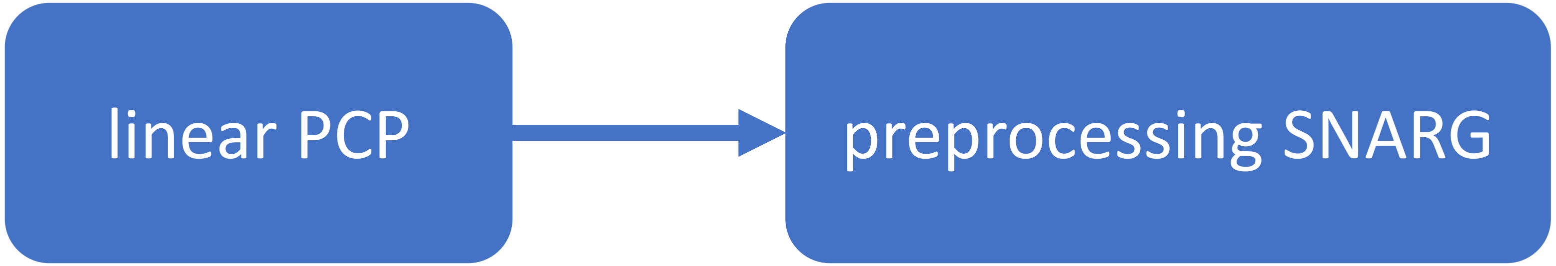



Our construction



# Comparison with [BCIOP13]

Preprocessing SNARGs from [BCIOP13]:

linear PCP → 2-round linear → preprocessing SNARG

stronger cryptographic assumption, but enables new constructions with better efficiency

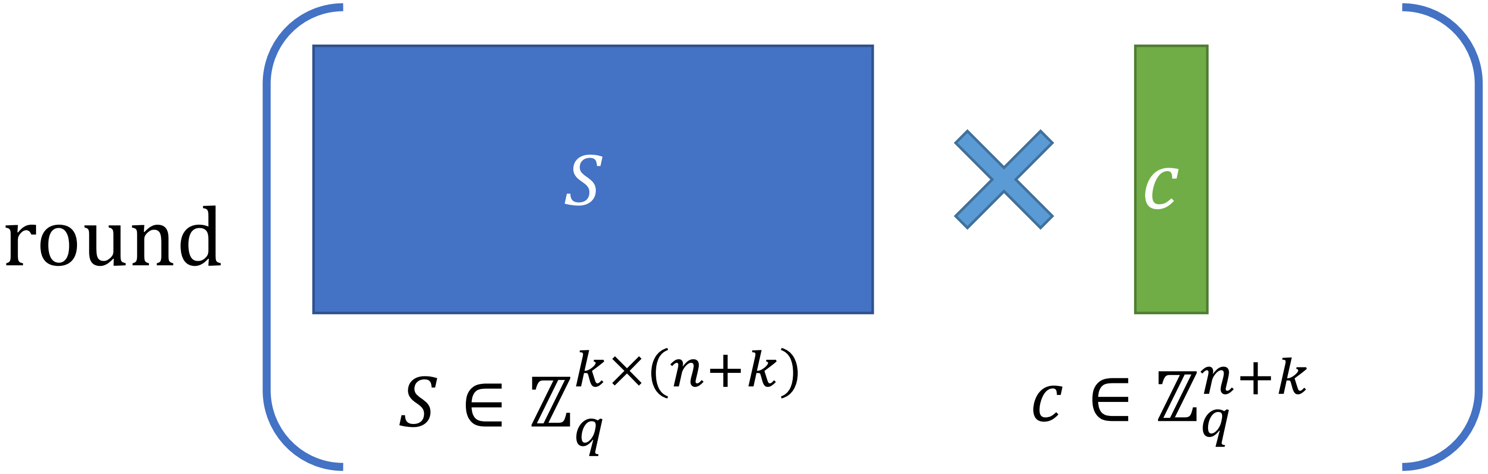
Our construction

linear PCP → preprocessing SNARG

# Instantiating Linear-Only Vector Encryption

<u>Conjecture</u>: Regev-based encryption (specifically, the [PVW08] variant) is a linear-only vector encryption scheme.

PVW decryption (for plaintexts with dimension $k$):




Each row of $S$ can be is an independent Regev secret key

# Concrete Comparisons

| Construction | Public vs. Designated | Prover Complexity | Proof Size | Assumption |
|---|---|---|---|---|
| CS Proofs [Mic00] | Public | $\tilde{O}(\|C\| + \lambda^2)$ | $\tilde{O}(\lambda^2)$ | Random Oracle |
| Groth [Gro10] | Public | $\tilde{O}(\|C\|^2\lambda + \|C\|\lambda^2)$ | $\tilde{O}(\lambda)$ | Knowledge of Exponent |
| GGPR [GGPR12] | Public | $\tilde{O}(\|C\|\lambda)$ | $\tilde{O}(\lambda)$ | |
| BCIOP (Pairing) [BCIOP13] | Public | $\tilde{O}(\|C\|\lambda)$ | $\tilde{O}(\lambda)$ | Linear-Only Encryption |
| BCIOP (LWE) [BCIOP13] | Designated | $\tilde{O}(\|C\|\lambda)$ | $\tilde{O}(\lambda)$ | |
| Our Construction (LWE) | Designated | $\tilde{O}(\|C\|\lambda)$ | $\tilde{O}(\lambda)$ | Linear-Only Vector Encryption [See paper.] |
| Our Construction (RLWE) | Designated | $\tilde{O}(\|C\|)$ | $\tilde{O}(\lambda)$ | |

Only $\mathrm{negl}(\lambda)$-soundness (instead of $2^{-\lambda}$-soundness) against $2^{\lambda}$-bounded provers

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| Our Construction (RLWE) | Designated | $\tilde{O}(\lvert C\rvert)$ | $\tilde{O}(\lambda)$ | |

Post-quantum resistant!

# Back to Obfuscation…

For bootstrapping obfuscation…

- Obfuscate FHE decryption and SNARG verification
- Degree of multilinearity: $\approx 2^{12}$
- Number of encodings: $\approx 2^{44}$

Many optimizations. [See paper for details.]

Still infeasible, but much, much better than $2^{100}$ for previous black-box constructions!

Looking into obfuscation gave us new insights into constructing better SNARGs:

- More direct framework of building SNARGs from linear PCPs
- First quasi-succinct construction from standard lattices
- First quasi-optimal construction from ideal lattices [See paper.]

# Open Problems

Publicly-verifiable SNARGs from lattice-based assumptions?

Stronger notion of quasi-optimality (achieve $2^{-\lambda}$ soundness rather than $\mathrm{negl}(\lambda)$ soundness)?

Concrete efficiency of new lattice-based SNARGs?

**Thank you!**

`http://eprint.iacr.org/2017/240`