Problem Set 9

CS 331

Due Tuesday, December 5

1. Consider an interval scheduling problem where we have multiple machines, and each interval can specify which machines it can run on. That is, you have n jobs, and each job j is described by (s_j, f_j, M_j) where $M_j \subseteq [m]$ is a subset of machines. Job j must be scheduled on a single machine in M_j, will occupy that machine for the full interval (s_j, f_j), and no other job can use that machine during that time.

   The goal of the optimization version of multiple-machine interval scheduling (MMIS) is to maximize the number of jobs that can be scheduled. The goal of the decision version of MMIS is to determine whether at least k jobs can be scheduled, for a given parameter k.

   (a) Show that the decision version of MMIS is NP-hard via a reduction from maximum independent set. Hint: Given the graph as input to maximum independent set, associate each vertex with a machine, and each edge $e_i = (u_i, v_i) \in E$ with the job $(i, i+1, \{u_i, v_i\})$, and additionally for each vertex u create the job $(1, |E|+1, \{u\})$.

   (b) Show that the multiple-machine interval scheduling problem is NP-complete.