Problem Set 4

CS 331

Due Wednesday, February 16

1. You are planning to buy a set of fancy lenses for your DSLR camera. The camera store has n lenses, where lens i has a cost c_i and works over a range of focal lengths $[s_i, f_i)$. (If you're unfamiliar with cameras, you can think of "focal length" as "zoom level".) You would like to buy a set of lenses that covers a wide range $[L, H)$ of focal lengths, so every length in this range is supported by at least one of your lenses.

   You may assume all the parameters are integers.

   (a) Give an $O(n(H - L))$ time dynamic programming algorithm to find the cheapest set of lenses that covers the entire range $[L, H)$.

   Recall that you should:

   - Define a subproblem, which you describe in English.
   - Give a recurrence to solve the subproblem.
   - Show that the recurrence is correct.
   - Describe how to solve the problem quickly, and analyze the resulting running time.

   (b) The camera store is now having an amazing sale, where every lens has the same cost $c_i = \$100$! Give a simple *greedy* algorithm that finds the answer in $O(n \log n)$ time.

   (c) (Optional): Show how to solve part (a) in $O(n \log n)$ time.

2. There's a Jupyter Notebook linked from the class webpage.