Problem Set 4

Randomized Algorithms

Due Friday, September 27

1. In class we proved that the two-choices approach improves the maximum load to $O(\log \log n)$. A generalization is that choosing the least loaded of d choices reduces the maximum load to $O(\log_d \log n)$. Explain what changes to the proof are needed to derive this result. Give only the diffs; do not bother writing a complete proof.

2. Probability practice.

   (a) You flip an unbiased coin until you get two heads in a row. What is the expected number of coin flips you make?

   (b) You flip an unbiased coin m times. During this process, we say that the state is "balanced" if you have observed equally many heads and tails. Up to constant factors, how many times in expectation will the state be balanced?

3. Consider the following iterative balls-in-bins process. We have n bins. At the start of the first iteration, we have $n/2$ balls. At a general iteration, we throw each of the remaining balls independently and uniformly at random into the bins. We then remove every ball that is alone in its bin. The remaining balls survive to the next iteration. The process stops when there are no balls remaining. How many rounds does this take in expectation, up to constant factors?