Problem Set 7

Randomized Algorithms

Due Friday, October 28

1. Let S be an unknown set of n items (with n known). Suppose that you receive a sample T of k items chosen from S uniformly at random *without replacement*. Show how to construct a sample T' of k items from S, whose distribution is identical to a uniform sample of k items from S drawn *with replacement*.

2. Consider the example given in class for how online bipartite matching using random edges achieves a competitive ratio of $R = 1/2$: each arriving vertex x_i has an edge to y_i as well as all of $y_{n/2}, \ldots, y_n$. Show that the algorithm that the algorithm given in class, which randomly ranks the right vertices y_i, has $R \leq 3/4 + o(1)$ on this example.

3. [Moshkovitz] Suppose that you have a giant (i.e., infinite) bag of coins. You know that 90% of the coins are highly biased, and come up heads 90% of the time. The other 10% of coins are unbiased, and come up heads 50% of the time. You do not know which coins are which, and you would like to find *one* of the biased coins.

   You are allowed to flip coins n times – each coin you flip can be either a fresh random coin from the bag, or a coin that you have flipped before. At the end of n coin flips, you must output a coin. You succeed if the coin is biased, and fail if the coin is unbiased. What is the minimum probability of failure, and how can you achieve this?

   (a) Show that the failure probability must be at least $\exp(-O(n))$.

   (b) Suppose that the biased coins were actually 100% biased. Show how to achieve $\exp(-\Omega(n))$ failure probability.

   (c) Show how to achieve $\exp(-\Omega(n))$ failure probability in the setting described, where the biased coins are 90% biased.

   Hint (rot13): Gur nytbevguz vf fvzvyne gb gur bar sbe ebohfg ovanel frnepu ba ceboyrz frg 2. Lbh fubhyq pbafgehpg n enaqbz jnyx ba fbzr tencu fhpu gung obgu ovnfrq naq haovnfrq pbvaf zbir lbh va gur pbeerpg qverpgvba zber bsgra guna abg.