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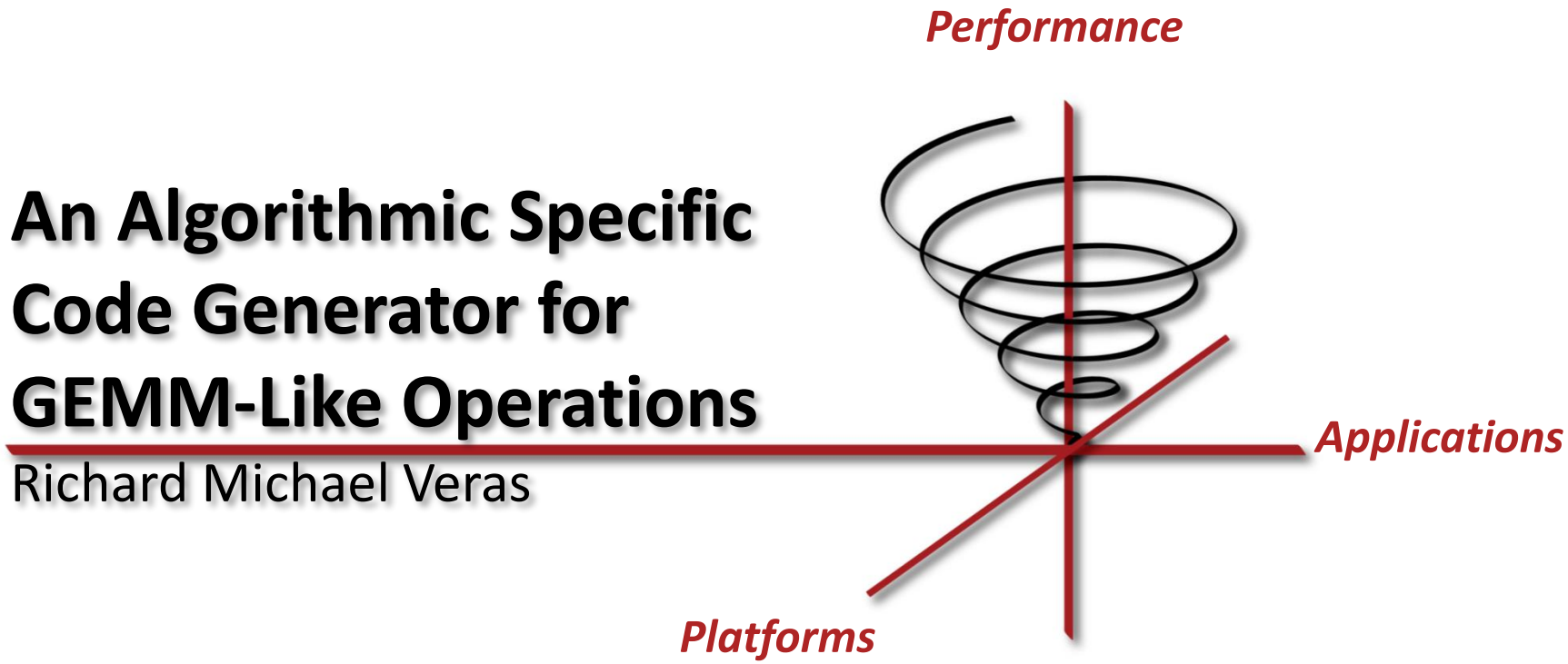
An Algorithmic Specific Code Generator for GEMM-Like Operations

Richard Michael Veras



Want Automatic High Performance

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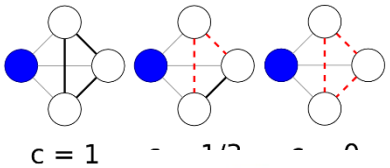
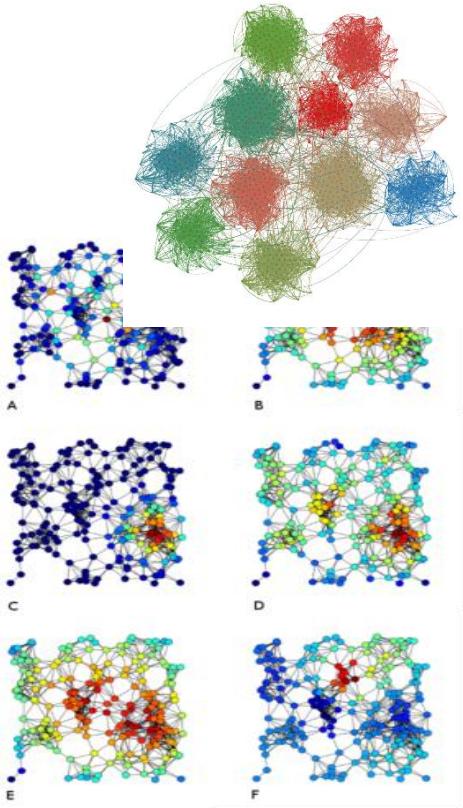
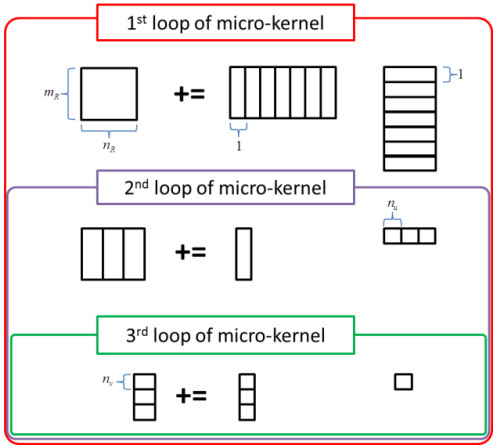
GEMM-Like Operations

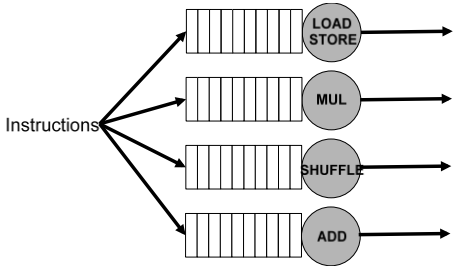




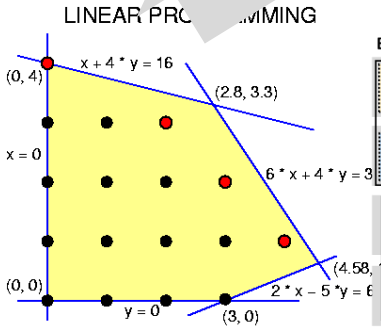
Model Driven approach for generating DGEMM:

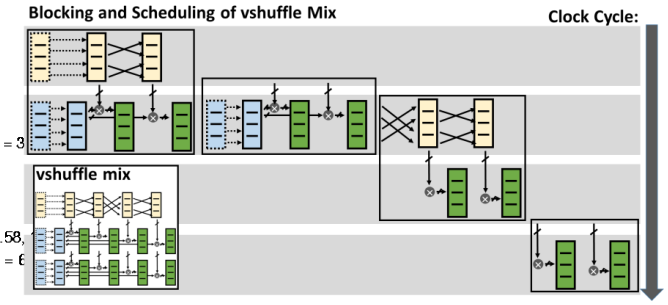


High Perf.



Compiler Techniques:





Function to maximize: f(x, y) = 6 * x + 5 * y
Optimum LP solution (x, y) = (2.4, 3.4)
Pareto optima: (0, 4), (2, 3), (3, 2), (4, 1)
Optimum ILP solution (x, y) = (4, 1)

GEMM Like Operations

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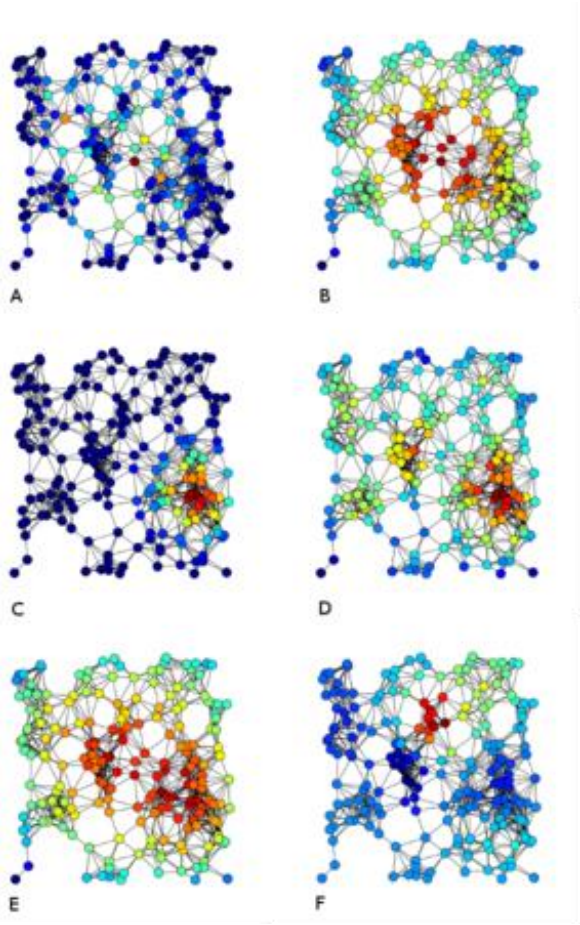
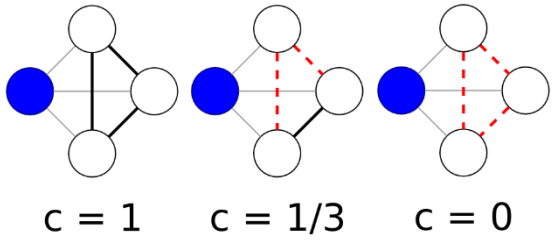
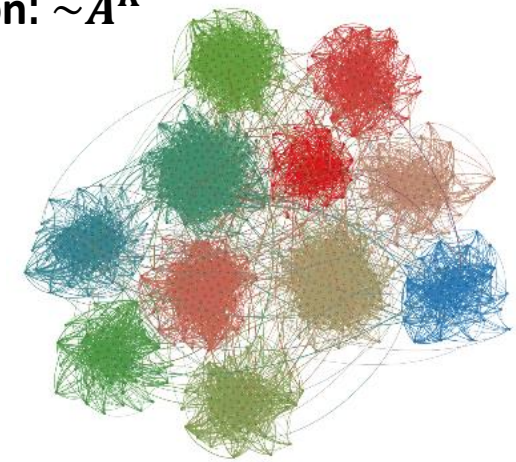
Centrality: $\sim A^T A$
[betweeness]
$(Z, +, MIN, 0, 1)$



Clustering: $\sim AA^T A$
(triangle)
$(Z, \wedge, +, 1, 0)$



Community Detection: $\sim A^K$



Check out GraphBLAS

High Performance Micro-Kernels

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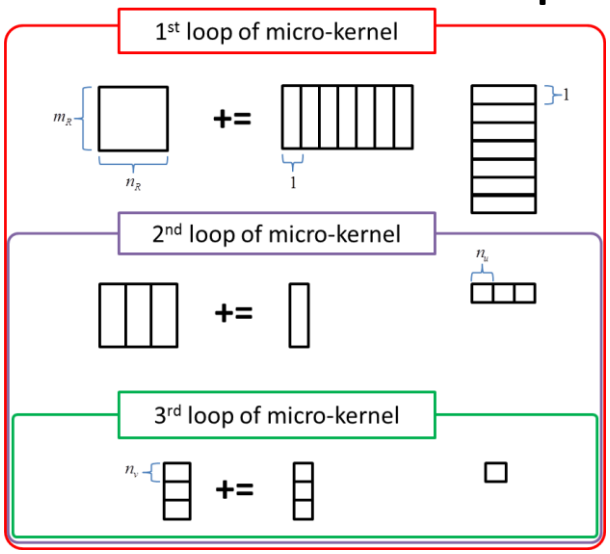
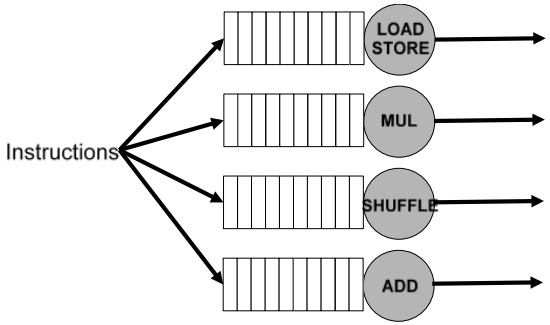
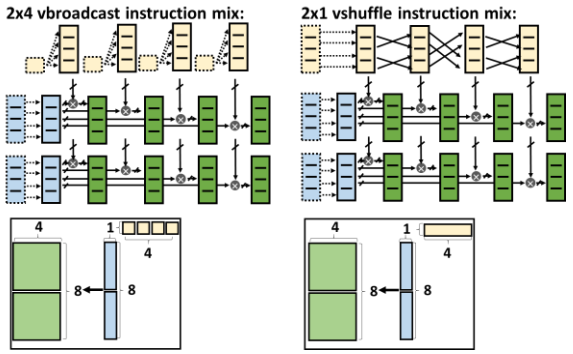
Cast micro kernel as outer product:



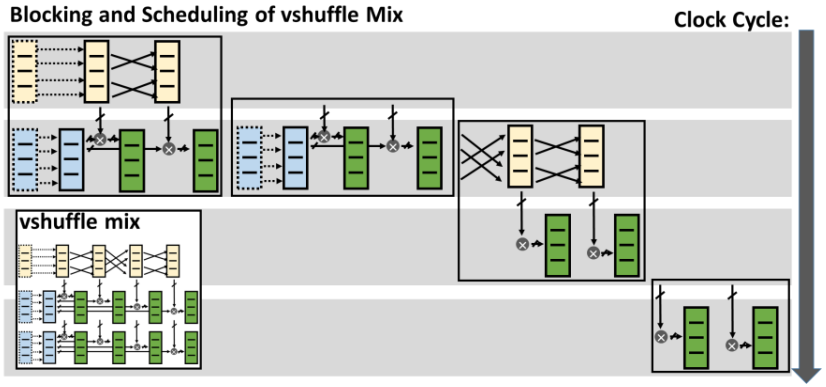
Use Models to Select from Design Space:



Enumerate all possible tilings given ISA



Aggressively Schedule and Optimize:



Veras, R. , Smith T., Low T.M., Franchetti, F. van de Geijn, R. [CGO 2017 Submitted]

High Performance Micro-Kernels

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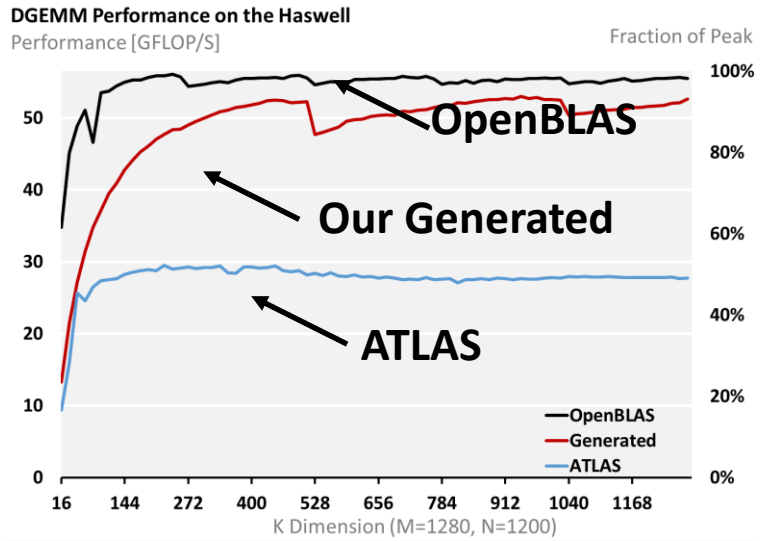

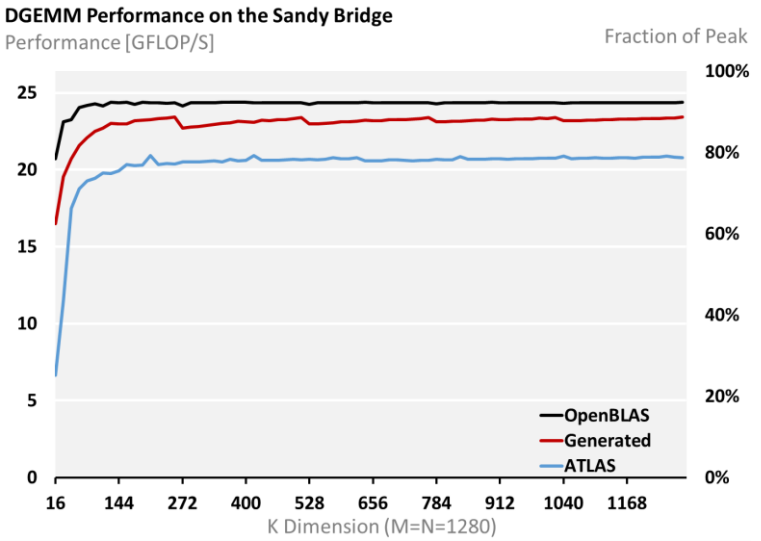

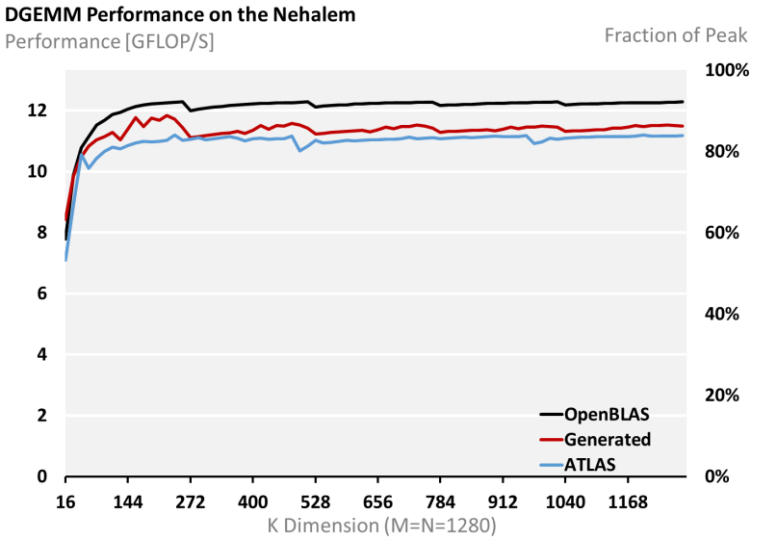

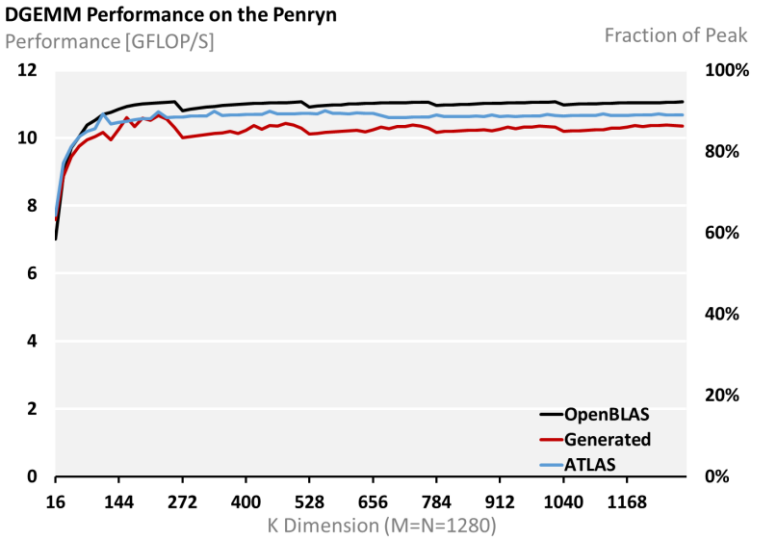

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Automating with Compiler Techniques

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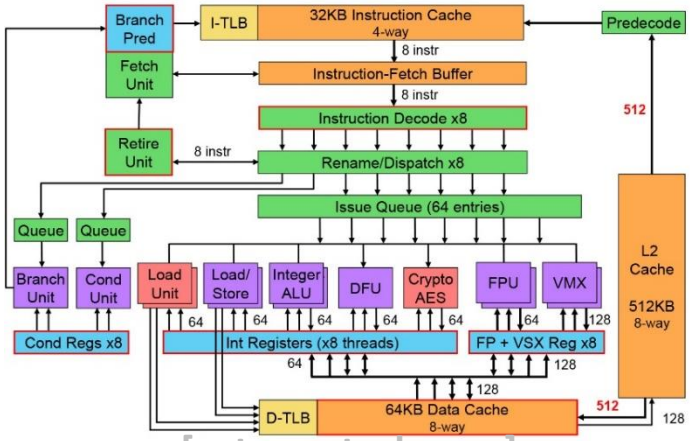
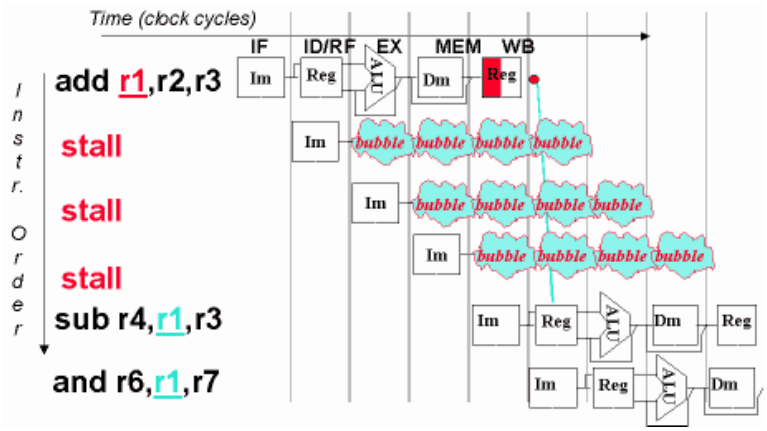
Subtle Semiring Changes Impact Performance:



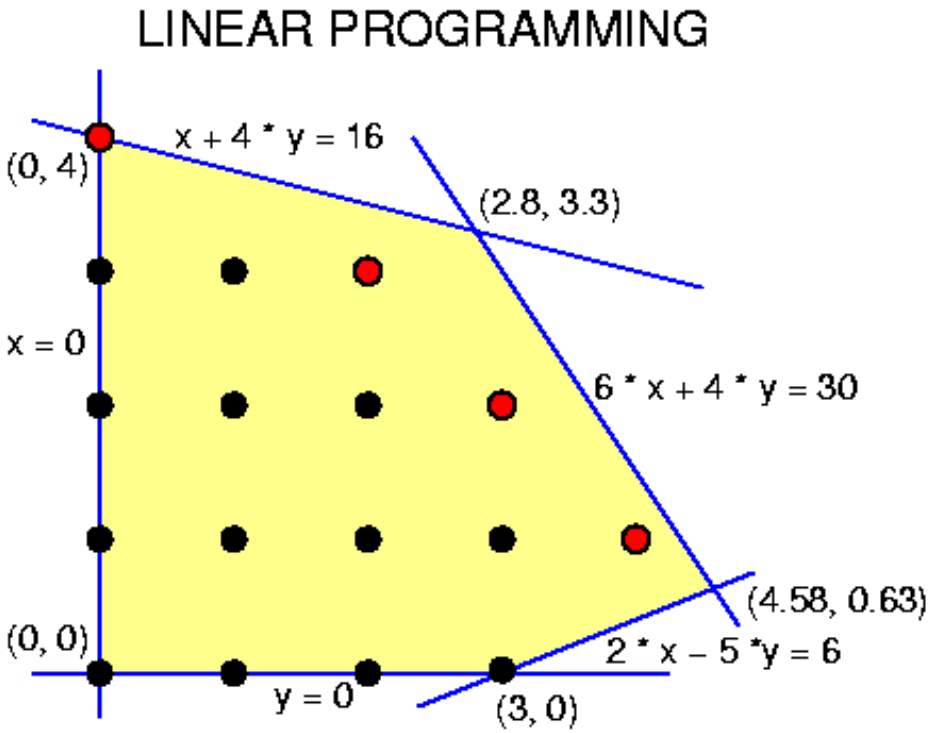
[extremetech.com]

Minimizing Stalls



[massey.ac.nz]

Cast as ILP Problem



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Optimum ILP solution (x, y) = (4, 1)

[cs.duke.edu]

A Generator for GEMM-Like Kernels

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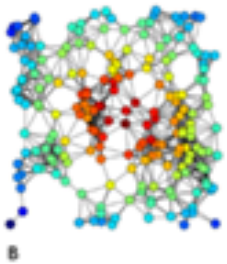
Input:

Betweeness Centrality (Floyd-Warshall)



GEMM Like:

$C \leftarrow AB + C$

$C_{acc} \leftarrow 0$ Initialize

$C_{acc} \leftarrow \sum_p ab^T$ Compute

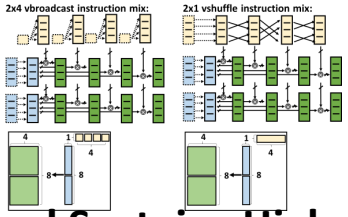
$C \leftarrow f(C_{acc})$ Accumulate

Semiring

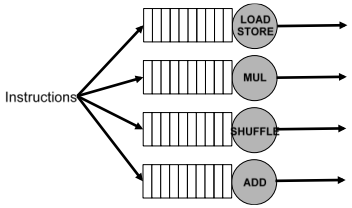
$(Z, +, MIN, 0, 1)$

Ouput:

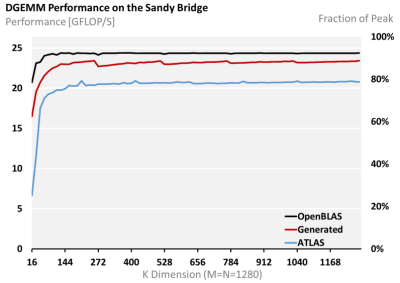
Kernel Algorithm from our design space:



Kernel Sustains High Throughput:



Tuned to the Target Architecture



Our GEMM Generator Pipeline

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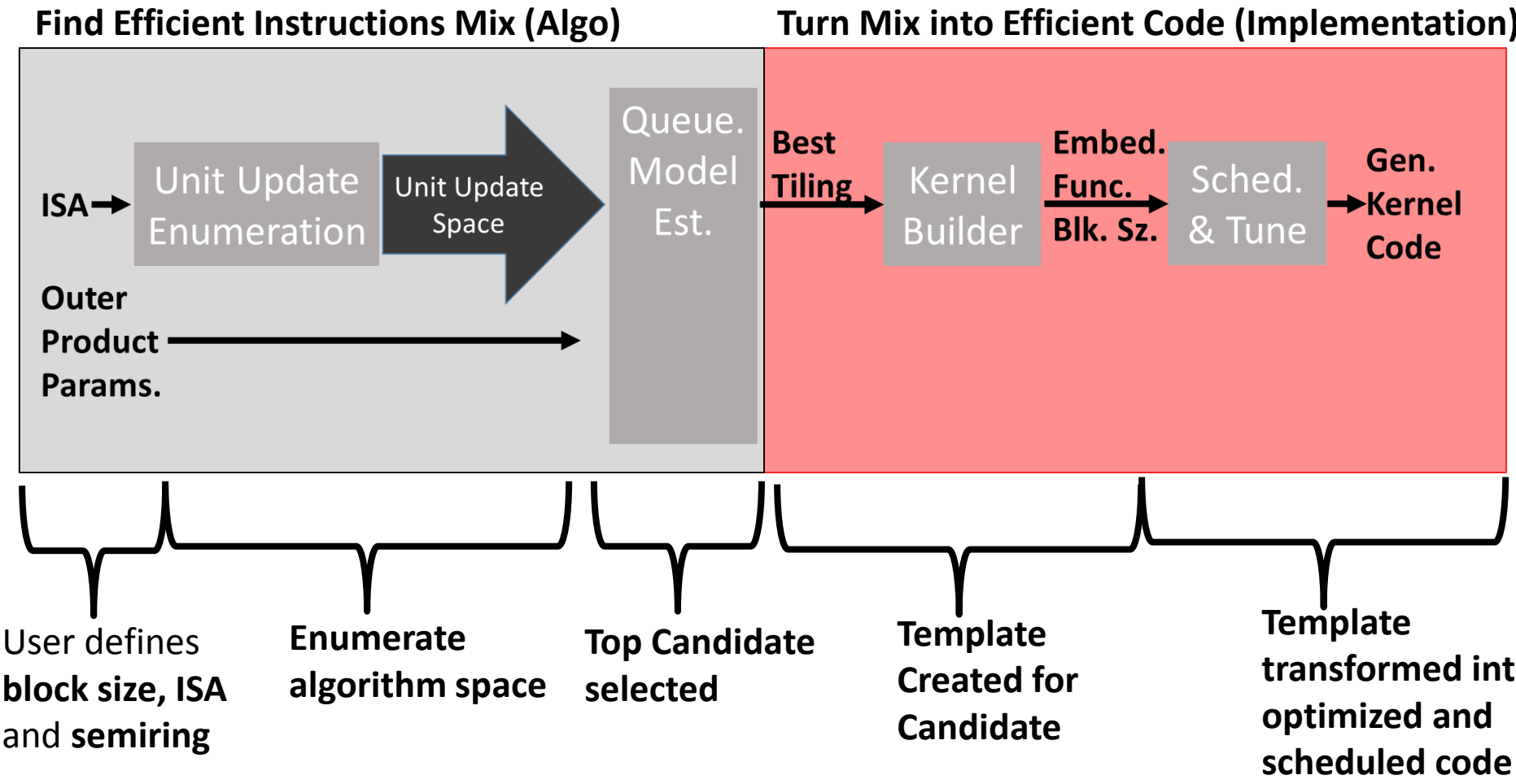


From Math to Tiling

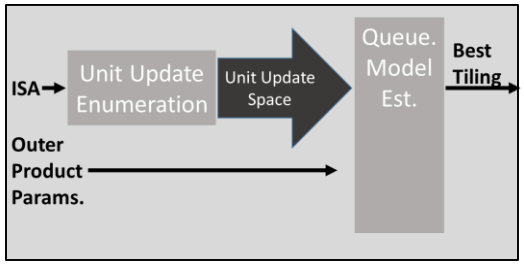
$C \leftarrow AB + C$

$C_{acc} \leftarrow 0$ Initialize

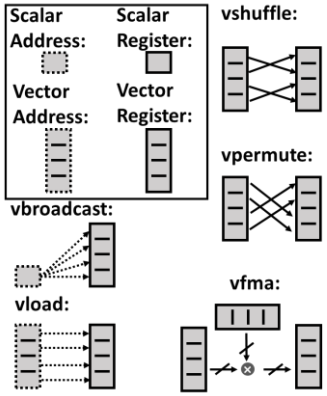
$C_{acc} \leftarrow \sum_p ab^T$ Compute

$C \leftarrow f(C_{acc})$ Accumulate

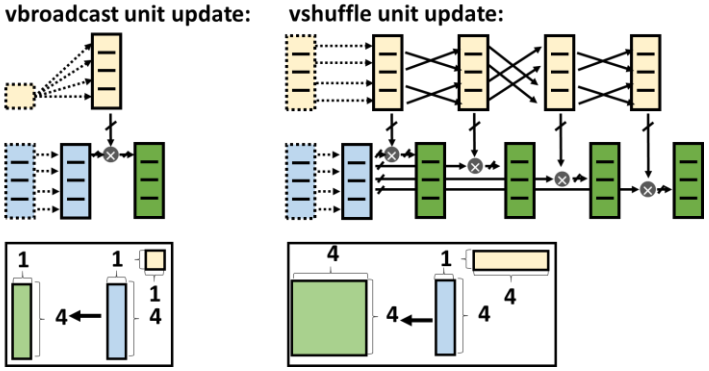
A High Throughput Mix



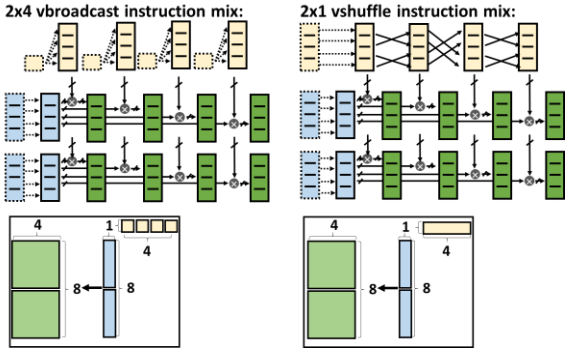
Start with ISA



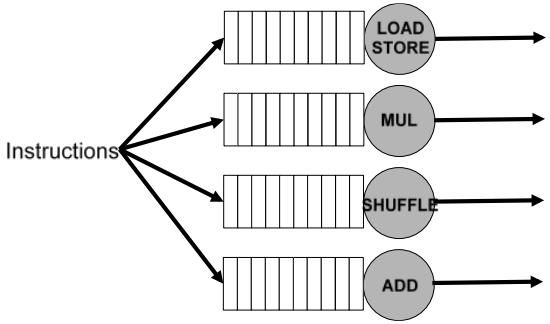
Identify Small Outer Products from ISA



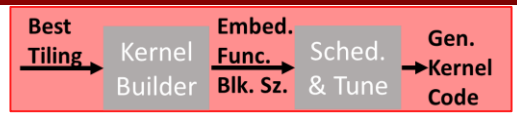
Enumerate Space of Outer Products



Select Best Mix with Queueing Model



From Tiling to Template



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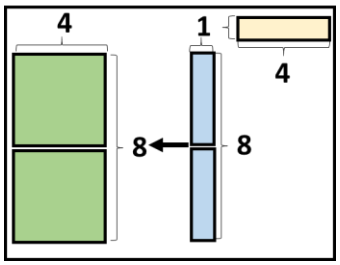
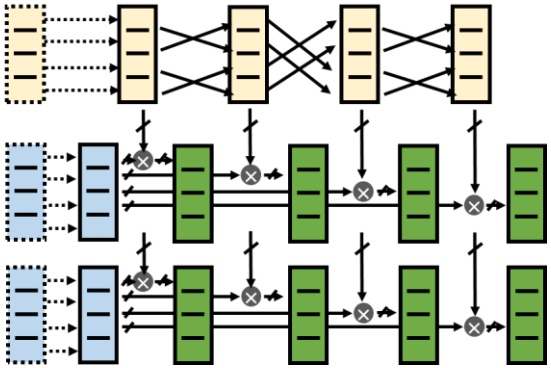
Selected Outer Product:



2x1 vshuffle instruction mix:



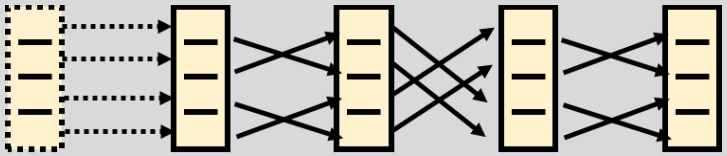
$C_{acc} \leftarrow 0$

```
for( i = 0; i <  m_r; i++ )
 for( j = 0; j <  n_r; j++ )
  init(c_reg, ii,jj )
```

$C_{acc} \leftarrow \sum_{p} ab^T$

```
for( pp = 0; pp < k_b; pp++ )
 /* perform the outer products */
 for( i = 0; i <  m_r; i+=m_s )
  for( j = 0; j <  n_r; j+=n_s )
   for( ii = i; ii < i+m_s; ii++ )
    get_a_elem(a_reg, ii,j );
    for( jj = j; jj < j+n_s; jj++ )
     get_b_elem(b_reg, ii,jj );
     apply(c_reg,a_reg,b_reg,ii,jj,pp);
```

Embedding Function (get_b):



```
def get_b_element( var array b_reg[][],
                   ptr B, ii, jj )
  opts = {
    0: assign( b_reg[jj], vload(B,jj)),
    1: assign( b_reg[jj], shuffle(b_reg[jj-1]) ),
    2: assign( b_reg[jj], permute(b_reg[jj-1]) ),
    3: assign( b_reg[jj], shuffle(b_reg[jj-1]) )}
  if ii mod v = 0
    return opts[jj]
```

$C \leftarrow f(C_{acc})$

```
for( i = 0; i <  m_r; i++ )
 for( j = 0; j <  n_r; j++ )
    accumulate(C, c_reg, ii, jj );
```

Floyd-Warshall Embedded in GEMM

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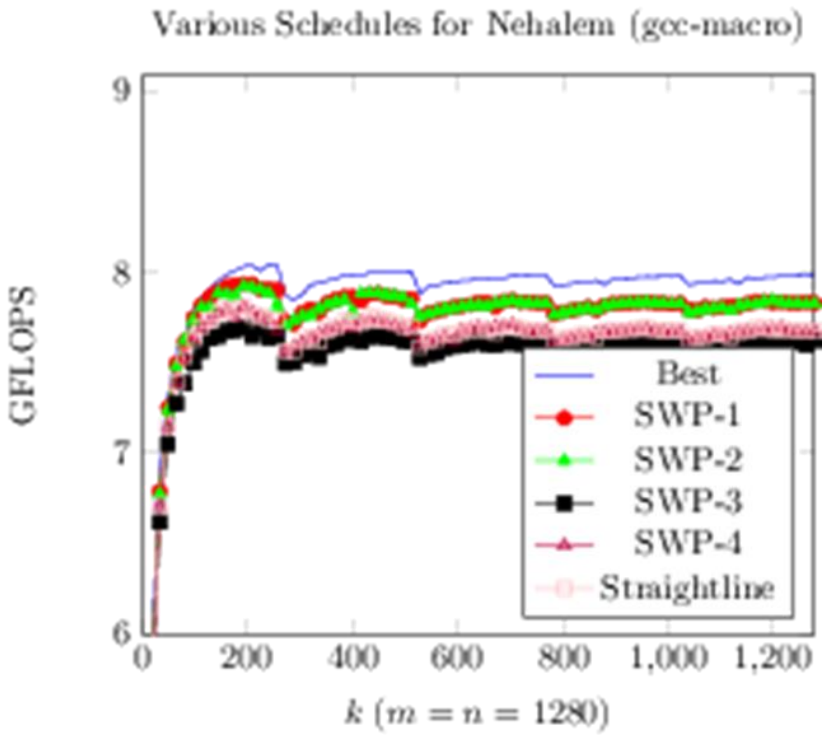
	DGEMM	Floyd-Warshall
Semiring:	$(R, *, +, 1, 0)$	$(Z, +, MIN, 0, \infty)$
Initialize: $C_{acc} \leftarrow 0$	`assign( c_reg[ii,jj], 0)`	`assign( c_reg[ii,jj], (ii==jj)? 0 : INFINITY)`
Compute: $C_{acc} \leftarrow \sum_p ab^T$	`assign( c_reg[ii,jj], a_reg[ii,pp]* b_reg[pp,jj], +c_reg[ii,jj]))`	`assign( c_reg[ii,jj], MIN(a_reg[ii,pp]+ b_reg[pp,jj], c_reg[ii,jj]))`
Accumulate: $C \leftarrow f(C_{acc})$	`assign( C[(ii,jj)], c_reg[ii,jj]+ C[(ii,jj)]`	`assign( C[(ii,jj)], MIN(c_reg[ii,jj], C[(ii,jj)] )`

Scheduling the Problem

We have built the kernel code,
Now we need to schedule:

Static Scheduling still matters on OOO Processors:



Pipeline for Scheduling:
Built Kernel +
Embedding Func.

↓

Express as Decision Vars

↓

Formulate Constraints over Vars

↓

Minimize with ILP Solver

↓

Scheduled Kernel Code

OASIC approach for ILP Scheduling

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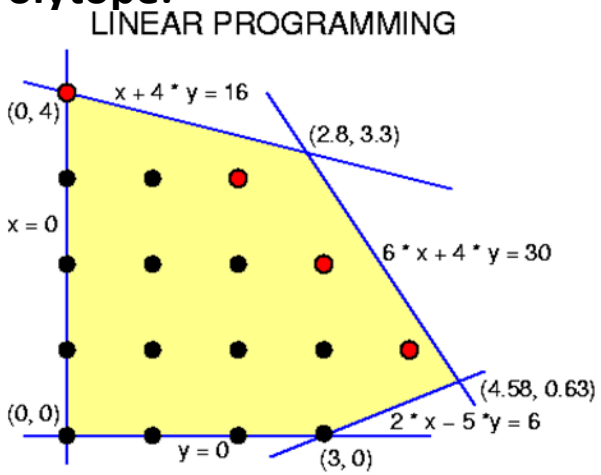
Representing Design Space as Polytope:



Decision Variable: instruction n is executed on functional unit k at time step t

x_{nt}^{k}

Functional unit

cycle

Instruction label

Expressing Constraints in terms of X:

Every instruction **n is executed once**

$$\sum_k \sum_t x_{nt}^k = 1$$

At any timestep t, functional unit k is used no more than it can

$$\sum_k x_{nt}^k \leq R_k$$

If t_m depends on t_n, then t_m will not execute until l cycles after t_n

$$\sum_k \sum_t x_{nt_m}^k + \sum_k \sum_t x_{nt_m}^k \leq 1$$

But wait, there's more!

Emitting The Code

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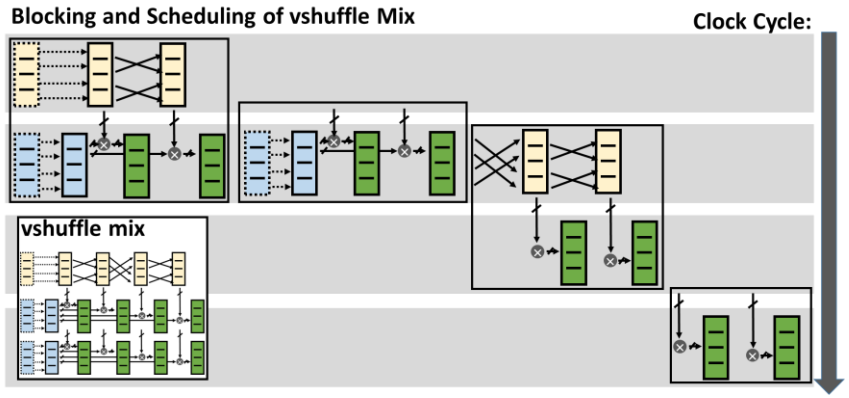
Code is now Scheduled



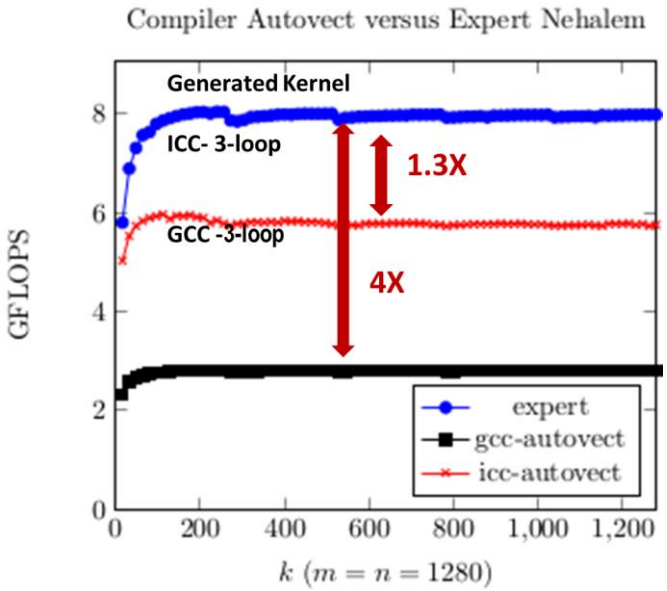
Code is emitted:

```
for( pp = 0; pp < k_b; pp+=KUNR )
 /* STEADY STATE CODE */
 VLOAD_IA(GET_A_ADDR(0),GET_A_REG(0))
 VLOAD_IA(GET_A_ADDR(1),GET_A_REG(1))
 VLOAD_IA(GET_B_ADDR(0),GET_B_REG(0))
 VSHUFFLE_IA(GET_B_REG(0),GET_B_REG(1))
 VFMA(GET_A_REG(0),
  GET_B_REG(0),GET_C_REG(0,0))
 VFMA(GET_A_REG(0),GET_B_REG(1),GET_C_REG(0,1))
 VPERM2F128_IA(0x01,GET_B_REG(1),GET_B_REG(2))
 VSHUFFLE_IA(0x05,GET_B_REG(2),GET_B_REG(3))
 VFMA(GET_A_REG(1),GET_B_REG(0),GET_C_REG(0,0))
 VFMA(GET_A_REG(1),GET_B_REG(1),GET_C_REG(0,1))
```

Need Custom ANSI C compliant SIMD wrappers to schedule in Compiler:

```
#define VADD(srca,srcb,dest)
  asm volatile(
  "vaddpd %[vsrca],%[vsrcb],
          %[vdest]"
  : [vdest] "=x"(dest)
  : [vsrca] "x"(srca),
    [vsrcb] "x"(srcb));
```

Is this necessary?



Putting it All Together

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Have Operation that we can express like GEMM:

$$C \leftarrow AB + C$$

$C_{acc} \leftarrow 0$ Initialize

$C_{acc} \leftarrow \sum_p ab^T$ Compute

$C \leftarrow f(C_{acc})$ Accumulate

	DGEMM	Floyd-Warshall
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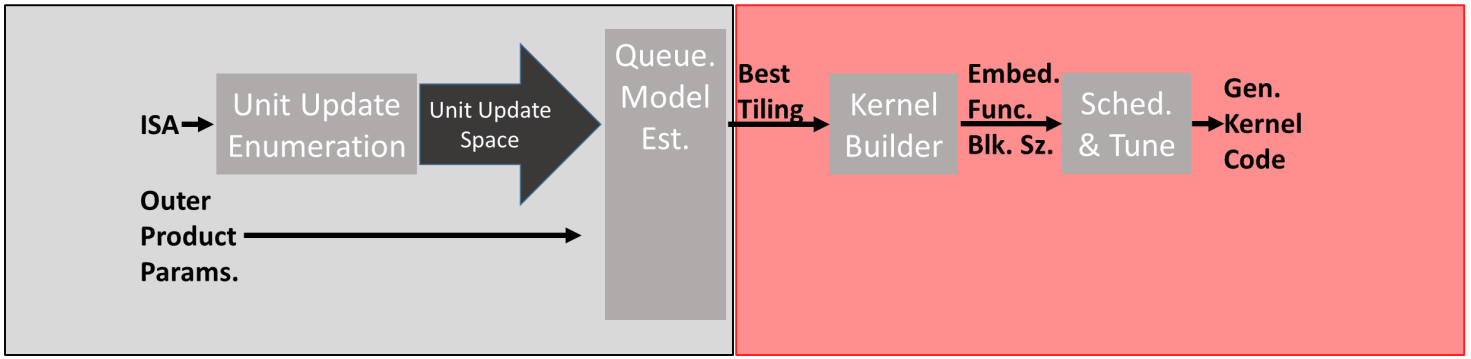
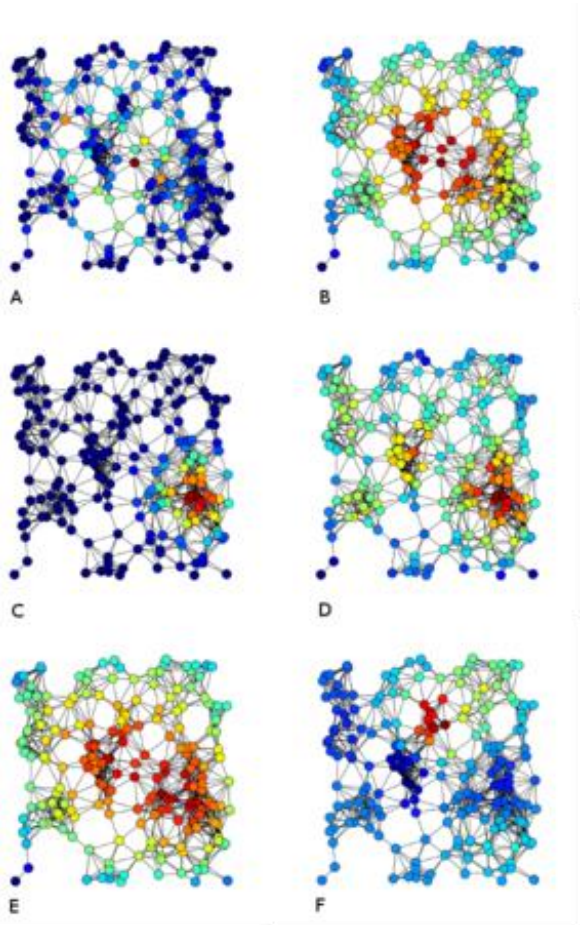
Compute: $C_{acc} \leftarrow \sum_p ab^T$	`assign( c_reg[ii,jj], a_reg[ii,pp]* b_reg[pp,jj], +c_reg[ii,jj]))`	`assign( c_reg[ii,jj], MIN(a_reg[ii,pp]+ b_reg[pp,jj], c_reg[ii,jj]))`
Accumulate: $C \leftarrow f(C_{acc})$	`assign( C[(ii,jj)], c_reg[ii,jj]+ C[(ii,jj)]`	`assign( C[(ii,jj)], MIN(c_reg[ii,jj], C[(ii,jj)] )`

Run through our Pipeline:

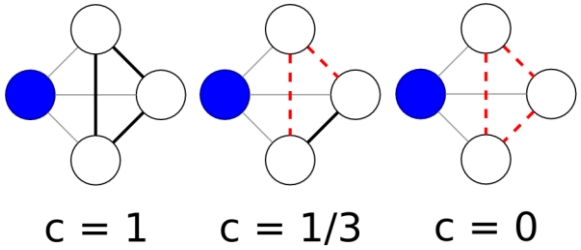


Moving Forward

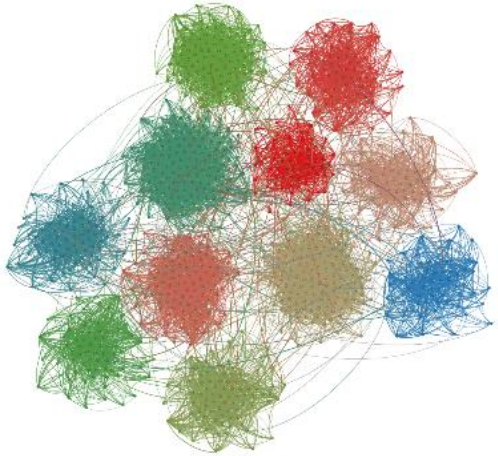
Centrality: $\sim A^T A$ (betweeness)



Clustering: $\sim AA^T A$ (triangle)



Community Detection: $\sim A^K$



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Summary

- There exists a **large class** of **GEMM-like Operations**
- Obtaining DGEMM level **performance** for each of these operations **requires automation**
- We have a **systematic approach** for automatically **generating DGEMM**
- We are extending it by allowing the user to **define a semi-ring** with an initialize and accumulate function