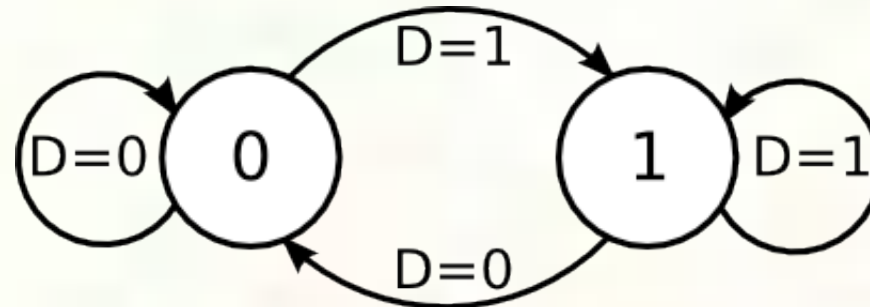


# State Machines



# Another look at D latch/flip-flop



$q_{old}$	D	$q_{new}$
0	0	0
0	1	1
1	0	0
1	1	1

This is an example of a **state diagram**  
more specifically a **Moore** machine

$$q_{new} = D$$

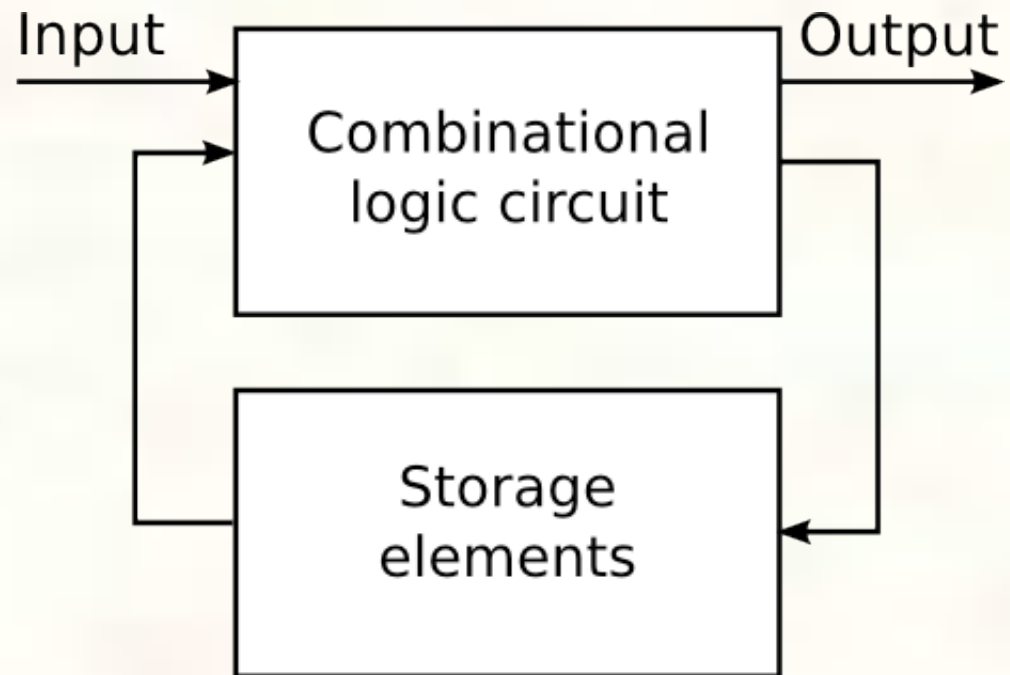


# Synchronous state machines

If a system can both process and store information, then the values stored in the memory elements depend on both the inputs and the previous values in these elements. This is called a **sequential** system.

Such a system is also called a **finite-state machine (FSM)**.

If all changes to memory values happen at the same time as determined by a global system clock, we have a **synchronous FSM**.





# FSM definition

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An FSM has the following components:

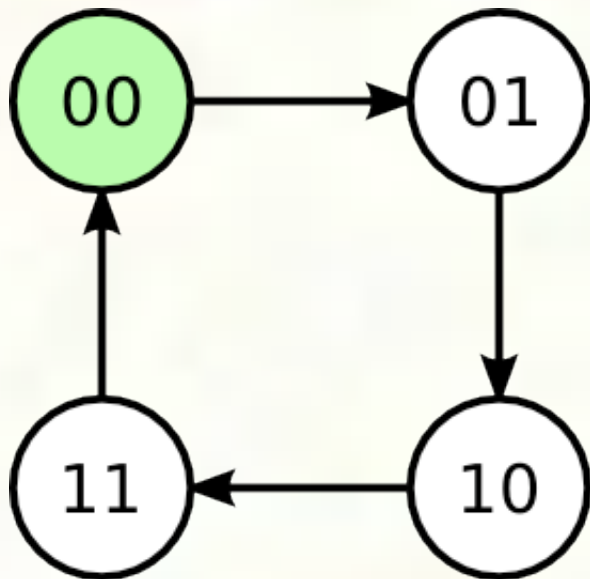
- A set of states
- A set of inputs
- A set of outputs
- A state-transition function (of the states and inputs)
- An output function (of the states and maybe inputs)
  - Moore machine - function of states only
  - Mealy machine - function of states and inputs

This can be represented by a **state diagram**

- States are circles
- Arcs show the state transition function
- Arcs are labeled with input values
- Outputs are labels on states (Moore) or arcs (Mealy)



# Another example - 2-bit counter



Counter starts at 0 (green) and increments each time the clock cycles, until it gets to 3 and then overflows back to 0.

Only input is the clock, we don't show that.

$H_{old}$	$L_{old}$	$H_{new}$	$L_{new}$
0	0	0	1
0	1	1	0
1	0	1	1
1	1	0	0

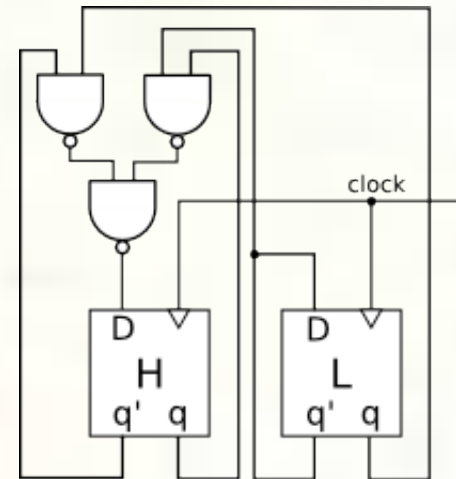
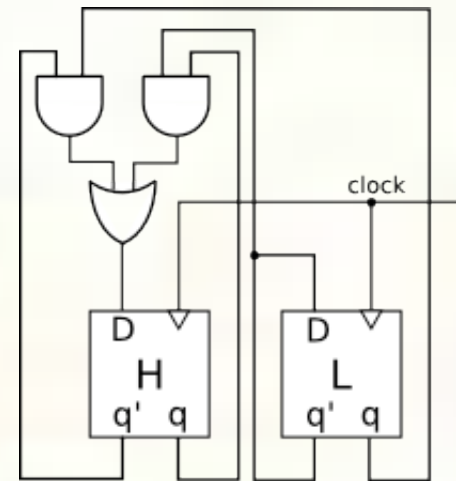


# 2-bit counter

$H_{old}$	$L_{old}$	$H_{new}$	$L_{new}$
0	0	0	1
0	1	1	0
1	0	1	1
1	1	0	0

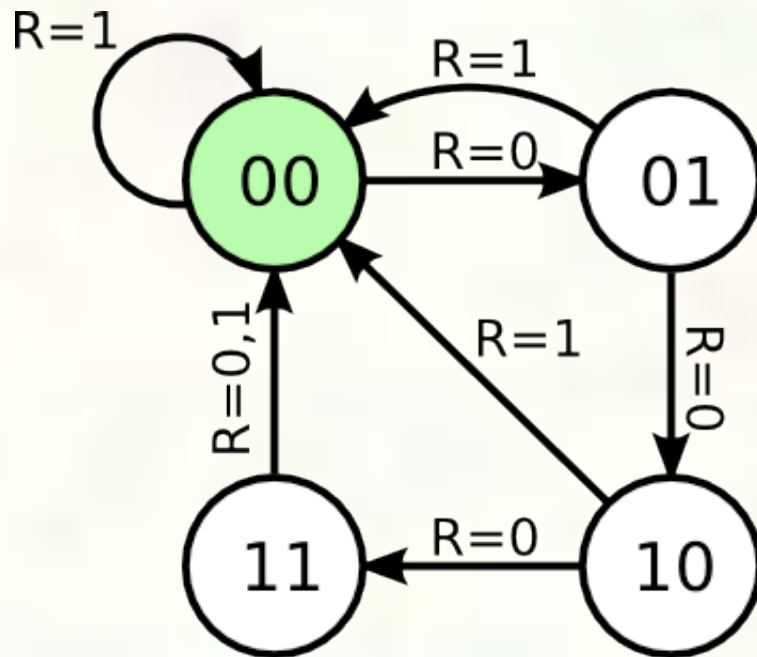
$$L_{new} = H_{old}'L_{old}' + H_{old}L_{old}' = L_{old}'$$

$$H_{new} = H_{old}'L_{old} + H_{old}L_{old}'$$





# 2-bit counter with reset



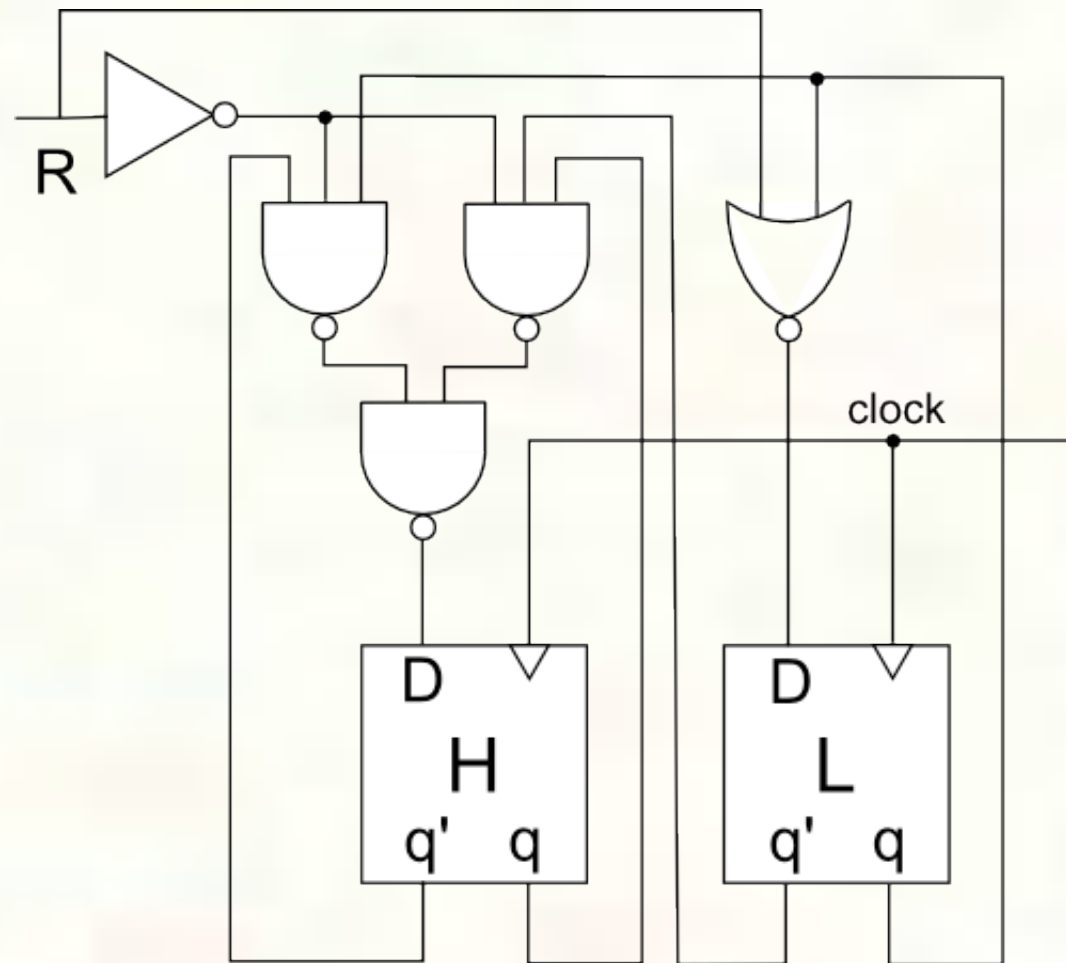
R	H <sub>old</sub>	L <sub>old</sub>	H <sub>new</sub>	L <sub>new</sub>
0	0	0	0	1
0	0	1	1	0
0	1	0	1	1
0	1	1	0	0
1	x	x	0	0

$$\begin{aligned}
 L_{\text{new}} &= R'H_{\text{old}}'L_{\text{old}}' + R'H_{\text{old}}L_{\text{old}}' \\
 &= R'L_{\text{old}}' = (R + L_{\text{old}})'
 \end{aligned}$$

$$\begin{aligned}
 H_{\text{new}} &= R'H_{\text{old}}'L_{\text{old}} + R'H_{\text{old}}L_{\text{old}}' \\
 &= R'(H_{\text{old}}'L_{\text{old}} + H_{\text{old}}L_{\text{old}}')
 \end{aligned}$$



# 2-bit counter with reset



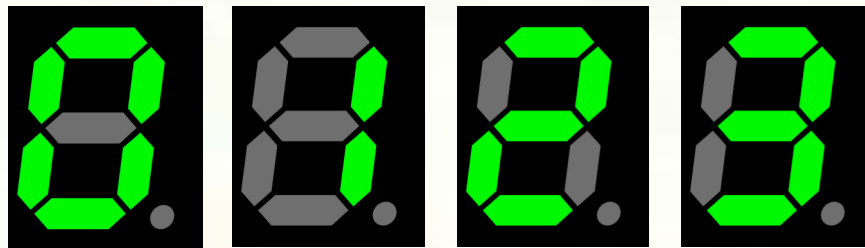
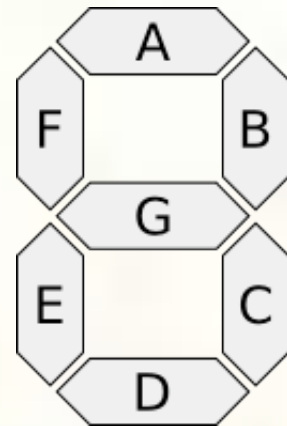




# Counter with 7-segment display

Each segment in the display can be lit independently to allow all 10 decimal digits to be displayed (also hex)

2-bit counter will need to display digits 0-3, so will output a 1 for each segment to be lit for a given state





# Counter with output functions

R	H <sub>0</sub>	L <sub>0</sub>	H <sub>n</sub>	L <sub>n</sub>	A	B	C	D	E	F	G
0	0	0	0	1	1	1	1	1	1	1	0
0	0	1	1	0	0	1	1	0	0	0	0
0	1	0	1	1	1	1	0	1	1	0	1
0	1	1	0	0	1	1	1	1	0	0	1
1	X	X	0	0	0	0	0	0	0	0	0

$$A = D = R'H_0'L_0' + R'H_0L_0' + R'H_0L_0 = R'(H_0'L_0)'$$

$$B = R'$$

$$C = R'(H_0L_0)'$$

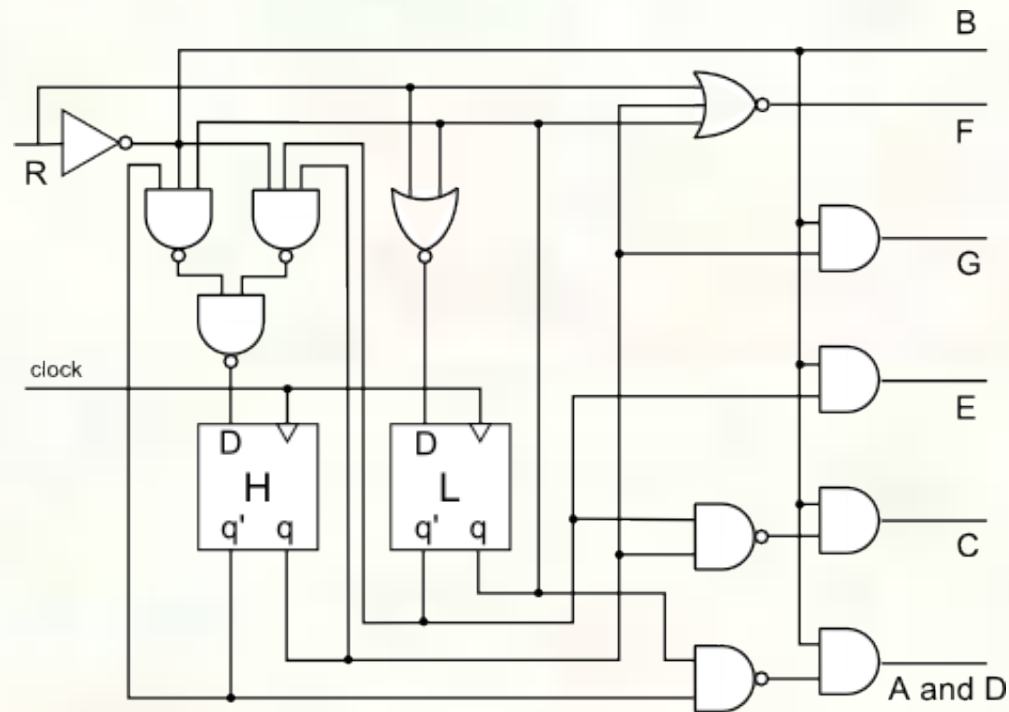
$$E = R'L_0'$$

$$F = R'H_0'L_0' = (R+H_0+L_0)'$$

$$G = R'H_0$$



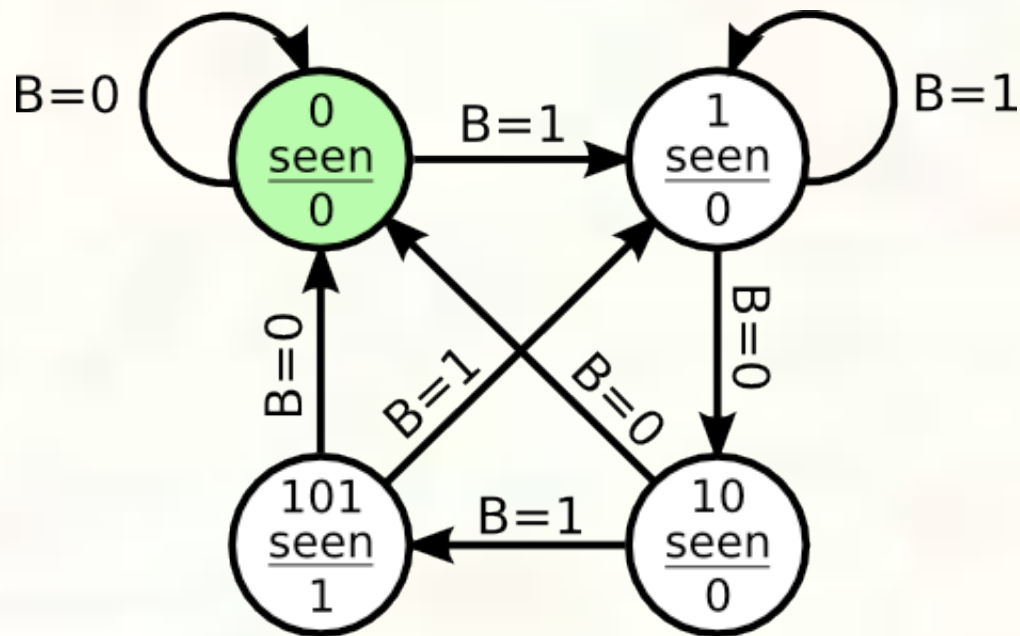
# 7-segment output logic





# Example - 101 lock

Combination lock with 101  
being the combination



B is input signal to the lock,  
X is output signal to unlock

B	H <sub>0</sub>	L <sub>0</sub>	H <sub>n</sub>	L <sub>n</sub>	X
0	0	0	0	0	0
0	0	1	1	0	0
0	1	0	0	0	0
0	1	1	0	0	1
1	0	0	0	0	0
1	0	1	0	1	0
1	1	0	1	1	1
1	1	1	0	1	0



# 101 combination lock

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$$X = H_0 L_0$$

$$H_n = B'H_0'L_0 + BH_0L_0'$$

$$\begin{aligned} L_n &= BH_0'L_0 + BH_0L_0' + BH_0L_0 \\ &= BH_0'L_0 + BH_0L_0 + BH_0L_0' + BH_0L_0 \\ &= BL_0 + BH_0 \end{aligned}$$



# LC-3 datapath

