CS354 Computer Graphics
Surface Representation IV

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Today’s Topic

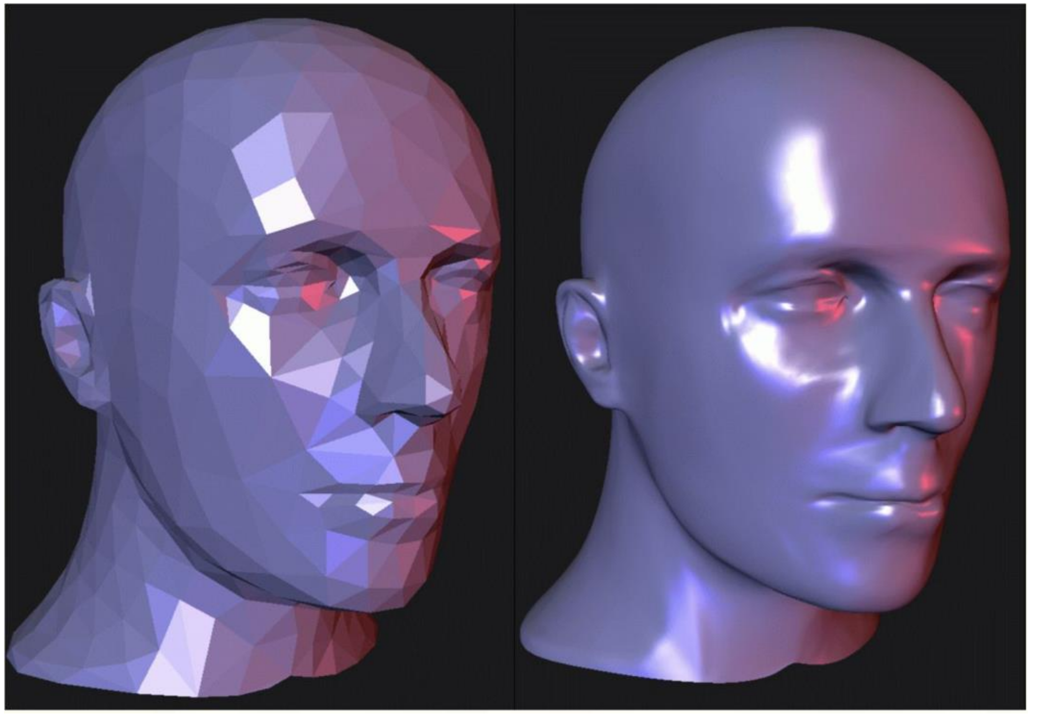
- Subdivision surfaces

- Implicit surface representation

Subdivision Surfaces

Building complex models

- We can extend the idea of subdivision from curves to surfaces…

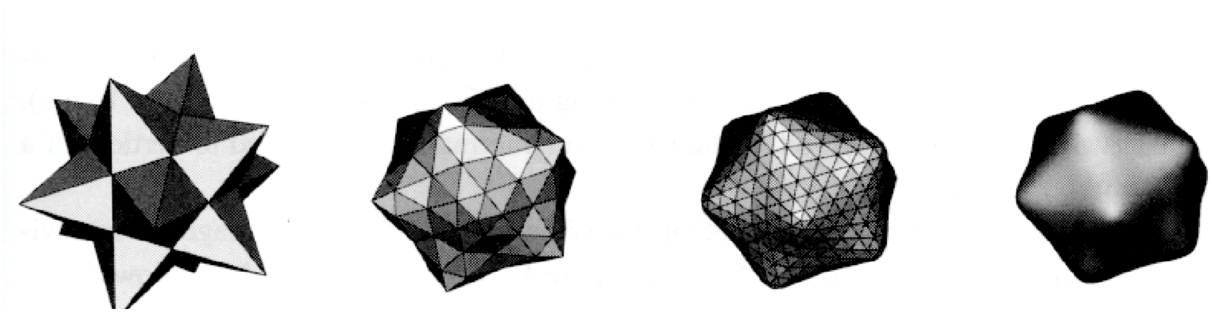


Subdivision surfaces

- Chaikin's use of subdivision for curves inspired similar techniques for subdivision surfaces
- Iteratively refine a control polyhedron (or control mesh) to produce the limit surface

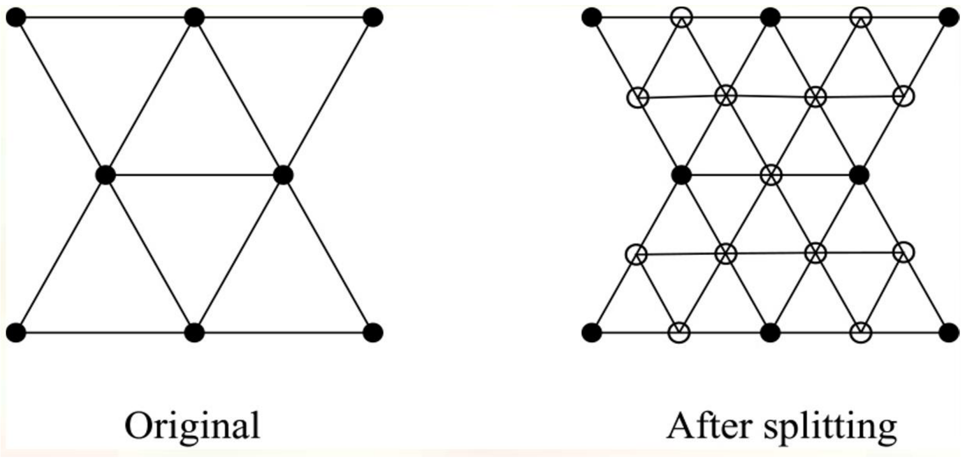
$$\sigma = \lim_{j \to \infty} M^j$$

using splitting and averaging steps



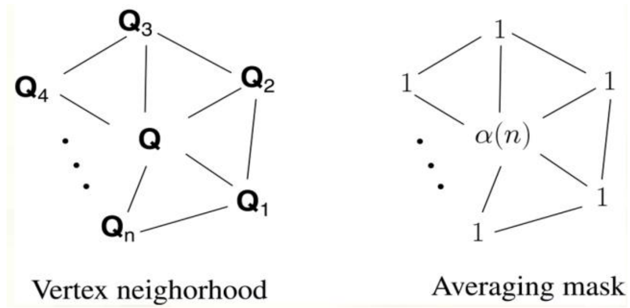
Triangular subdivision

- There are a variety of ways to subdivide a poylgon mesh.
- A common choice for triangle meshes is 4:1 subdivision – each triangular face is split into four subfaces:



Loop averaging step

- Once again we can use masks for the averaging step:



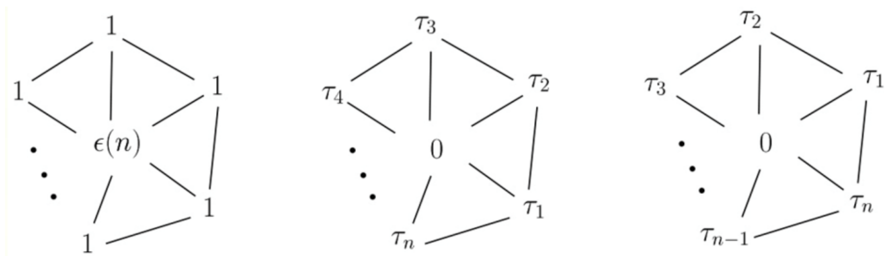
- Where

$$\mathbf{Q} \leftarrow \frac{\alpha(n)\mathbf{Q} + \mathbf{Q}_1 + \cdots + \mathbf{Q}_n}{\alpha(n) + n} \quad \alpha(n) = \frac{n(1-\beta(n))}{\beta(n)} \quad \beta(n) = \frac{5}{4} - \frac{(3 + 2\cos(2\pi/n))^2}{32}$$

- These values, due to Charles Loop, are carefully chosen to ensure smoothness – namely, tangent plane or normal continuity
- Note: tangent plane continuity is also known as G1 continuity for surfaces

Loop evaluation and tangent masks

- As with subdivision curves, we can split and average a number of times and then push the points to their limit positions.



Evaluation mask — Tangent masks

$$\mathbf{Q}^{\infty} = \frac{\varepsilon(n)\mathbf{Q} + \mathbf{Q}_1 + \cdots + \mathbf{Q}_n}{\varepsilon(n) + n}$$

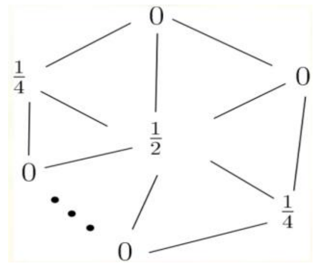
$$\mathbf{T}_1^{\infty} = \tau_1(n)\mathbf{Q}_1 + \tau_2(n)\mathbf{Q}_2 + \cdots + \tau_n(n)\mathbf{Q}_n$$

$$\mathbf{T}_2^{\infty} = \tau_n(n)\mathbf{Q}_1 + \tau_1(n)\mathbf{Q}_2 + \cdots + \tau_{n-1}(n)\mathbf{Q}_n$$

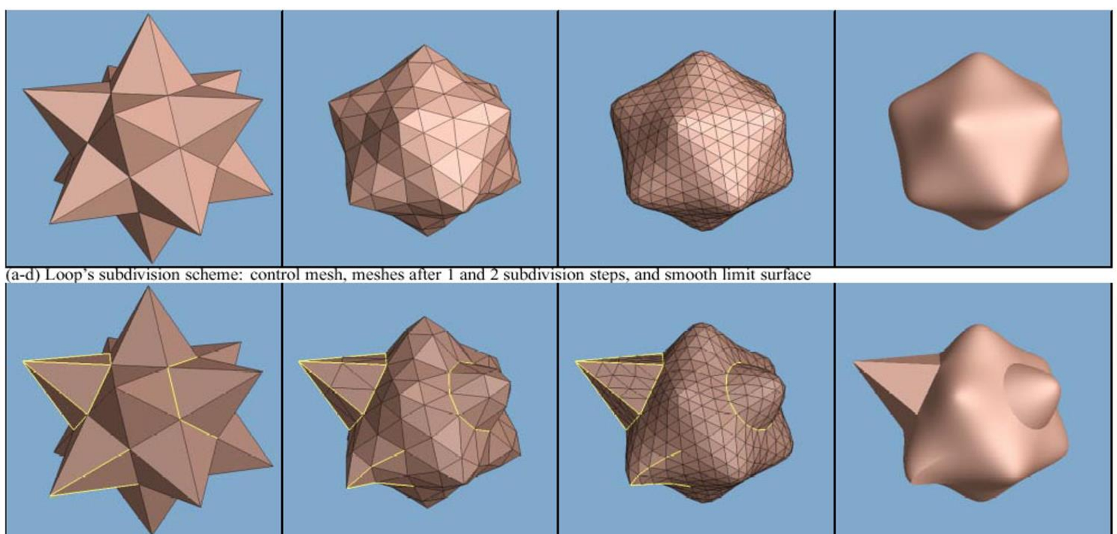
- Where $\varepsilon(n) = \dfrac{3n}{\beta(n)} \quad \tau_i(n) = \cos(2\pi i / n)$
- How do we compute the normal?

Adding creases without trim curves

- For subdivision surfaces, we can just modify the subdivision mask:



- This gives rise to G^0 continuous surfaces (i.e., having positional but not tangent plane continuity)



(a-d) Loop's subdivision scheme: control mesh, meshes after 1 and 2 subdivision steps, and smooth limit surface

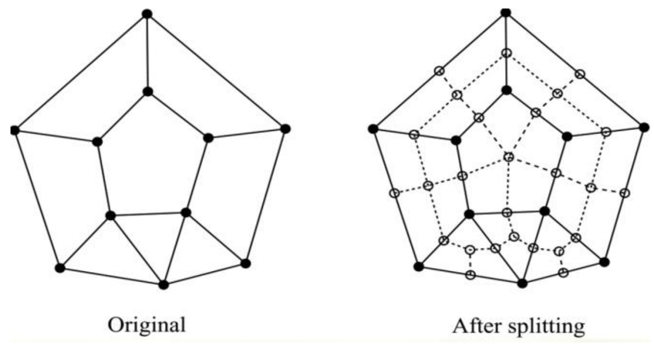
(e-h) Our piecewise smooth subdivision scheme: tagged control mesh, meshes after 1 and 2 subdivision steps, and piecewise smooth limit surface

Creases without trim curves, cont

- Here’s an example using Catmull-Clark surfaces (based on subdividing quadrilateral meshes):

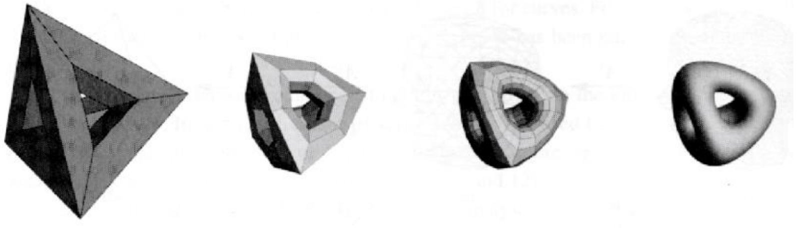
Face schemes

- 4:1 subdivision of triangles is sometimes called a face scheme for subdivision, as each face begets more faces.



- An alternative face scheme starts with arbitrary polygon meshes and inserts vertices along edges and at face centroids:

Catmull-Clark:



- Note: after the first subdivision, all polygons are quadilaterals in this scheme.

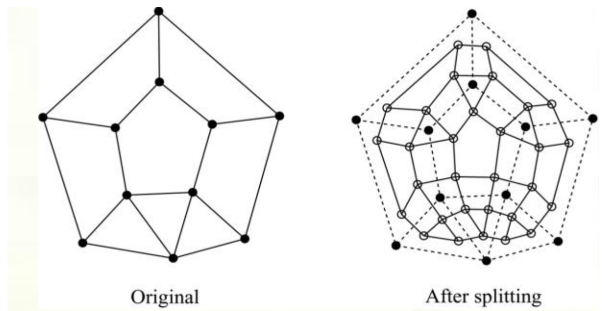
Subdivision=tensor-product patches!

- For a regular quadrilateral mesh, Catmull-Clark subdivision produces the same surface as tensor product cubic B-splines

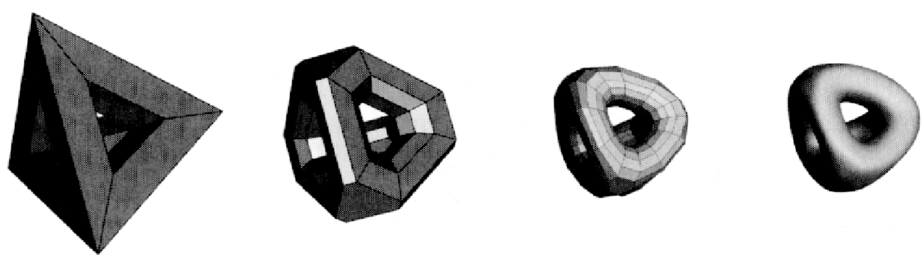
- But – it handles irregular meshes as well
  - There are similar correspondences between other subdivision schemes and other tensor-product patch schemes

Vertex schemes

- In a vertex scheme, each vertex begets more vertices. In particular, a vertex surrounded by n faces is split into n sub-vertices, one for each face:



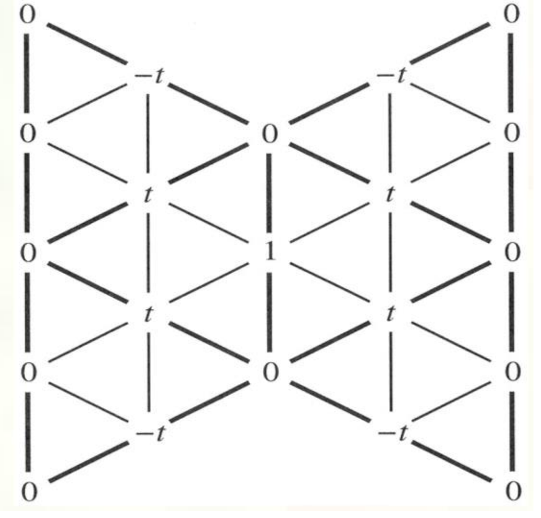
- Doo-Sabin subdivision:



- The number edges (faces) incident to a vertex is called its valence. Edges with only once incident face are on the boundary. After splitting in this subdivision scheme, all nonboundary vertices are of valence 4.

Interpolating subdivision surfaces

- Interpolating schemes are defined by
  - splitting
  - averaging only new vertices
- The following averaging mask is used in butterfly subdivision:

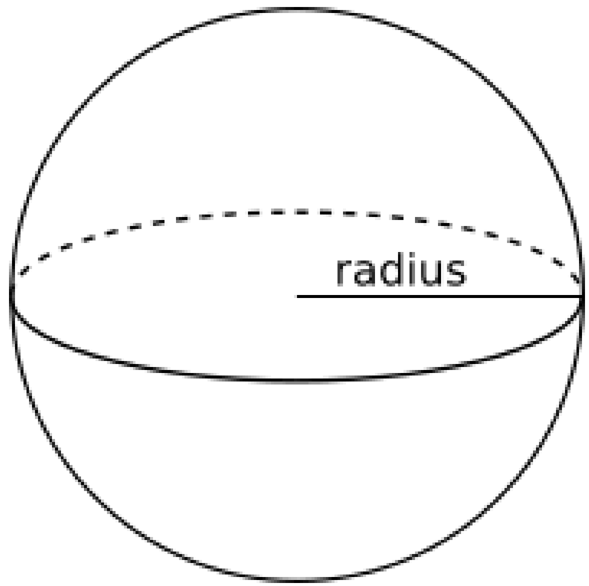


- Setting *t=0* gives the original polyhedron, and increasing small values of t makes the surface smoother, until *t=1/8* when the surface is provably G^1

Implicit Surfaces

What is implicit surface?

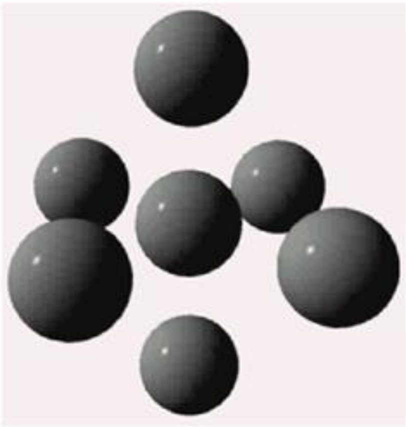
- A sphere $x^2 + y^2 + z^2 = radius^2$ is an implicit surface



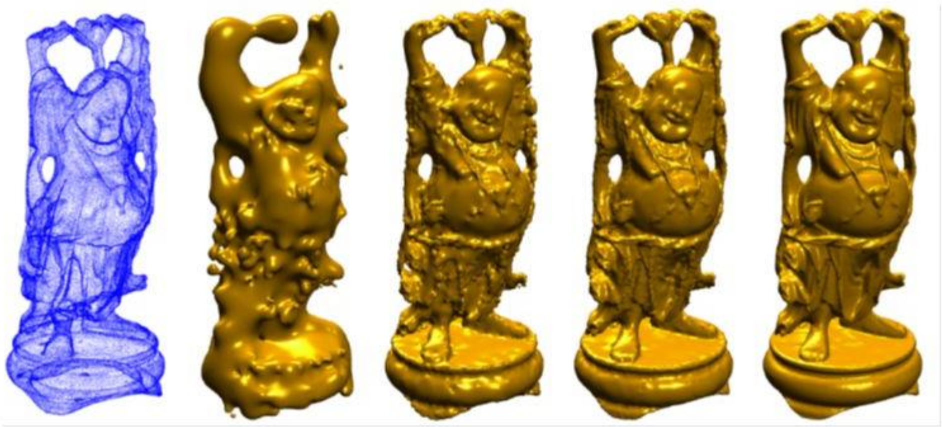
What is implicit surface?

- Implicit surfaces are two-dimensional, geometric shapes that exist in three dimensional space
  - Defined in R^3
  - 2D Manifold if no singular points
  - A surface embedded in R^3

Examples of implicit surfaces



Metaball



Radial Basis Function
[Carr et al. 01]

Definition of implicit surface

- Definition

$$\{p=(x,y,z): f(p)=0,\ p\in\mathbf{R}^3\}$$

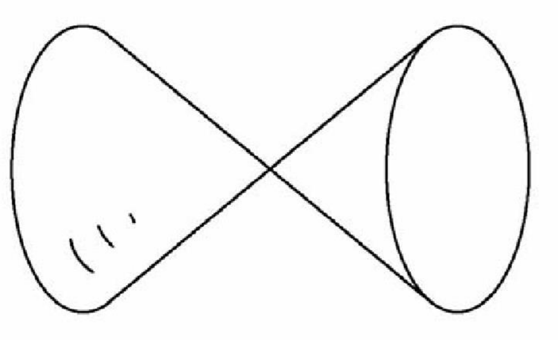
- When *f* is algebraic function, i.e., polynomial function
  - Note that *f* and *c*f* specify the same curve
  - Algebraic distance: the value of *f(p)* is the approximation of distance from *p* to the algebraic surface *f=0*

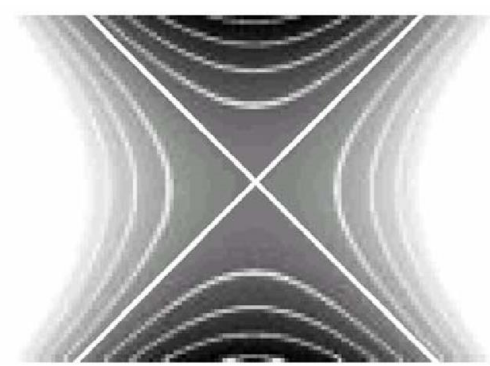
Definition of implicit surface

- Regular point *p* on the surface

$$\nabla f(p) = (\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}) \neq 0$$

- Consider cone $z^2=x^2+y^2$
  - (0,0,0) is not a regular point





Implicit function theorem

Let f: $\mathbf{R}^{n+m} \to \mathbf{R}^m$ be a continuously differentiable function, and let $\mathbf{R}^{n+m}$ have coordinates ($\mathbf{x}$, $\mathbf{y}$). Fix a point ($\mathbf{a}$, $\mathbf{b}$) = (a_1, ..., a_n, b_1, ..., b_m) with $f(\mathbf{a}, \mathbf{b}) = \mathbf{0}$, where $\mathbf{0} \in \mathbf{R}^m$ is the zero vector. If the Jacobian matrix $J_{f,\mathbf{y}}(\mathbf{a}, \mathbf{b}) = [(\partial f_i / \partial y_j)(\mathbf{a}, \mathbf{b})]$ is invertible, then there exists an open set U of $\mathbf{R}^n$ containing $\mathbf{a}$, and such that there exists a unique continuously differentiable function g: $U \to \mathbf{R}^m$ such that

$$g(\mathbf{a}) = \mathbf{b}$$

and

$$f(\mathbf{x}, g(\mathbf{x})) = \mathbf{0} \text{ for all } \mathbf{x} \in U.$$

Moreover, the partial derivatives of g in U are given by

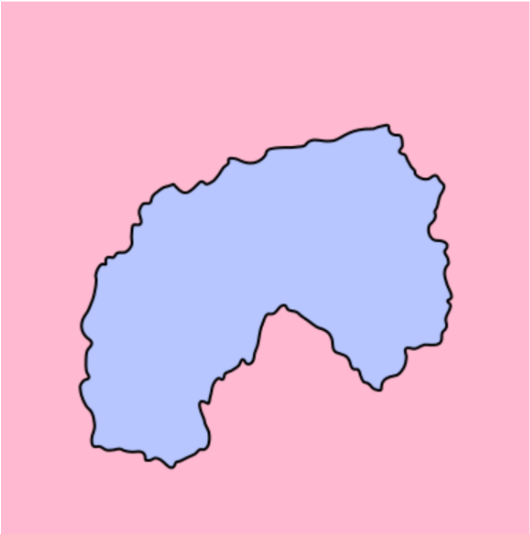
$$\frac{\partial g}{\partial x_j}(\mathbf{x}) = -\sum_i \left(J_{f,\mathbf{y}}(\mathbf{x}, g(\mathbf{x}))^{-1}\right)_{ji} \frac{\partial f}{\partial x_i}(\mathbf{x}, g(\mathbf{x})).$$

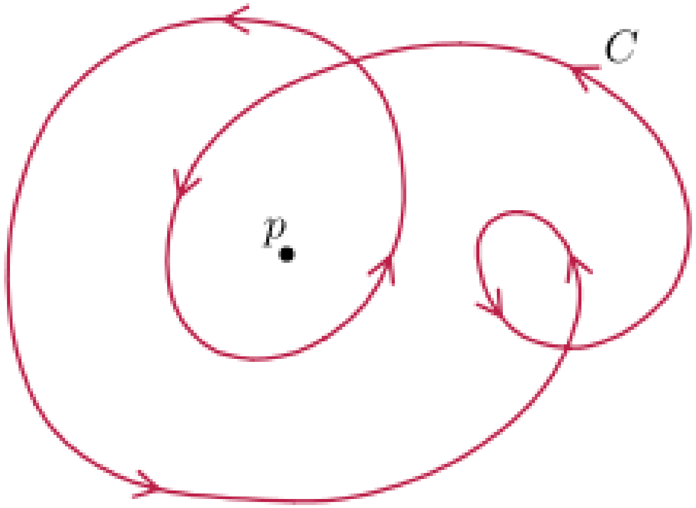
No singular points then an implicit surface is a manifold

From https://en.wikipedia.org/wiki/Implicit_function_theorem

Jordan-Brouwer Separation Theorem

- Any compact, connected hyper-surface X in R^n will divide R^n into two connected regions: the "outside" D_0 and the "inside" D_1. Furthermore, D_1 is itself a compact manifold with boundary X





Implicit v.s. Parametric Surfaces

- Implicit surfaces
  - Pros: Point classification (solid modeling, interfence check) is easy
  - Pros: Intersections/offsets can be represented
  - Cons: Difficult to fit and manipulate free-from shapes
  - Cons: Axis dependent
  - Cons: Complex to trace

Implicit v.s. Parametric Surfaces

- Parametric surfaces
  - Pros: Axis independent
  - Pros: Easy to generate composite curves
  - Pros: Easy to trace
  - Pros: Easy in fitting and manipulating free-from shaps
  - Cons: High flexibility complicates intersections and point classification

Next Lecture

- Blobby (metaball, soft objects)
- Implicit surface defined by skeletons
  - Distance surface
  - Convolution surface
- Variational Implicit Surfaces
- Level-Set Methods
- Procedural Models
- Animation applications

Next Lecture

- Implicitization
  - Parametric representation to implicit representation
- Parameterization
  - Implicit representation to parametric representation

- Implicit surface to triangular mesh

Questions?