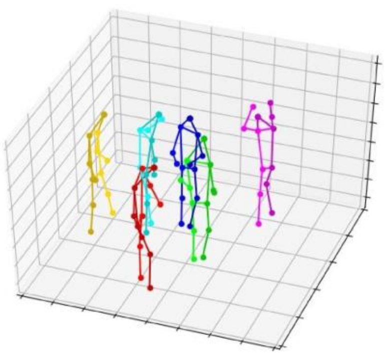
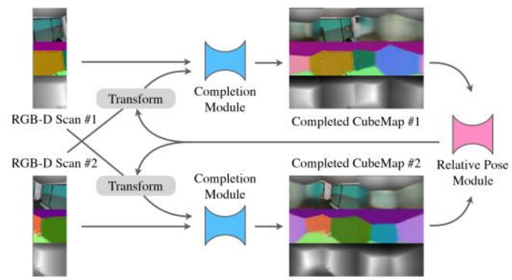
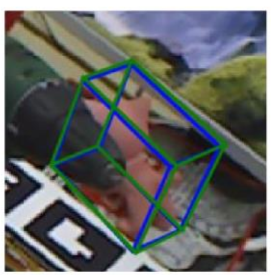
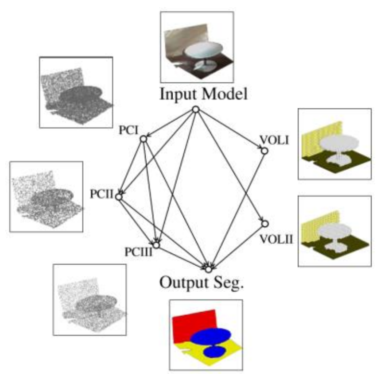
CS376 Computer Vision
Lecture 9: Active Contours









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Feb. 20th 2019

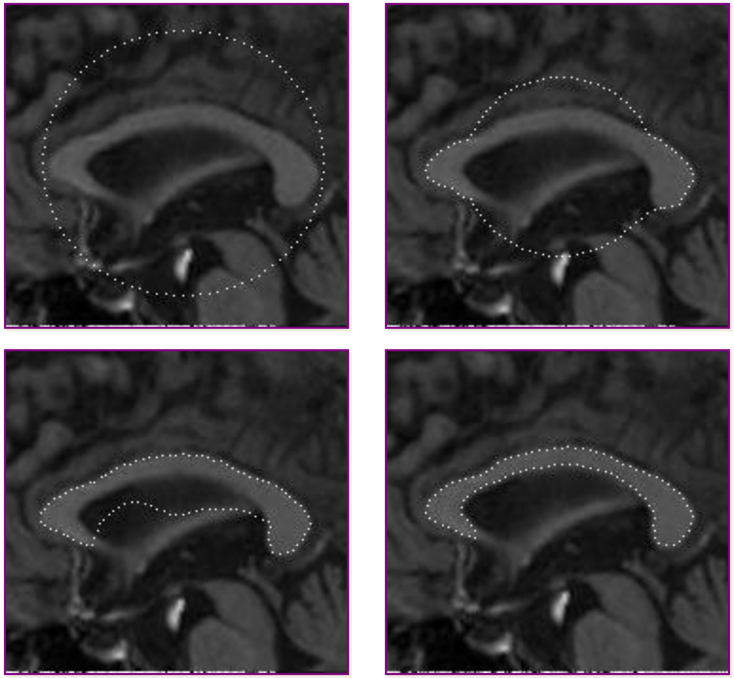
Previous Lecture

- RANSAC

- Robust fitting

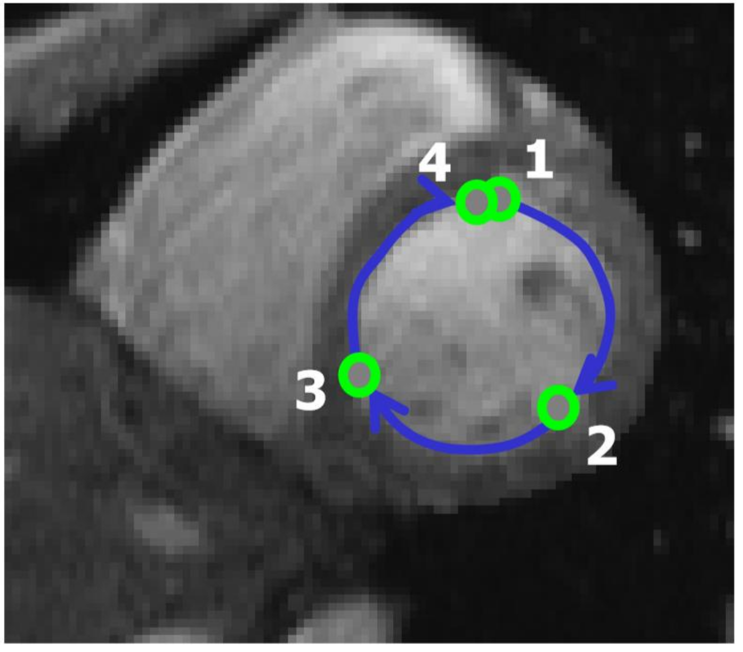
This Lecture

- Active contour and its variants
  - Widely used in computer vision and beyond



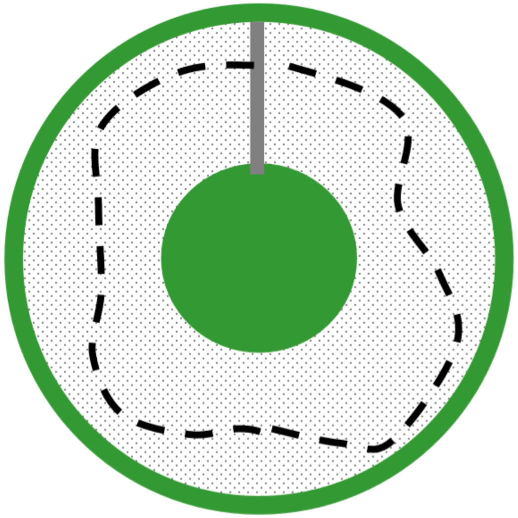
Shortest paths segmentation

- “Intelligent scissors”or “Live wire”
  - Shortest paths on image graph connect seeds, which the user places on the boundary



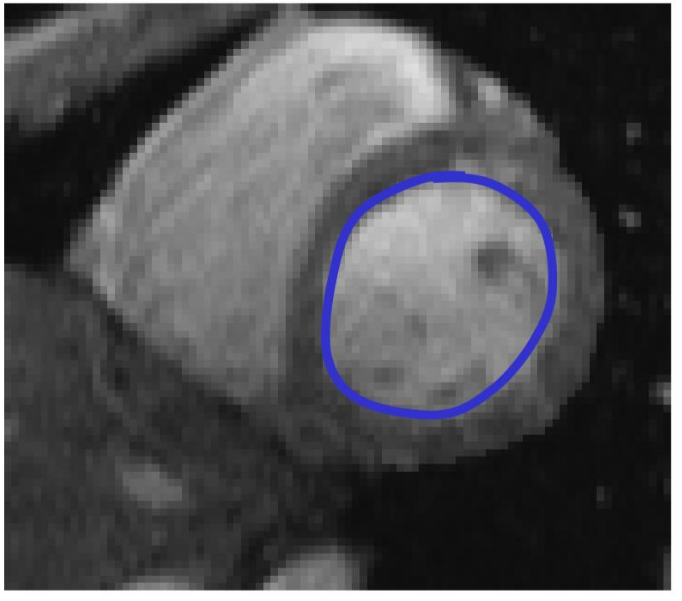
Minimizing user interaction

- Suppose we only know roughly where the object is. Can we do without placing sample points



Active Contours (snakes)

- Start with a curve near the object
  - Evolve the curve to fit the boundary
  - Application is in segmentation and tracking



Snakes: Active Contour Models

- Introduced by Kass, Witkin, and Terzopoulos in 1988
- Framework: energy minimization
  - Bending and stretching curve = more energy
  - Good features = less energy
  - Curve evolves to minimize energy
- Also “Deformable Contours”

Snake energy function

- Energy function on a snake has two terms (one prior + one data)

$$E_{total} = E_{in} + E_{ex}$$

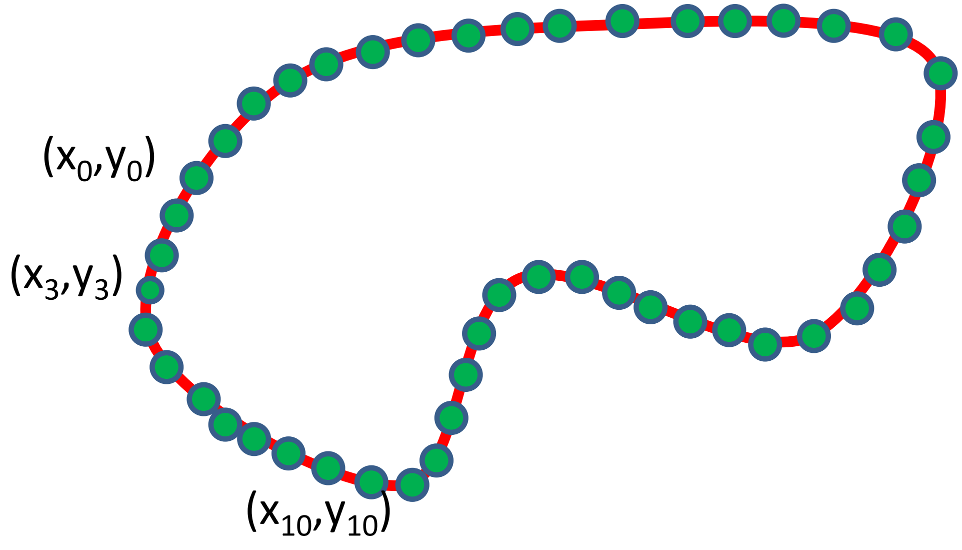
Internal energy encourages smoothness or any particular shape

External energy encourages curve onto image structures (e.g. image edges)

Internal energy incorporates prior knowledge about object boundary allowing to extract boundary even if some image data is missing

Discrete snake formulation

- Use a spline with control points $v_i = (x_i, y_i)$



Discrete external energy

- Want to attract the snake to edges

$$E_{ex} = -\sum_i |G_x(x_i, y_i)|^2 + |G_y(x_i, y_i)|^2$$

$$G_x = \frac{\partial}{\partial x} G_\sigma \otimes I$$

$$G_y = \frac{\partial}{\partial y} G_\sigma \otimes I$$

Other external energy-corner attraction

- Can use corner detector (e.g., when talking about the optical flow)
- Alternatively, let $\theta = \tan^{-1} I_y / I_x$
  and let $\mathbf{n}_\perp$ be a unit vector perpendicular to the gradient. Then

$$E_{img} = w \cdot |\frac{\partial \theta}{\partial n_\perp}|$$

Other external energy-constraint forces

- Spring

$$E_{con} = k \cdot \|\mathbf{v} - \mathbf{x}\|^2$$

- Repulsion

$$E_{con} = \frac{k}{\|\mathbf{v} - \mathbf{x}\|^2}$$

Slide Credit: Ramin Zabih

Discrete internal energy

$$\frac{dv}{ds} \approx v_{i+1} - v_i \qquad \frac{d^2v}{ds^2} \approx (v_{i+1} - v_i) - (v_i - v_{i-1}) = v_{i+1} - 2v_i + v_{i-1}$$

$$E_{in} = \sum_{i=0}^{n-1} \alpha \|\mathbf{v}_{i+1} - \mathbf{v}_i\|^2 + \beta \|\mathbf{v}_{i+1} - 2\mathbf{v}_i + \mathbf{v}_{i-1}\|^2$$

Elasticity term Stiffness term

Slide Credit: Ramin Zabih

Relative weighting of terms

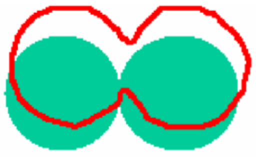
- Notice that the strength of the internal elastic component can be controlled by the parameter alpha

$$E_{in} = \alpha \cdot \sum_{i=0}^{n-1} L_i^{\,2}$$

- Increasing this increases curve stiffness

Large α

Medium α

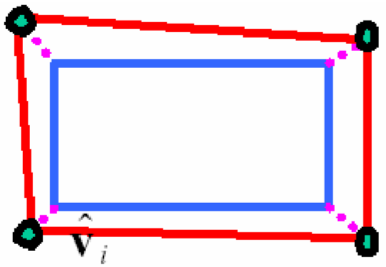


Small α

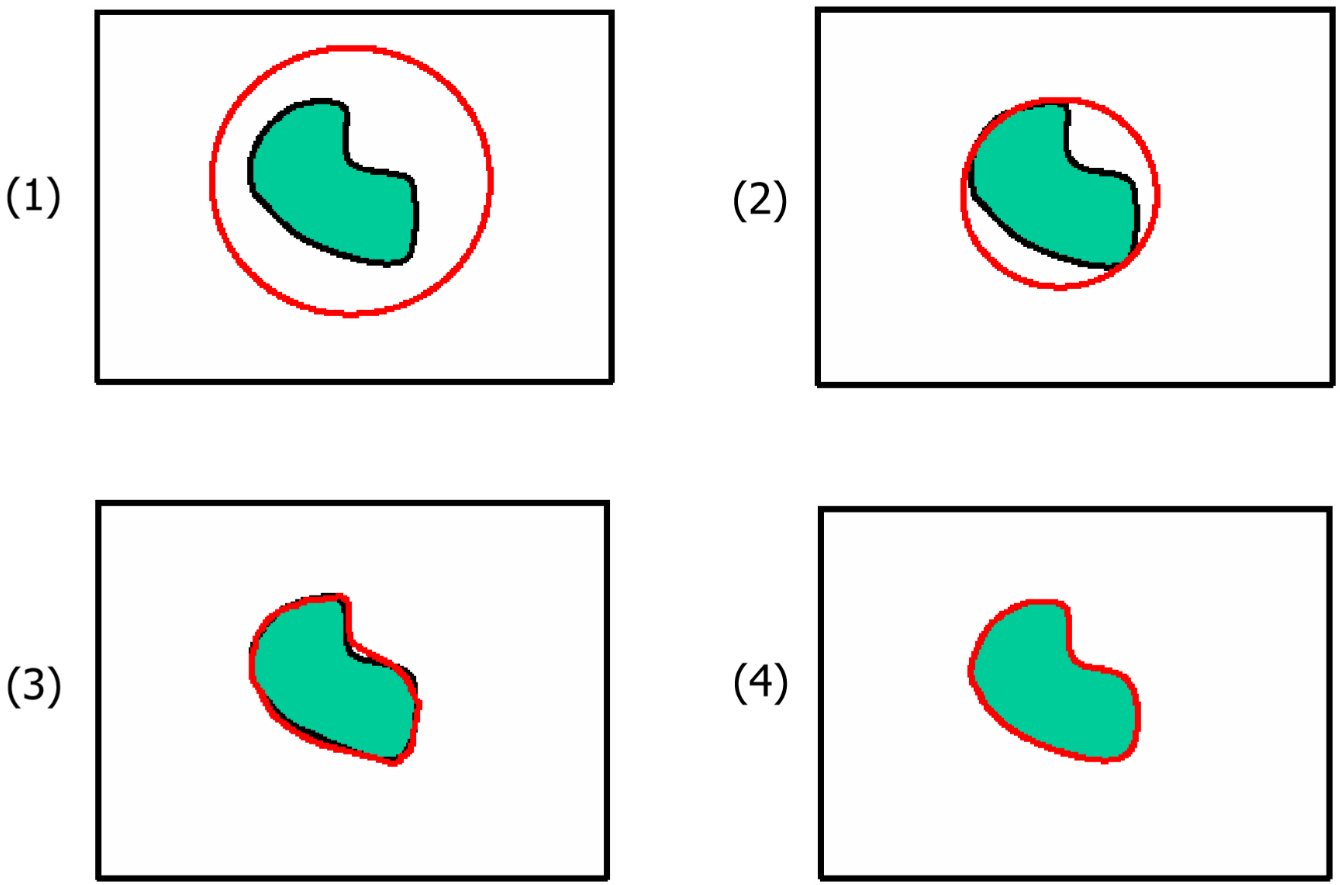
Some variants

Avoid shrinkage: $E_{in} = \alpha \cdot \sum_{i=0}^{n-1} (L_i - \hat{L}_i)^2$

Prefer known shape: $E_{in} = \alpha \cdot \sum_{i=0}^{n-1} ($



Synthetic example



Slide Credit: Ramin Zabih

Snake energy

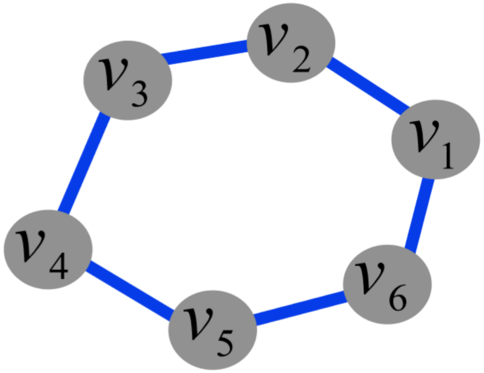
$$E_{total}(v_0, \ldots, v_{n-1}) = -\sum_{i=0}^{n-1} \| G(v_i) \|^2 + \alpha \cdot \sum_{i=0}^{n-1} \| v_{i+1} - v_i \|^2$$

$$E_{total}(v_0, \ldots, v_{n-1}) = \sum_{i=0}^{n-1} E_i(v_i, v_{i+1})$$

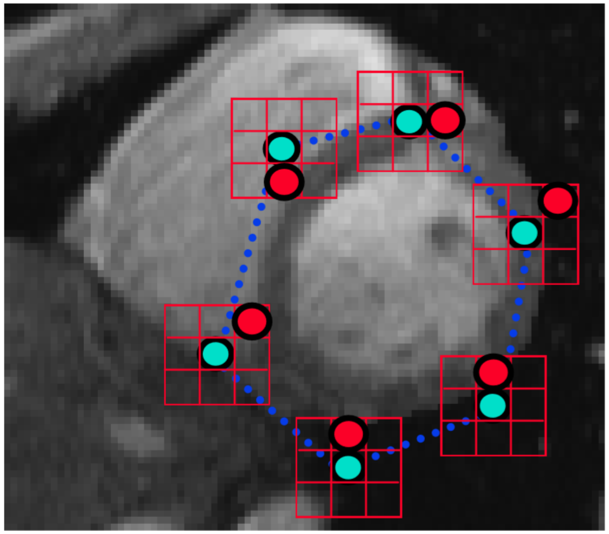
where $E_i(v_i, v_{i+1}) = -\| G(v_i) \|^2 + \alpha \| v_i - v_{i+1} \|^2$

Slide Credit: Ramin Zabih

Evolving strategy - dynamic programming



control points



First-order interactions

$$E(v_1, v_2, \ldots, v_n) = E_1(v_1, v_2) + E_2(v_2, v_3) + \ldots + E_{n-1}(v_{n-1}, v_n)$$

Energy E is minimized via Dynamic Programming

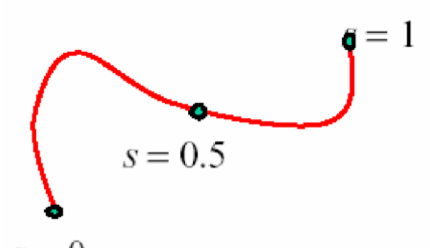
Other evolving strategies (Not required)

- Think about the continuous formulation

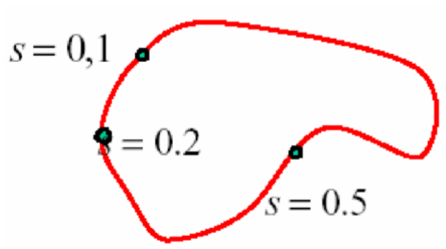
$$E = \int \Big(E_{int}(\mathbf{v}(s)) + E_{img}(\mathbf{v}(s)) + E_{con}(\mathbf{v}(s)) \Big) ds$$

- A curve can be represented parametrically

$$v(s) = (x(s), y(s)) \qquad 0 \le s \le 1$$



open curve



closed curve

Evolving curve

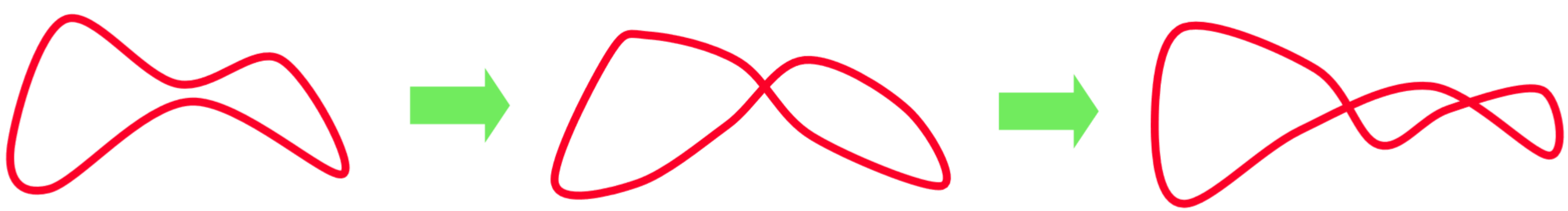
- Exact solution: calculus of variations
- Write equations directly in terms of forces, not energy

$$\frac{\partial}{\partial s^2}\left(\frac{\partial E}{\partial \ddot{\mathbf{v}}}\right) + \frac{\partial}{\partial s}\left(\frac{\partial E}{\partial \dot{\mathbf{v}}}\right) + \frac{\partial E}{\mathbf{v}} = 0$$

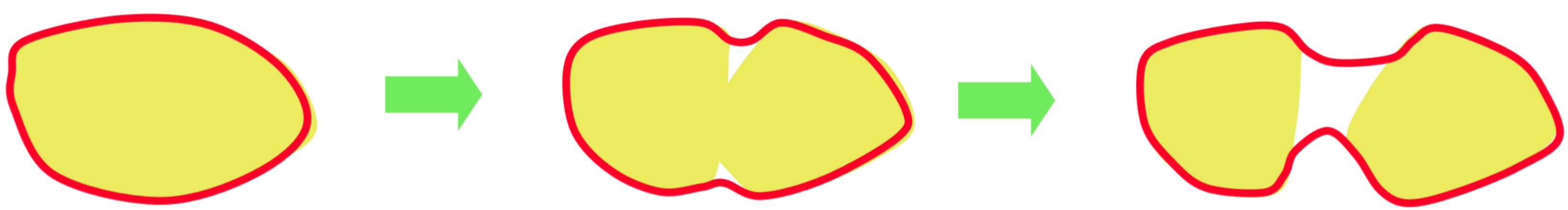
- Most follow-up works use this formulation
- Implicit equation solver
  - More complicated implementations

Limitations of snakes

- Get stuck in local minimum
- Often miss indentations in objects
- Hard to prevent self-intersections



- Cannot follow topological changes!



Variants on snakes

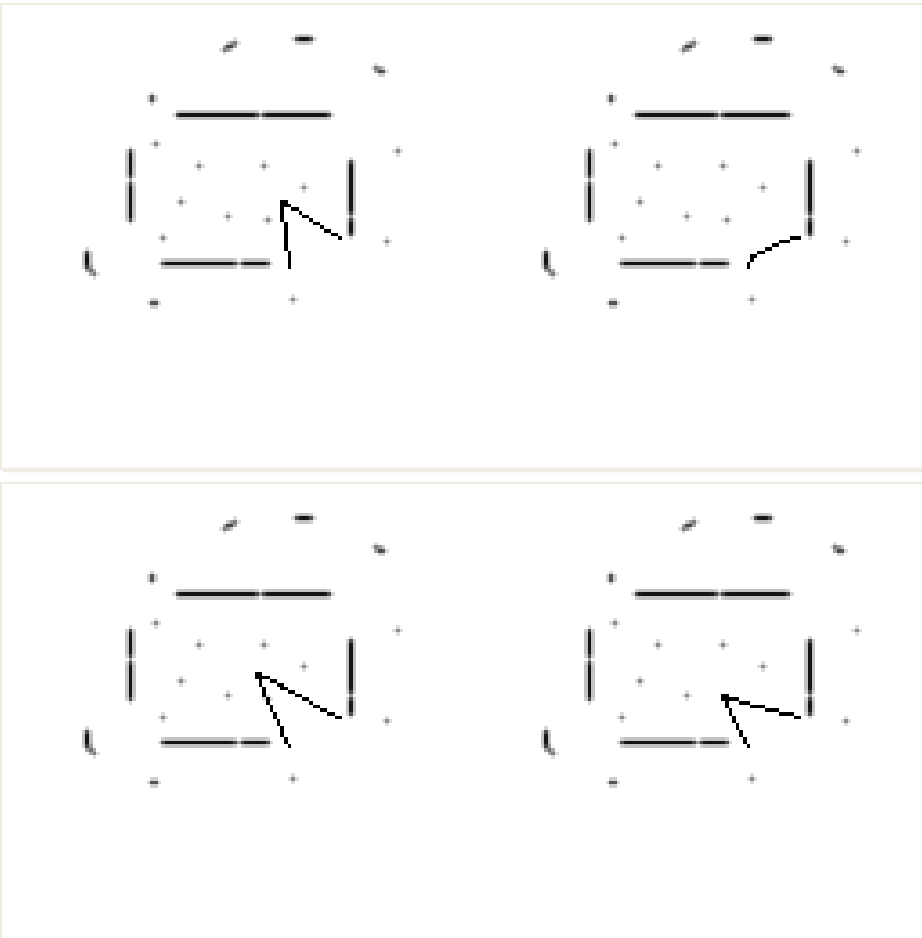
- Balloons [Cohen 91]
  - Add inflation force

$$F_{infl} = k\,\mathbf{n}(s)$$

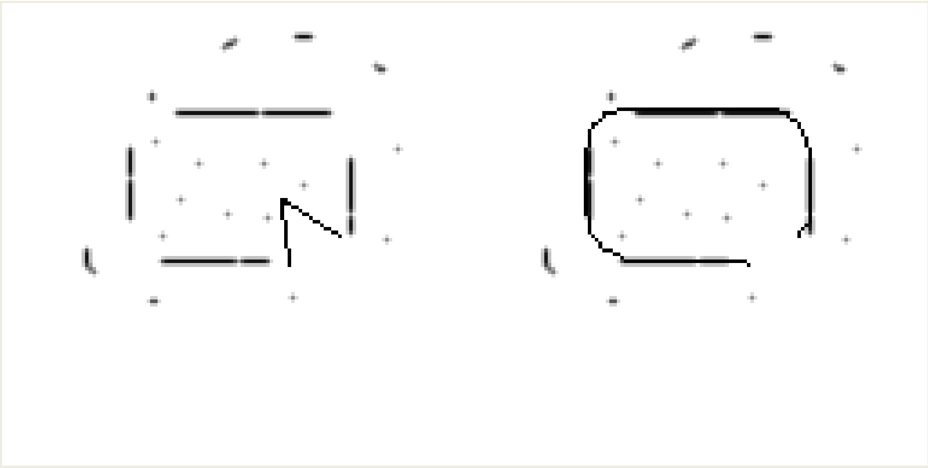
  - Helps avoid getting stuck on small features

Slide Credit: Szymon Rusinkiewicz

Balloons



Snakes

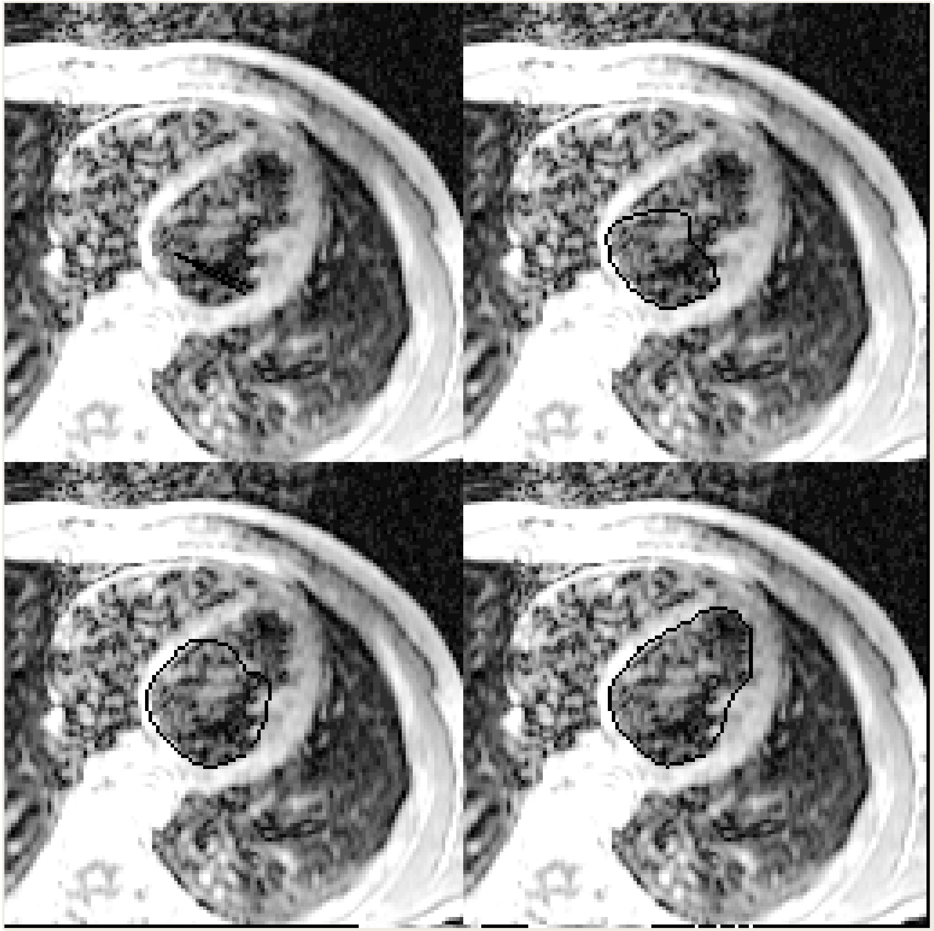


Balloons

[Cohen 91]

Slide Credit: Szymon Rusinkiewicz

Balloons



[Cohen 91]

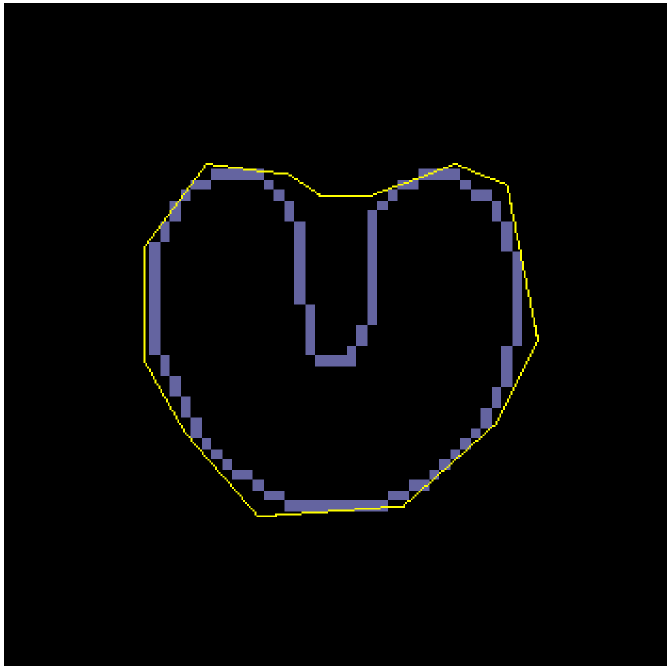
Slide Credit: Szymon Rusinkiewicz

Diffusion-Based Methods

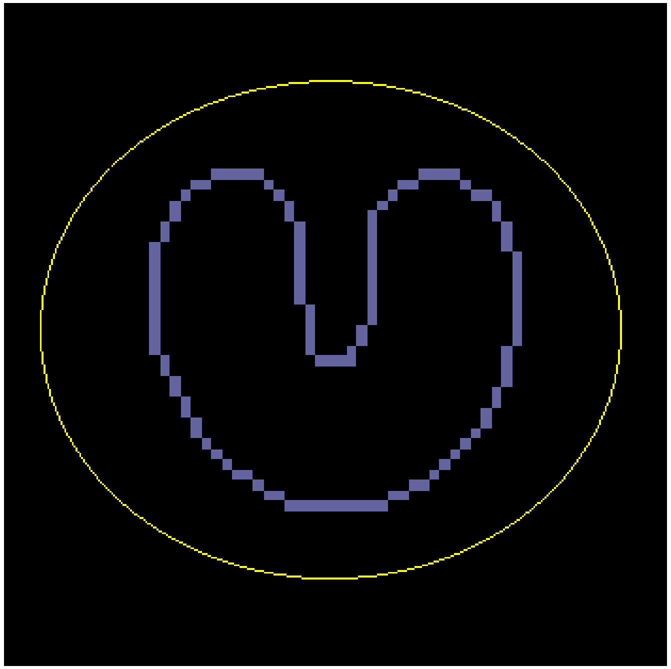
- Another way to attract curve to localized features: vector flow or diffusion methods
- Example:
  - Find edges using Canny
  - For each point, compute distance to nearest edge
  - Push curve along gradient of distance field

Slide Credit: Szymon Rusinkiewicz

Gradient Vector Fields



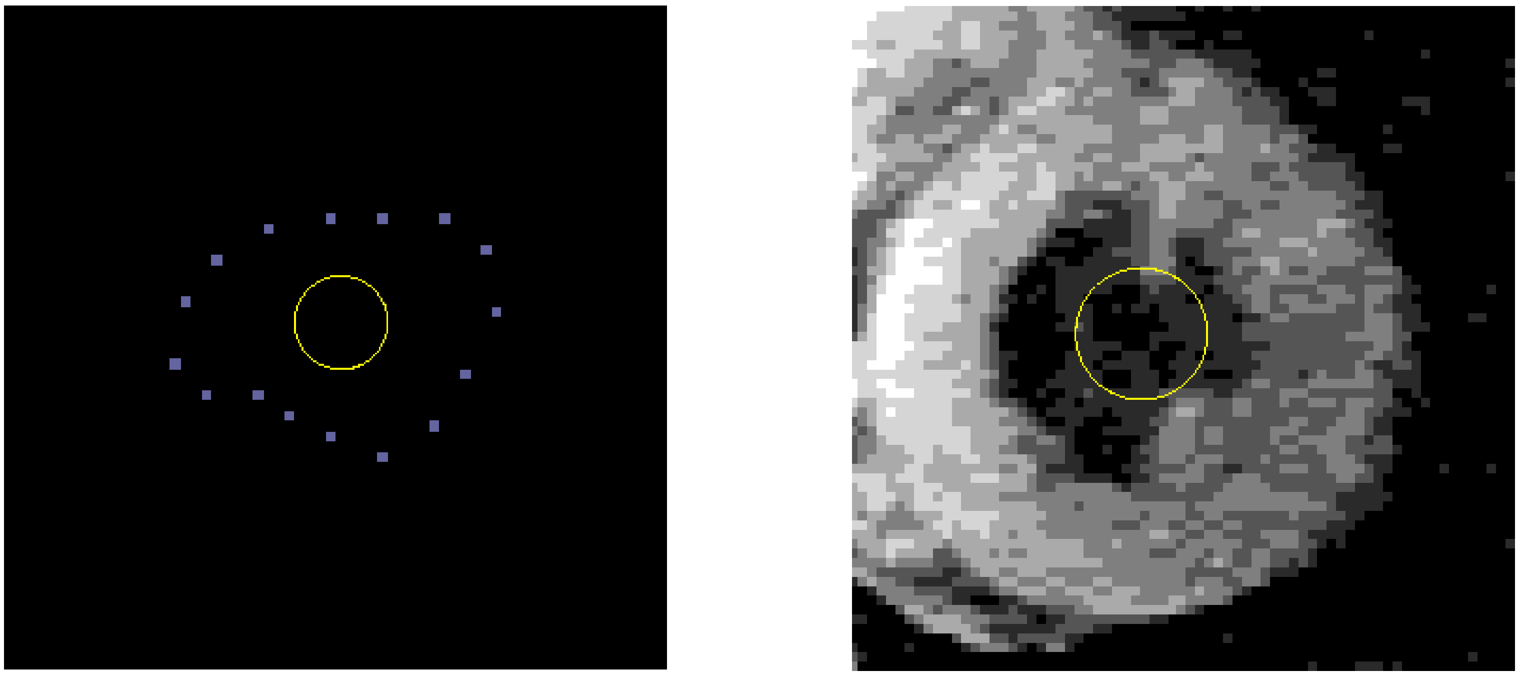
Simple Snake



With Gradient Vector Field

Xu and Prince 98

Gradient Vector Fields



Xu and Prince 98

User-Visible Options

- Initialization: user-specified, automatic
- Curve properties: continuity, smoothness
- Image features: intensity, edges, corners, …
- Other forces: hard constraints, springs, attractors, repulsors, …
- Scale: local, multiresolution, global