ILP Modulo Data

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Northeastern University

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October 23, 2014

Id	Cap	Sector
1 (EMC)	large	tech
2 (FII)	medium	financials
3 (AKR)	small	retail
…	…	…

`stocks`

Id	Cap	Sector
1 (EMC)	large	tech
2 (FII)	medium	financials
3 (AKR)	small	retail
...	...	...

`stocks`

Id	Diff
1	128
2	117
3	89
...	...

`quotes`

Id	Cap	Sector
1 (EMC)	large	tech
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3 (AKR)	small	retail
…	…	…

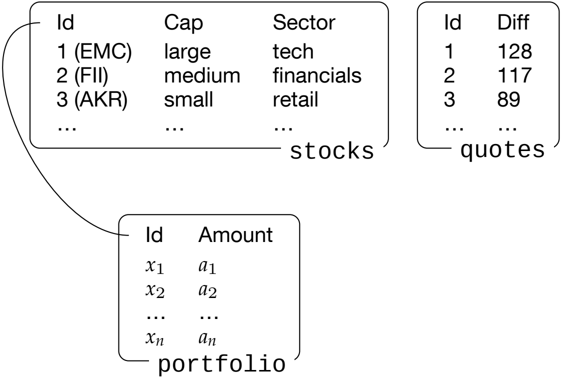
`stocks`

Id	Diff
1	128
2	117
3	89
…	…

`quotes`

Id	Amount
x_1	a_1
x_2	a_2
…	…
x_n	a_n

`portfolio`


Id Cap Sector
1 (EMC) large tech
2 (FII) medium financials
3 (AKR) small retail
... ...
stocks
Id Diff
1 128
2 117
3 89
... ...
quotes
Id Amount
x_1 a_1
x_2 a_2
... ...
x_n a_n
portfolio

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`stocks`

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%

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...	...	...

`stocks`

Id	Diff
1	128
2	117
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`quotes`

%

maximize

$\Sigma_{1 \leq i \leq n} a_i \cdot d_i$

subject to

$(x_i, c_i, s_i) \in \texttt{stocks}, \qquad 1 \leq i \leq n$

$(x_i, d_i) \in \texttt{quotes}, \qquad 1 \leq i \leq n$

Id	Cap	Sector
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$x_i \neq x_j, \qquad 1 \le i < j \le n$

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$(x_i, c_i, s_i) \in \texttt{stocks}$,	$1 \le i \le n$
$(x_i, d_i) \in \texttt{quotes}$,	$1 \le i \le n$
$x_i \neq x_j$,	$1 \le i < j \le n$
$\sum_{\{i \mid 1 \le i \le n, s_i = s\}} a_i \le \sum_{1 \le i \le n} a_i / 3$,	for every sector s

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3 (AKR)	small	retail
...	...	...

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$\sum_{\{i \ \mid \ 1 \leq i \leq n, s_i = s\}} a_i \leq \sum_{1 \leq i \leq n} a_i / 3,$ $\quad$ for every sector s

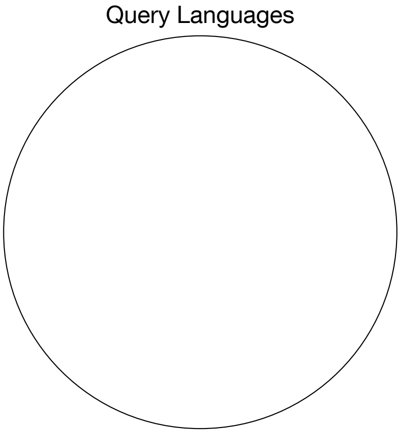
$\sum_{\{i \ \mid \ 1 \leq i \leq n, c_i = \text{small}\}} a_i \leq \sum_{1 \leq i \leq n} a_i / 4$

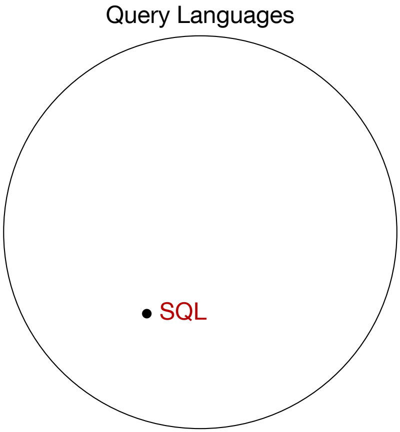
Motivation

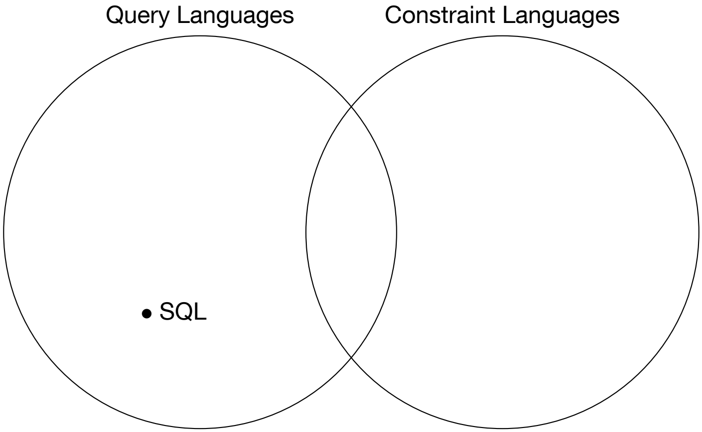
The Logic Δ

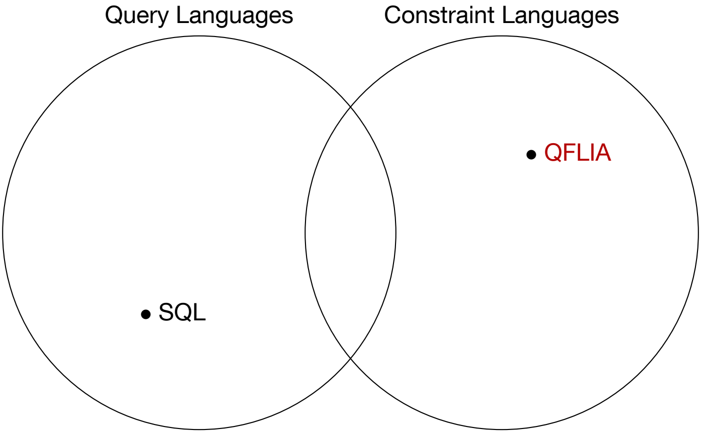
Efficient Solving

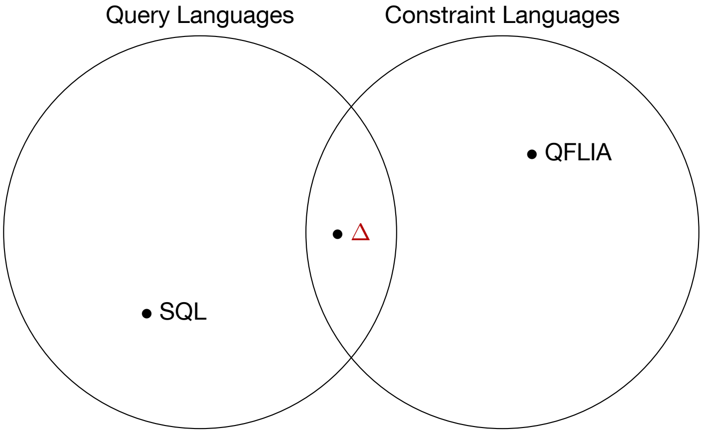
Experimental Evaluation


Query Languages


Query Languages
SQL


Query Languages
Constraint Languages
SQL


Query Languages
Constraint Languages
QFLIA
SQL


Query Languages
Constraint Languages
QFLIA
Δ
SQL

$\Delta ::=$ QFLIA % Relational Operators

Input Table

$$\texttt{portfolio} = \boxed{\begin{array}{l} \{(1, (x_1, a_1)), \\ \quad (2, (x_2, a_2)), \\ \quad \ldots, \\ \quad (n, (x_n, a_n))\} \end{array}}$$

Input Table

unique ID

$$\texttt{portfolio} = \boxed{\begin{aligned}\{&(1, (x_1, a_1)),\\ &(2, (x_2, a_2)),\\ &\ldots,\\ &(n, (x_n, a_n))\}\end{aligned}}$$

Input Table

stock ID

unique ID

$$\texttt{portfolio} = \boxed{\begin{array}{l} \{(1, (x_1, a_1)), \\ \ (2, (x_2, a_2)), \\ \ \ldots, \\ \ (n, (x_n, a_n))\} \end{array}}$$

Input Table

amount

stock ID

unique ID

$$\texttt{portfolio} = \boxed{\begin{array}{l} \{(1, (x_1, a_1)), \\ \ (2, (x_2, a_2)), \\ \ \ldots, \\ \ (n, (x_n, a_n))\} \end{array}}$$

Cross Product

$$\texttt{portfolio} \times \texttt{portfolio}$$

σ

$$\left\langle \sigma\, p : \begin{array}{l} \mathsf{first}(\,\mathsf{left}(p)) \neq \mathsf{first}(\,\mathsf{right}(p)) \wedge \\ \mathsf{second}(\,\mathsf{left}(p)) = \mathsf{second}(\,\mathsf{right}(p)) \end{array} : \boxed{\texttt{portfolio}} \times \boxed{\texttt{portfolio}} \right\rangle$$

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i.e.,

$$\begin{aligned} \{(q,r) \mid\ & q = (q_1,(q_2,q_3)) \in \texttt{portfolio}, \\ & r = (r_1,(r_2,r_3)) \in \texttt{portfolio}, \\ & q_1 \neq r_1, \\ & q_2 = r_2\} \end{aligned}$$

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Constraints

$$\neg\,\exists \boxed{\begin{array}{ll} \langle \sigma\, p : & \textsf{first}(\textsf{left}(p)) \neq \textsf{first}(\textsf{right}(p)) \wedge \\ & \textsf{second}(\textsf{left}(p)) = \textsf{second}(\textsf{right}(p)) \\ \quad : & \boxed{\texttt{portfolio}} \times \boxed{\texttt{portfolio}} \qquad \rangle \end{array}}$$

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$\exists\, D$: D is not empty

Constraints

$$\neg \exists \boxed{\langle \sigma\, p : \begin{array}{l} \mathsf{first}(\mathsf{left}(p)) \neq \mathsf{first}(\mathsf{right}(p)) \wedge \\ \mathsf{second}(\mathsf{left}(p)) = \mathsf{second}(\mathsf{right}(p)) \end{array} : \boxed{\texttt{portfolio}} \times \boxed{\texttt{portfolio}} \rangle}$$

$$\neg \bigvee_{\substack{(q_1,(q_2,q_3)) \in \texttt{portfolio}, \\ (r_1,(r_2,r_3)) \in \texttt{portfolio}}} [q_1 \neq r_1 \wedge q_2 = r_2]$$

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$O(n^2)$ entries (general case: $O(n^n)$)

Motivation

The Logic Δ

Efficient Solving

Experimental Evaluation

$$F \in \exists\Delta$$

iff

$$\begin{cases} F \in \Delta \\ \exists \text{ only appears with positive polarity in } F \end{cases}$$

$$\exists\ \boxed{\begin{array}{ll}\langle \sigma\ p : & \mathsf{first}(\mathsf{left}(p)) \neq \mathsf{first}(\mathsf{right}(p)) \wedge \\ & \mathsf{second}(\mathsf{left}(p)) = \mathsf{second}(\mathsf{right}(p)) \\ \quad : & \boxed{\texttt{portfolio}} \times \boxed{\texttt{portfolio}} \qquad \rangle\end{array}}$$

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$$\begin{aligned} & p \in [\texttt{portfolio} \times \texttt{portfolio}] \\ \wedge \quad & \mathsf{first}(\mathsf{left}(p)) \neq \mathsf{first}(\mathsf{right}(p)) \\ \wedge \quad & \mathsf{second}(\mathsf{left}(p)) = \mathsf{second}(\mathsf{right}(p)) \end{aligned}$$

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$$p \mapsto (q, r)$$

$$\begin{array}{ll} & q \in \texttt{portfolio} \\ \wedge & r \in \texttt{portfolio} \\ \wedge & \mathsf{first}(\mathsf{left}((q, r))) \neq \mathsf{first}(\mathsf{right}((q, r))) \\ \wedge & \mathsf{second}(\mathsf{left}((q, r))) = \mathsf{second}(\mathsf{right}((q, r))) \end{array}$$

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$q \mapsto (q_1, (q_2, q_3))$

$r \mapsto (r_1, (r_2, r_3))$

$$\exists \boxed{\begin{array}{ll} \langle \sigma\, p : & \mathsf{first}(\mathsf{left}(p)) \neq \mathsf{first}(\mathsf{right}(p)) \wedge \\ & \mathsf{second}(\mathsf{left}(p)) = \mathsf{second}(\mathsf{right}(p)) \\ \quad : & \boxed{\texttt{portfolio}} \times \boxed{\texttt{portfolio}} \quad \rangle \end{array}}$$

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$q \mapsto (q_1, (q_2, q_3))$

$r \mapsto (r_1, (r_2, r_3))$

Table Membership

$$(x_1, \ldots, x_k) \quad \in \quad \{(y_{1,1}, \ldots, y_{1,k}), \ldots, (y_{l,1}, \ldots, y_{l,k})\}$$

Table Membership

$$\underbrace{(x_1, \ldots, x_k)}_{} \quad \in \quad \underbrace{\{(y_{1,1}, \ldots, y_{1,k}), \ldots, (y_{l,1}, \ldots, y_{l,k})\}}_{\text{table (concrete } \textit{and} \text{ symbolic data)}}$$

ILP $\%$ T [CAV 2013]

$$\overbrace{\text{ILP} \quad\quad \% \quad\quad T}^{\text{stably-infinite}}$$

[CAV 2013]

$$\text{ILP} \quad \% \quad \overbrace{T}^{\text{stably-infinite}} \underset{\text{Data}}{\sim}$$

[CAV 2013]

stably-infinite

$\overbrace{\hphantom{T}}$

ILP $\quad \% \quad T$ [CAV 2013]

$\geq \qquad\qquad \sim$

QFLIA $\qquad\qquad$ Data

$$\overbrace{\qquad}^{\text{stably-infinite}}$$

$$\text{ILP} \quad \% \quad T$$

[CAV 2013]

$$\geq \qquad \sim$$

$$\underbrace{\text{QFLIA} \qquad \text{Data}}_{\exists \Delta}$$

Branching and Propagation Modulo Data

Input

$$\begin{aligned} C(x, y, a, b, \ldots) &\quad \wedge \\ (x, y) \in \{(1,2), (2,3), (3,2), (a,b)\} &\quad \wedge \\ \ldots & \end{aligned}$$

Branching and Propagation Modulo Data

$(x, y) \in$

$$\{(1,2),\ (2,3),\ (3,2),\ (a,b)\}$$

Input

$$\begin{aligned} C(x, y, a, b, \ldots) \quad & \wedge \\ (x, y) \in \{(1,2), (2,3), (3,2), (a,b)\} \quad & \wedge \\ \ldots \quad & \end{aligned}$$

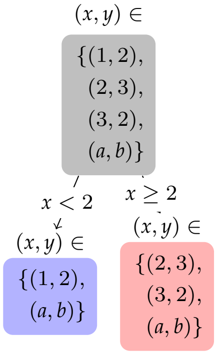
$\{C\}$

Branching and Propagation Modulo Data

Input

$$
\begin{aligned}
C(x, y, a, b, \ldots) & \quad \wedge \\
(x, y) \in \{(1,2), (2,3), (3,2), (a,b)\} & \quad \wedge \\
\ldots &
\end{aligned}
$$

$$\{C\} \xrightarrow[\texttt{Branch}]{} \{C \cdot x < 2, C \cdot x \geq 2\}$$

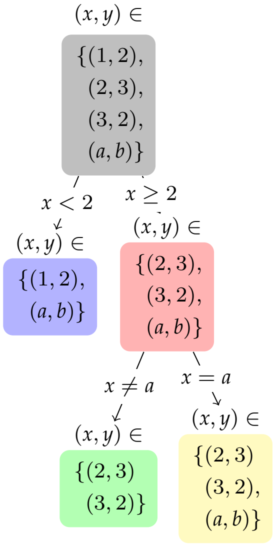


Branching and Propagation Modulo Data

Input

$$\begin{aligned} C(x, y, a, b, \ldots) &\quad \wedge \\ (x, y) \in \{(1,2), (2,3), (3,2), (a,b)\} &\quad \wedge \\ \ldots & \end{aligned}$$

$$\{C\}$$
$$\xrightarrow{\texttt{Branch}}$$
$$\{C \cdot x < 2, C \cdot x \geq 2\}$$
$$\xrightarrow{\texttt{Branch}}$$
$$\{C \cdot x < 2, C \cdot x \geq 2 \cdot x \neq a, C \cdot x \geq 2 \cdot x = a\}$$



Branching and Propagation Modulo Data

Input

$$\begin{array}{rl} C(x, y, a, b, \ldots) & \wedge \\ (x, y) \in \{(1,2), (2,3), (3,2), (a,b)\} & \wedge \\ \ldots & \end{array}$$

$$\{C\}$$

$$\xrightarrow{\texttt{Branch}}$$

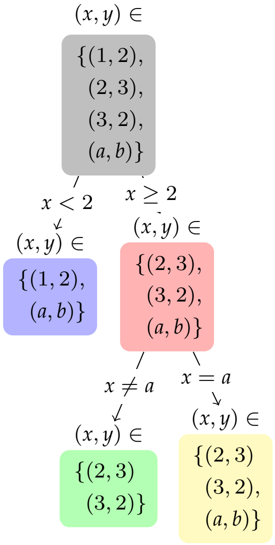
$$\{C \cdot x < 2, C \cdot x \geq 2\}$$

$$\xrightarrow{\texttt{Branch}}$$

$$\{C \cdot x < 2, C \cdot x \geq 2 \cdot x \neq a, C \cdot x \geq 2 \cdot x = a\}$$

$$\xrightarrow{D\texttt{-Learn}}$$

$$\{C \cdot x < 2, C \cdot 2 \leq x \leq 3 \cdot x \neq a, C \cdot x \geq 2 \cdot x = a\}$$



Branching and Propagation Modulo Data

Input

$$
\begin{array}{rl}
C(x, y, a, b, \ldots) & \wedge \\
(x, y) \in \{(1,2), (2,3), (3,2), (a,b)\} & \wedge \\
\ldots &
\end{array}
$$

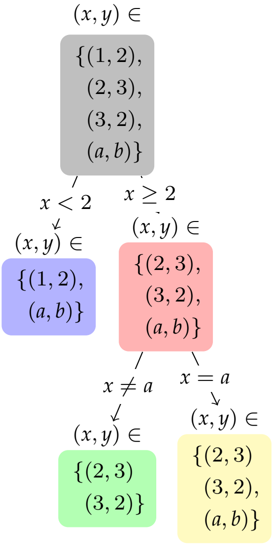
$\{C\}$

$\xrightarrow[\texttt{Branch}]{}$
$\{C \cdot x < 2, C \cdot x \geq 2\}$

$\xrightarrow[\texttt{Branch}]{}$
$\{C \cdot x < 2, C \cdot x \geq 2 \cdot x \neq a, C \cdot x \geq 2 \cdot x = a\}$

$\xrightarrow[D\texttt{-Learn}]{}$
$\{C \cdot x < 2, C \cdot 2 \leq x \leq 3 \cdot x \neq a, C \cdot x \geq 2 \cdot x = a\}$

$\longrightarrow \ldots$



Motivation

The Logic Δ

Efficient Solving

Experimental Evaluation

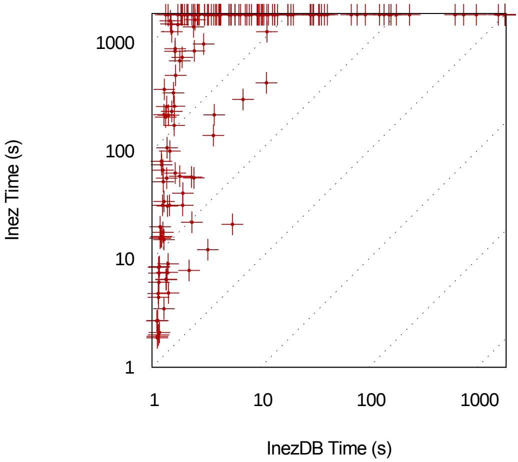
http://scip.zib.de

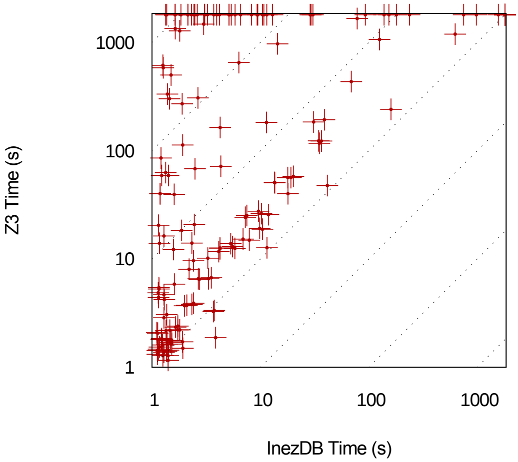
http://caml.inria.fr

https://github.com/vasilisp/inez

Benchmarks

http://www.ccs.neu.edu/home/vpap/fmcad-2014.html


Inez Time (s)
1000
100
10
1
1
10
100
1000
InezDB Time (s)



1000
100
10
1
1
10
100
1000
Z3 Time (s)
InezDB Time (s)

The End