Accurate ICP-based Floating-Point Reasoning

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Chair of Computer Architecture

FMCAD 2016

Context of this Work

Context of this Work (1)

Cooperation with Industrypartners (AVACS Transfer Project 1):
“Accurate Dead Code Detection in Embedded C Code by Arithmetic Constraint Solving”

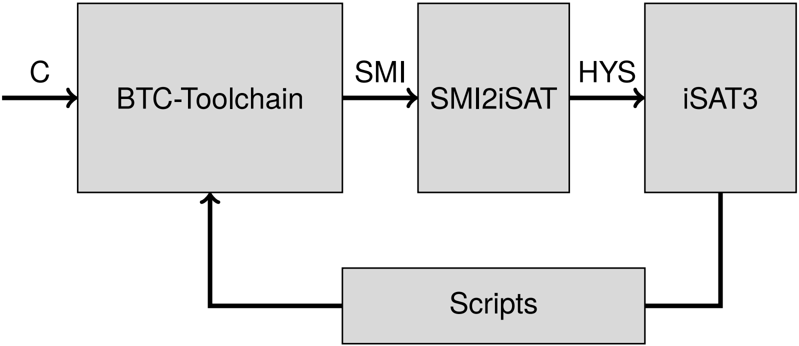
University of Oldenburg:
- Ahmed Mahdi
- Martin Fränzle

University of Freiburg:
- Felix Neubauer
- Karsten Scheibler
- Bernd Becker

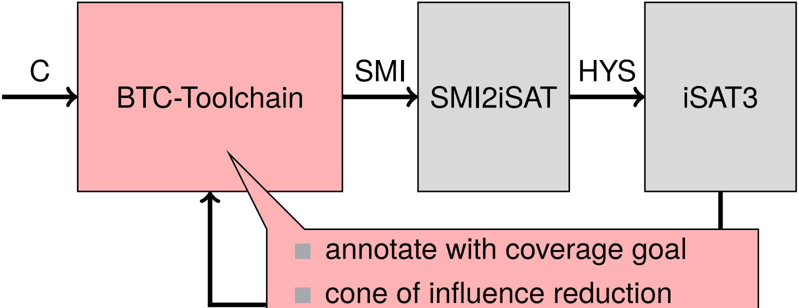
BTC-ES (Oldenburg):
- Tino Teige
- Tom Bienmüller

SICK (Waldkirch):
- Detlef Fehrer

Context of this Work (2)

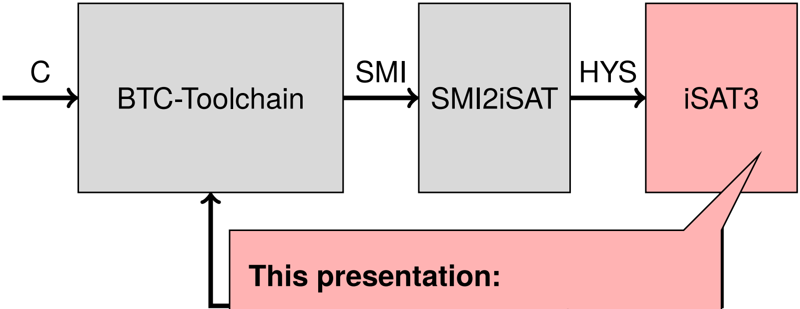


Context of this Work (3)



- annotate with coverage goal
- cone of influence reduction
- resolve loops and functions
- flatten data types
- static single assignment form
- **BMC problem**

Context of this Work (4)



This presentation:

- accurate reasoning for floating-point arithmetic
- support for bitwise integer operations

How does iSAT3 Work

iSAT3 = CDCL + ICP

CDCL: conflict-driven clause learning
ICP: interval constaint propagation

CDCL (1)

CNF

$(\neg b \vee \neg h_1) \wedge$
$(c \vee \neg h_1) \wedge$
$(b \vee \neg c \vee h_1) \wedge$
$(a \vee h_1 \vee \neg h_2) \wedge$
$(a \vee \neg h_1 \vee h_2) \wedge$
$(\neg a \vee h_1 \vee h_2) \wedge$
$(\neg a \vee \neg h_1 \vee \neg h_2) \wedge$
(h_2)

CDCL (1)

CNF		Boolean Formula
$(\neg b \vee \neg h_1) \wedge$ $(c \vee \neg h_1) \wedge$ $(b \vee \neg c \vee h_1) \wedge$ $(a \vee h_1 \vee \neg h_2) \wedge$ $(a \vee \neg h_1 \vee h_2) \wedge$ $(\neg a \vee h_1 \vee h_2) \wedge$ $(\neg a \vee \neg h_1 \vee \neg h_2) \wedge$ (h_2)	⟵ Tseitin-Transformation $(h_1 \Leftrightarrow (\neg b \wedge c))$ $(h_2 \Leftrightarrow (a \oplus h_1))$	$(a \oplus (\neg b \wedge c))$

CDCL (2)

CNF

$(\neg b \vee \neg h_1) \wedge$
$(c \vee \neg h_1) \wedge$
$(b \vee \neg c \vee h_1) \wedge$
$(a \vee h_1 \vee \neg h_2) \wedge$
$(a \vee \neg h_1 \vee h_2) \wedge$
$(\neg a \vee h_1 \vee h_2) \wedge$
$(\neg a \vee \neg h_1 \vee \neg h_2) \wedge$
(h_2)

CDCL (3)

CNF

$(\neg b \vee \neg h_1) \wedge$
$(c \vee \neg h_1) \wedge$
$(b \vee \neg c \vee h_1) \wedge$
$(a \vee h_1 \vee \neg h_2) \wedge$
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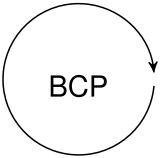
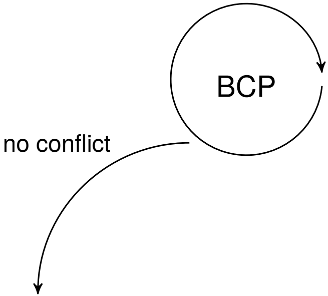
$(\neg a \vee h_1 \vee h_2) \wedge$
$(\neg a \vee \neg h_1 \vee \neg h_2) \wedge$
(h_2)



CDCL (4)

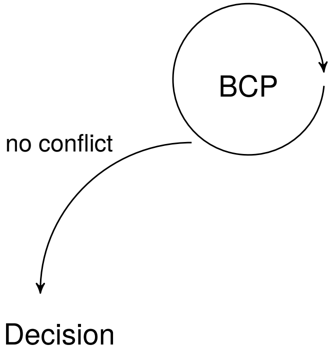
CNF

$(\neg b \vee \neg h_1) \wedge$
$(c \vee \neg h_1) \wedge$
$(b \vee \neg c \vee h_1) \wedge$
$(a \vee h_1 \vee \neg h_2) \wedge$
$(a \vee \neg h_1 \vee h_2) \wedge$
$(\neg a \vee h_1 \vee h_2) \wedge$
$(\neg a \vee \neg h_1 \vee \neg h_2) \wedge$
(h_2)



CNF

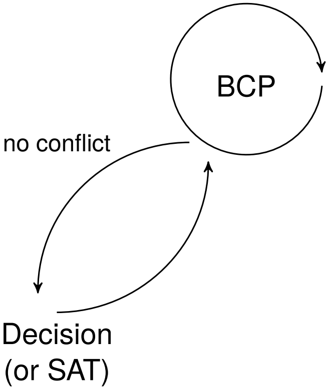
$(\neg b \vee \neg h_1) \wedge$
$(c \vee \neg h_1) \wedge$
$(b \vee \neg c \vee h_1) \wedge$
$(a \vee h_1 \vee \neg h_2) \wedge$
$(a \vee \neg h_1 \vee h_2) \wedge$
$(\neg a \vee h_1 \vee h_2) \wedge$
$(\neg a \vee \neg h_1 \vee \neg h_2) \wedge$
(h_2)



CDCL (4)

CNF

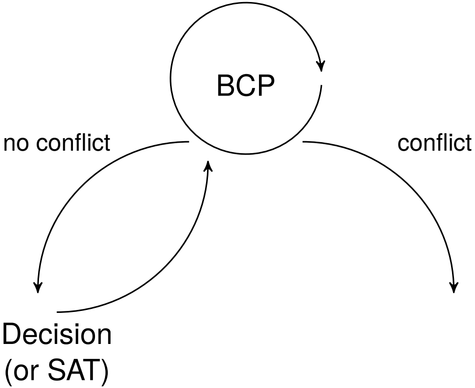
$(\neg b \vee \neg h_1) \wedge$
$(c \vee \neg h_1) \wedge$
$(b \vee \neg c \vee h_1) \wedge$
$(a \vee h_1 \vee \neg h_2) \wedge$
$(a \vee \neg h_1 \vee h_2) \wedge$
$(\neg a \vee h_1 \vee h_2) \wedge$
$(\neg a \vee \neg h_1 \vee \neg h_2) \wedge$
(h_2)



CDCL (5)

CNF

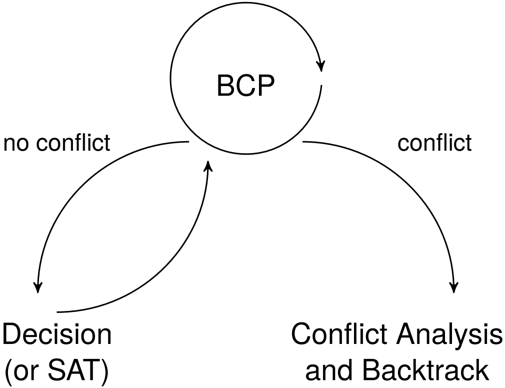
$(\neg b \vee \neg h_1) \wedge$
$(c \vee \neg h_1) \wedge$
$(b \vee \neg c \vee h_1) \wedge$
$(a \vee h_1 \vee \neg h_2) \wedge$
$(a \vee \neg h_1 \vee h_2) \wedge$
$(\neg a \vee h_1 \vee h_2) \wedge$
$(\neg a \vee \neg h_1 \vee \neg h_2) \wedge$
(h_2)



CDCL (5)

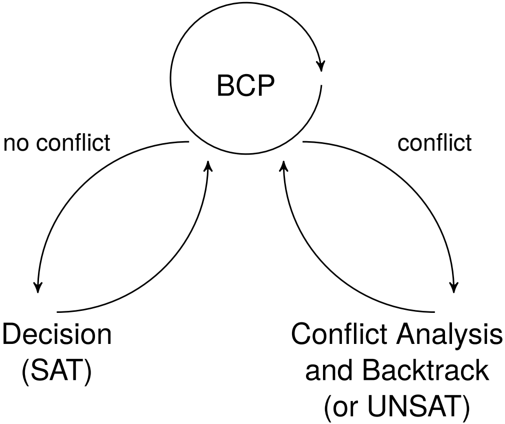
CNF

$(\neg b \vee \neg h_1) \wedge$
$(c \vee \neg h_1) \wedge$
$(b \vee \neg c \vee h_1) \wedge$
$(a \vee h_1 \vee \neg h_2) \wedge$
$(a \vee \neg h_1 \vee h_2) \wedge$
$(\neg a \vee h_1 \vee h_2) \wedge$
$(\neg a \vee \neg h_1 \vee \neg h_2) \wedge$
(h_2)



CNF

$(\neg b \vee \neg h_1) \wedge$
$(c \vee \neg h_1) \wedge$
$(b \vee \neg c \vee h_1) \wedge$
$(a \vee h_1 \vee \neg h_2) \wedge$
$(a \vee \neg h_1 \vee h_2) \wedge$
$(\neg a \vee h_1 \vee h_2) \wedge$
$(\neg a \vee \neg h_1 \vee \neg h_2) \wedge$
(h_2)



iSAT3 (1)

PC + MAP + CNF

$$
\begin{aligned}
&(h_1 = y^2) \land \\
&(h_2 = x + h_1) \land \\
&(h_3 \Leftrightarrow (h_2 < 5)) \land \\
&(a \lor h_3 \lor \neg h_4) \land \\
&(a \lor \neg h_3 \lor h_4) \land \\
&(\neg a \lor h_3 \lor h_4) \land \\
&(\neg a \lor \neg h_3 \lor \neg h_4) \land \\
&(h_4)
\end{aligned}
$$

iSAT3 (1)

PC + MAP + CNF

$$(h_1 = y^2) \wedge$$
$$(h_2 = x + h_1) \wedge$$
$$(h_3 \Leftrightarrow (h_2 < 5)) \wedge$$
$$(a \vee h_3 \vee \neg h_4) \wedge$$
$$(a \vee \neg h_3 \vee h_4) \wedge$$
$$(\neg a \vee h_3 \vee h_4) \wedge$$
$$(\neg a \vee \neg h_3 \vee \neg h_4) \wedge$$
$$(h_4)$$

Tseitin-like Transformation

$$(h_1 = y^2)$$
$$(h_2 = x + h_1)$$
$$(h_3 \Leftrightarrow (h_2 < 5))$$
$$(h_4 \Leftrightarrow (a \oplus h_3))$$

SMT Formula

$$(a \oplus (x + y^2 < 5))$$

linear and nonlinear real arithmetic

with transcendental functions

- maintain interval for every real- or integer-valued variable
- PC: primitive constraints: $(h_1 = y^2)$, $(h_2 = x + h_1)$
- MAP: map literals to simple bounds: $(h_3 \Leftrightarrow (h_2 < 5))$

iSAT3 (1)

PC + [partially hidden]

$(h_1 = y$ …
$(h_2 = x$ …
$(h_3 \Leftrightarrow$ …
$(a \vee h$ …
$(a \vee \neg$ …
$(\neg a \vee$ …
$(\neg a \vee$ …
(h_4)

…ula

…$< 5))$

…al arithmetic

…functions

Assignment

Variable	Type	Value
a	bool	false
x	real	...
y	real	...
h_1	real	...
h_2	real	h_3
h_3	bool simple bound $(h_2 < 5)$	true
h_4	bool	true

- maint… …able
- PC: p…
- MAP: map literals to simple bounds: $(h_3 \Leftrightarrow (h_2 < 5))$

iSAT3 (2)

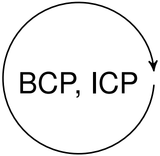
PC + MAP + CNF

$$
\begin{aligned}
&(h_1 = y^2) \wedge \\
&(h_2 = x + h_1) \wedge \\
&(h_3 \Leftrightarrow (h_2 < 5)) \wedge \\
&(a \vee h_3 \vee \neg h_4) \wedge \\
&(a \vee \neg h_3 \vee h_4) \wedge \\
&(\neg a \vee h_3 \vee h_4) \wedge \\
&(\neg a \vee \neg h_3 \vee \neg h_4) \wedge \\
&(h_4)
\end{aligned}
$$

iSAT3 (3)

PC + MAP + CNF

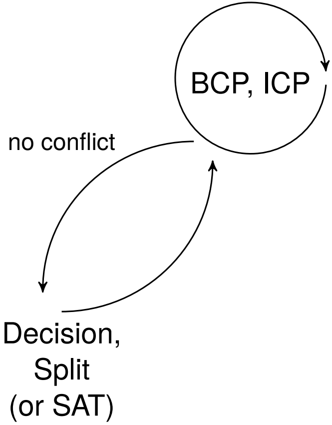
$(h_1 = y^2) \wedge$
$(h_2 = x + h_1) \wedge$
$(h_3 \Leftrightarrow (h_2 < 5)) \wedge$
$(a \vee h_3 \vee \neg h_4) \wedge$
$(a \vee \neg h_3 \vee h_4) \wedge$
$(\neg a \vee h_3 \vee h_4) \wedge$
$(\neg a \vee \neg h_3 \vee \neg h_4) \wedge$
(h_4)



iSAT3 (4)

PC + MAP + CNF

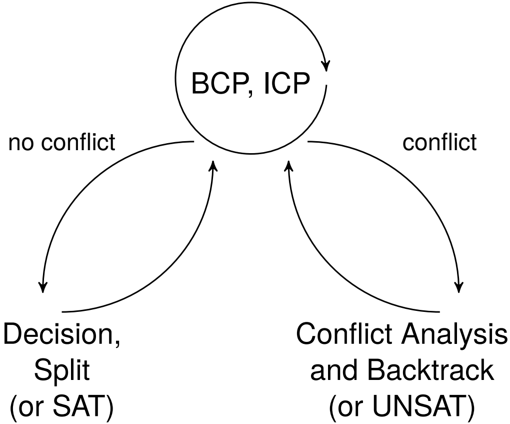
$$
\begin{aligned}
&(h_1 = y^2) \wedge \\
&(h_2 = x + h_1) \wedge \\
&(h_3 \Leftrightarrow (h_2 < 5)) \wedge \\
&(a \vee h_3 \vee \neg h_4) \wedge \\
&(a \vee \neg h_3 \vee h_4) \wedge \\
&(\neg a \vee h_3 \vee h_4) \wedge \\
&(\neg a \vee \neg h_3 \vee \neg h_4) \wedge \\
&(h_4)
\end{aligned}
$$



iSAT3 (5)

PC + MAP + CNF

$(h_1 = y^2) \wedge$
$(h_2 = x + h_1) \wedge$
$(h_3 \Leftrightarrow (h_2 < 5)) \wedge$
$(a \vee h_3 \vee \neg h_4) \wedge$
$(a \vee \neg h_3 \vee h_4) \wedge$
$(\neg a \vee h_3 \vee h_4) \wedge$
$(\neg a \vee \neg h_3 \vee \neg h_4) \wedge$
(h_4)



iSAT3 (6)

	SAT	iSAT3
Deductions	• BCP for clauses	• BCP for clauses evaluate simple bound literals $\rightsquigarrow$ implication clauses • ICP for PC $\rightsquigarrow$ arithmetic clauses
Decisions	• decide literals	• decide literals • generate new simple bound literals and decide them
Conflict Analyses	• traverse implication graph (1UIP) $\rightsquigarrow$ conflict clauses	• traverse implication graph (1UIP) $\rightsquigarrow$ conflict clauses

iSAT3 (6)

	SAT	iSAT3
Deductions	• BCP for clauses	• BCP for clauses evaluate simple bound literals ⇝ **implication clauses** • ICP for PC ⇝ **arithmetic clauses**
Decisions	• decide literals	• decide literals • generate new simple bound literals and decide them
Conflict Analyses	• traverse implication graph (1UIP) ⇝ conflict clauses	• traverse implication graph (1UIP) ⇝ conflict clauses

iSAT3 (7)

Implication Clauses:

- unassigned simple bound literals are evaluated lazily
- therefore implications possible: $(h_2 < 5) \Rightarrow (h_2 < 7)$

Arithmetic Clauses:

- result of interval constraint propagation (ICP)
- e.g. $h_2 = x + h_1$: $((x \leq 3) \wedge (h_1 < 2)) \Rightarrow (h_2 < 5)$
- redirect, e.g. $x = h_2 - h_1$: $((h_2 < 10) \wedge (h_1 \geq 1)) \Rightarrow (x < 9)$
- using floating-point numbers for interval bounds
- always round outwards for safe enclosing intervals
- generate new simple bound literals

iSAT3 Summarized

iSAT3 = CDCL + ICP, **goes beyond CDCL(T)**:

Boolean abstraction contains	
CDCL(T)	iSAT3
combinations of truth values of the theory atoms	interval bounds of theory variables and sub-expressions

iSAT3 Summarized

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Boolean abstraction contains	
CDCL(T)	iSAT3
combinations of truth values of the theory atoms	interval bounds of theory variables and sub-ex ssions

iSAT3 is the 3rd implementation of the iSAT algorithm. Abstract CDCL with interval abstraction has similarities to the iSAT algorithm

iSAT algorithm: *"Efficient Solving of Large Non-linear Arithmetic Constraint Systems with Complex Boolean Structure"*, JSAT 2007

Abstract CDCL: *"Deciding Floating-Point Logic with Systematic Abstraction"*, FMCAD 2012

iSAT3 Summarized

iSAT3 = CDCL + ICP, **goes beyond CDCL(T)**:

Boolean abstraction contains	
CDCL(T)	iSAT3
combinations of truth values of the theory atoms	interval bounds of theory variables and sub-expressions

1. **new arithmetic operations $\rightsquigarrow$ add ICP-contractors**
2. **need to adapt Boolean abstraction for floating-point**

Accurate Reasoning for Floating-Point Arithmetic

Accurate Reasoning for FP (1)

UNI FREIBURG

IEEE-754 Specification (float, 32 bits)

Bitpos $\rightarrow$ 31	30 ... 23	22 ... 0
sign	exponent	fraction / mantissa

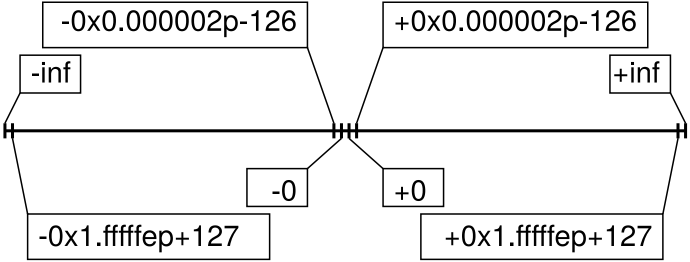
1. normal numbers:
   - mantissa bitpos 23 assumed to be 1
   - exponent $1 \rightsquigarrow -126 \quad \dots \quad 254 \rightsquigarrow +127$
   - sign $0 \rightsquigarrow$ positive $\quad 1 \rightsquigarrow$ negative
2. special numbers:
   - signed zeros (-0, $+0$)
   - $-\infty, +\infty$ (-inf, +inf)
   - subnormal numbers (leading zeros in mantissa)
   - not a number (NaN)
3. rounding modes (up, down, **nearest**)

Accurate Reasoning for FP (2)

32 bit floating-point values and their ordering

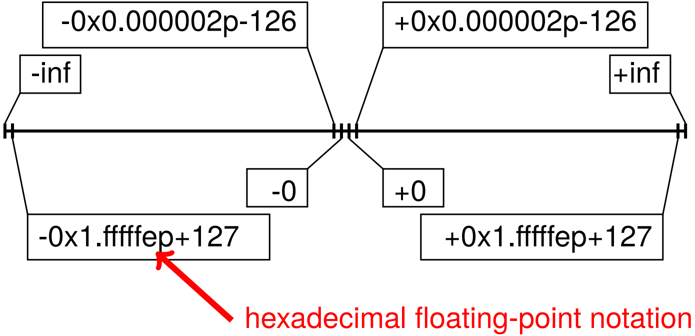
Accurate Reasoning for FP (2)

32 bit floating-point values and their ordering



Accurate Reasoning for FP (2)

32 bit floating-point values and their ordering



Accurate Reasoning for FP (3)

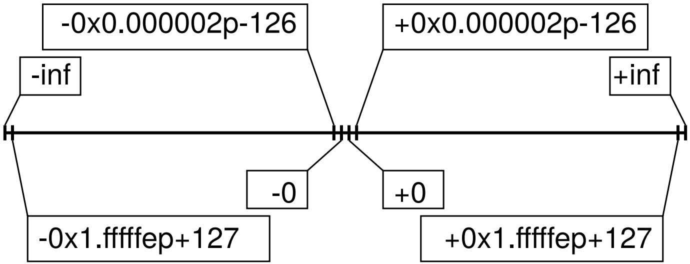
simple bound ordering:

-inf $<$ -0x1.fffffep+127 $< \ldots$
$\ldots <$ -0x0.000002p-126 $<$ -0 $<$ +0 $<$ +0x0.000002p-126 $< \ldots$
$\ldots <$ +0x1.fffffep+127 $<$ +inf

- no strict bounds needed:
  reals: $(x \leq 5) \Leftrightarrow \neg(x > 5)$
  floating-point: $(x \leq$-0x0.000002p-126$) \Leftrightarrow \neg(x \geq$-0$)$
- floating-point comparison operators and signed zeros:
  - `(x <= 0)` $\rightsquigarrow (x \leq +0)$
  - `(x >= 0)` $\rightsquigarrow (x \geq -0)$
  - `(x == 0)` $\rightsquigarrow (x \geq -0) \wedge (x \leq +0)$

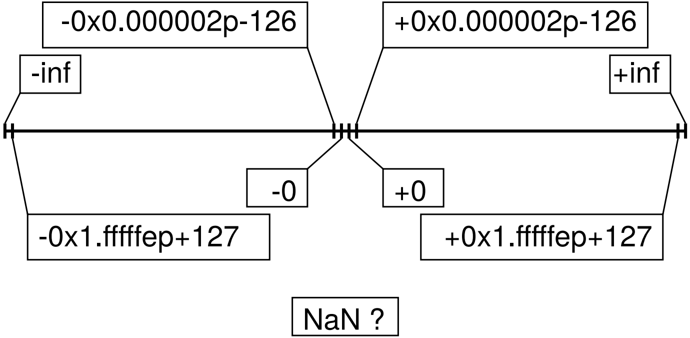
Accurate Reasoning for FP (3)

32 bit floating-point values and their ordering



Accurate Reasoning for FP (3)

32 bit floating-point values and their ordering



Accurate Reasoning for FP (4)

```
#include <math.h>
#include <stdio.h>
int main(void) {
    double a = sqrt(-1);
    printf("%1.2f\n", a);

    if (a < 0) printf("if\n");
    else printf("else\n");

    if (a >= 0) printf("if\n");
    else printf("else\n");
    return (0);
    }
```

```
-nan
else
else
```

Accurate Reasoning for FP (4)

```
#include <math.h>
#include <stdio.h>
int main(void) {
    double a = sqrt(-1);
    printf("%1.2f\n", a);

    if (a <= 0) printf("if\n");
    else printf("else\n");

    if (a > 0) printf("if\n");
    else printf("else\n");
    return (0);
    }
```

```
-nan
else
else
```

Accurate Reasoning for FP (4)

```
#include <math.h>
#include <stdio.h>
int main(void) {
    double a = sqrt(-1);
    printf("%1.2f\n", a);

    if (a == 0) printf("if\n");
    else printf("else\n");

    if (a != 0) printf("if\n");
    else printf("else\n");
    return (0);
    }
```

```
-nan
else
if
```

Accurate Reasoning for FP (5)

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Accurate Reasoning for FP (5)

- NaN incomparable against all other values:
  $(x \sim NaN)$, $\sim \in \{<, \leq, =, \geq, >\}$ is always false

- adapt Boolean encoding: special literal x_{NaN}

x_{NaN}	x is NaN
$\neg x_{NaN}$	x is determined by simple bound literals $(x \leq$-inf$) \ldots (x \leq$-0$) \ldots$

Accurate Reasoning for FP (5)

- implication clauses:
  $(\neg x_{NaN} \wedge (x \leq 5)) \Rightarrow (x \leq 7)$

- arithmetic clauses: $h = x + y$
  $(\neg x_{NaN} \wedge \neg y_{NaN} \wedge \neg h_{NaN} \wedge (x \leq 3) \wedge (y \leq 2)) \Rightarrow (h \leq 5)$

- not shown here, but x_{NaN} also relevant during Tseitin-like transformation

- besides $<, \leq, =, \geq, >$ operators, further operator to mimic behaviour of assignments: `x = y` vs. `x == y`

Accurate Reasoning for FP (6)

New ICP-Contractors for $+, -, *, /$ (round-to-nearest):

1. NaN cases: handled outside with separate clauses
2. forward deduction: execute operation with round-to-nearest
3. backward deduction: only redirecting the primitive constraint is not enough

Accurate Reasoning for FP (6)

New ICP-Contractors for $+, -, *, /$ (round-to-nearest):

1. NaN cases: handled outside with separate clauses
2. forward deduction: execute operation with round-to-nearest
3. backward deduction: only redirecting the primitive constraint is not enough

ICP-contractors called when NaN-literals of operands false (otherwise the created arithmetic clauses not unit)

Accurate Reasoning for FP (6)

1. Separate clauses for primitive constraint ($h = x + y$):

x or y is NaN	$\Rightarrow$	h is NaN
x and y are infinities with opposite signs	$\Rightarrow$	h is NaN
x and y are not NaN and x is never -inf or +inf	$\Rightarrow$	h is not NaN
x and y are not NaN and y is never -inf or +inf	$\Rightarrow$	h is not NaN
x and y are not NaN and x and y are never -inf	$\Rightarrow$	h is not NaN
x and y are not NaN and x and y are never +inf	$\Rightarrow$	h is not NaN

$$(\neg x_{NaN} \vee h_{NaN}) \wedge (\neg y_{NaN} \vee h_{NaN}) \wedge$$
$$(x_{NaN} \vee y_{NaN} \vee \neg(x \leq \text{-inf}) \vee \neg(y \geq \text{+inf}) \vee h_{NaN}) \wedge$$
$$(x_{NaN} \vee y_{NaN} \vee \neg(x \geq \text{+inf}) \vee \neg(y \leq \text{-inf}) \vee h_{NaN}) \wedge$$
$$(x_{NaN} \vee y_{NaN} \vee (x \leq \text{-inf}) \vee (x \geq \text{+inf}) \vee \neg h_{NaN}) \wedge$$
$$(x_{NaN} \vee y_{NaN} \vee (y \leq \text{-inf}) \vee (y \geq \text{+inf}) \vee \neg h_{NaN}) \wedge$$
$$(x_{NaN} \vee y_{NaN} \vee (x \leq \text{-inf}) \vee (y \leq \text{-inf}) \vee \neg h_{NaN}) \wedge$$
$$(x_{NaN} \vee y_{NaN} \vee (x \geq \text{+inf}) \vee (y \geq \text{+inf}) \vee \neg h_{NaN})$$

Accurate Reasoning for FP (6)

2 Forward deduction primitive constraint ($h = x + y$):

$h \in$ [-inf, +inf],
$x \in$ [0x1.1p+100, 0x1.1p+100],
$y \in$ [-0x1.1p+11, -0x1.1p+10]

$h_{lb} = x_{lb} + y_{lb}$ =0x1.1p+100 + -0x1.1p+11=0x1.1p+100
$h_{ub} = x_{ub} + y_{ub}$ =0x1.1p+100 + -0x1.1p+10=0x1.1p+100

apply operation with round-to-nearest

Accurate Reasoning for FP (6)

3 Backward deduction primitive constraint ($h = x + y$):

$h \in$ [0x1.1p+100, 0x1.1p+100],
$x \in$ [0x1.1p+100, 0x1.1p+100],
$y \in$ [-0x1.1p+11, -0x1.1p+10]

$$\begin{array}{lllllll} y_{lb} & = & h_{lb} - x_{ub} & = & \text{0x1.1p+100} - \text{0x1.1p+100} & = & 0 \\ y_{ub} & = & h_{ub} - x_{lb} & = & \text{0x1.1p+100} - \text{0x1.1p+100} & = & 0 \end{array}$$

[-0x1.1p+11, -0x1.1p+10] $\cap [0,0] = \emptyset$
simply redirecting and rounding outward is **WRONG!**

Accurate Reasoning for FP (6)

3 Backward deduction primitive constraint ($h = x + y$):

$h \in$ [0x1.1p+100, 0x1.1p+100],
$x \in$ [0x1.1p+100, 0x1.1p+100],
$y \in$ [-0x1.1p+11, -0x1.1p+10]

$$
\begin{array}{lllll}
y_{lb} & = & h_{lb} - x_{ub} & = & \mathit{prev}(\texttt{0x1.1p+100}) - \mathit{next}(\texttt{0x1.1p+100}) \\
 & & & = & \texttt{0x1.0ffffep+100 - 0x1.100002p+100} \\
 & & & = & \texttt{-0x1.000000p+78} \\
y_{ub} & = & h_{ub} - x_{lb} & = & \mathit{next}(\texttt{0x1.1p+100}) - \mathit{prev}(\texttt{0x1.1p+100}) \\
 & & & = & \texttt{0x1.100002p+100 - 0x1.0ffffep+100} \\
 & & & = & \texttt{0x1.000000p+78}
\end{array}
$$

Accurate Reasoning for FP Summarized

- floating-point arithmetic contains special values
- ordering possible, except NaN
- unordered NaN $\rightsquigarrow$ adapted Boolean encoding
  - implication clauses
  - arithmetic clauses
- new ICP-contractors for floating-point operations
  - NaN-cases handled with BCP
  - outward rounding not enough in backward deduction

ICP-Contractors for Bitwise Integer Operations

ICP-Contractors for Bitwise Operations (1)

- operating on intervals
- a bit-pattern can be interpreted as signed or unsigned

	0001 0001	1000 0001
`signed char`	17	-127
`unsigned char`	17	129

- need to know bitwidth and signedness of each operation
- S_NOT(*arg*,*bitwidth*), U_NOT(*arg*,*bitwidth*)
- S_AND(*arg*1,*arg*2,*bitwidth*), U_AND(*arg*1,*arg*2,*bitwidth*)
- S_OR(*arg*1,*arg*2,*bitwidth*), U_OR(*arg*1,*arg*2,*bitwidth*)
- S_XOR(*arg*1,*arg*2,*bitwidth*), U_XOR(*arg*1,*arg*2,*bitwidth*)
- S_CAST(*arg*,*bitwidth*), U_CAST(*arg*,*bitwidth*)

ICP-Contractors for Bitwise Operations (2)

- $(h = x + y)$, $x \in [1,7]$, $y \in [1,8]$:
  $$\begin{array}{lllll} h_{lb} & = & x_{lb} + y_{lb} & = & 1+1 & = & 2 \\ h_{ub} & = & x_{ub} + y_{ub} & = & 7+8 & = & 15 \end{array}$$
  $\rightsquigarrow$ operating on bounds **OK**

- $(h = \text{U\_AND}(x,y,8))$, $x \in [1,7]$, $y \in [1,8]$:
  $$\begin{array}{lllllll} h_{lb} & = & x_{lb} \,\&\, y_{lb} & = & 1 \,\&\, 1 & = & 1 & \quad (1 \,\&\, 2 = 0) \\ h_{ub} & = & x_{ub} \,\&\, y_{ub} & = & 7 \,\&\, 8 & = & 0 & \quad (7 \,\&\, 7 = 7) \end{array}$$
  $\rightsquigarrow$ operating on bounds **WRONG**

ICP-Contractors for Bitwise Operations (3)

UNI FREIBURG

1. use addition, subtraction, minimum and maximum to get safe overapproximations of the lower and upper bounds,
   e.g. ($h = \text{U\_AND}(x,y,8)$), $x \in [1,7]$, $y \in [1,8]$:

   $$h_{ub} \quad = \quad min(x_{ub}, y_{ub}) \quad = \quad min(7,8) \quad = \quad 7$$

2. exploit common bit-prefixes,
   e.g. ($h = \text{U\_AND}(x,y,8)$), $x \in [18,30]$, $y \in [89,92]$:

   $$\begin{array}{lllll} x_{lb} & = & 18 & = & 0001\,0010 \\ x_{ub} & = & 30 & = & 0001\,1110 \\ & & & & 0001 \end{array} \qquad \text{common bit-prefix for values in } x$$

   $$\begin{array}{lllll} y_{lb} & = & 89 & = & 0101\,1001 \\ y_{ub} & = & 92 & = & 0101\,1100 \\ & & & & 0101\,1 \end{array} \qquad \text{common bit-prefix for values in } y$$

   $$\begin{array}{lll} h_{lb} & = & 0001\,0000 \;\&\; 0101\,1000 \;=\; 0001\,0000 \;=\; 16 \qquad \text{trailing bits are 0} \\ h_{ub} & = & 0001\,1111 \;\&\; 0101\,1111 \;=\; 0001\,1111 \;=\; 31 \qquad \text{trailing bits are 1} \end{array}$$

ICP-Contractors for Bitwise Operations (3)

1. use addition, subtraction, minimum and maximum to get safe overapproximations of the lower and upper bounds,
   e.g. ($h = \text{U\_AND}(x, y, 8)$), $x \in [1,7]$, $y \in [1,8]$:

A detailed description of all operations can be found in AVACS Technical Report 116:
"Extending iSAT3 with ICP-Contractors for Bitwise Integer Operations"

$$
\begin{array}{lcccl}
y_{lb} & = & 89 & = & 01011001 \\
y_{ub} & = & 92 & = & 01011100 \\
 & & & & 01011
\end{array}
$$

common bit-prefix for values in y

$$
\begin{array}{lclcl}
h_{lb} & = & 00010000 \,\&\, 01011000 = 00010000 = 16 & & \text{trailing bits are 0} \\
h_{ub} & = & 00011111 \,\&\, 01011111 = 00011111 = 31 & & \text{trailing bits are 1}
\end{array}
$$

Optimizations

Intermediate Point-Splits

- decomposition into PCs might lead to coarser intervals, e.g. $((x+y)-x \leq 7) \rightsquigarrow (h_1 = x+y) \wedge (h_2 = h_1 - x)$
  $x, y \in [0,10] : h_1 \in [0,20], h_2 \in [-10,30] \supset [0,10]$

- tighter intervals if x is point interval

- change decision heuristic, every k-th interval split will assign a point interval ($k = 4$)

- might help to find a solution, **BUT:** detrimental for conflict clauses

Global-ICP (1)

$$\ldots \wedge (a \rightarrow (i_1 - i_2 = 0)) \wedge (i_1 \neq \text{S\_CAST}(\text{ITE}(b, i_2, 0), 32)) \wedge \ldots$$

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with $a = 1$:

$$\ldots \wedge (i_1 - i_2 = 0) \quad \wedge (i_1 \neq \text{S\_CAST}(\text{ITE}(b, i_2, 0), 32)) \wedge \ldots$$

$$\ldots \wedge (i_1 = i_2) \quad \wedge (i_1 \neq \text{S\_CAST}(\text{ITE}(b, i_2, 0), 32)) \wedge \ldots$$

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with $b = 1$:

$$\ldots \wedge (i_1 \neq \text{S\_CAST}(i_1, 32)) \wedge \ldots$$

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with $b = 1$:

$$\ldots \quad \wedge (i_1 \neq \text{S\_CAST}(i_1, 32)) \wedge \ldots$$

with $i_1 \in [0, 2^{31} - 1]$:

$$\ldots \quad \wedge (i_1 \neq i_1) \wedge \ldots$$

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but this symbolic dependency is not visible for ICP

$(h_1 = i_1 - i_2) \wedge$
$(h_2 = \text{ITE}(b, i_2, 0)) \wedge$
$(h_3 = \text{S\_SCAST}(h_2, 32)) \wedge$
$(h_4 = i_1 - h_3)$

just looking at these primitive constraints

Global-ICP (1)

$$\ldots \wedge (a \rightarrow (i_1 - i_2 = 0)) \wedge (i_1 \neq \text{S\_CAST}(\text{ITE}(b, i_2, 0), 32)) \wedge \ldots$$

with $a = 1$:

$$\ldots \wedge (i_1 - i_2 = 0) \qquad \wedge (i_1 \neq \text{S\_CAST}(\text{ITE}(b, i_2, 0), 32)) \wedge \ldots$$

$$\ldots \wedge (i_1 = i_2) \qquad \wedge (i_1 \neq \text{S\_CAST}(\text{ITE}(b, i_2, 0), 32)) \wedge \ldots$$

$$\ldots \qquad \wedge (i_1 \neq \text{S\_CAST}(\text{ITE}(b, i_1, 0), 32)) \wedge \ldots$$

with $b = 1$:

$$\ldots \qquad \wedge (i_1 \neq \text{S\_CAST}(i_1, 32)) \wedge \ldots$$

with $i_1 \in [0, 2^{31} - 1]$:

$$\ldots \qquad \wedge (i_1 \neq i_1) \wedge \ldots$$

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$(h_1 = i_1 - i_2) \wedge$
$(h_2 = \text{ITE}(b, i_2, 0)) \wedge$
$(h_3 = \text{S\_SCAST}(h_2, 32)) \wedge$
$(h_4 = i_1 - h_3)$

just looking at these primitive constraints

ICP with smallest possible bound improvement for i_1:

$$\rightsquigarrow [1, 2^{31} - 1] \rightsquigarrow [2, 2^{31} - 1] \rightsquigarrow [2, 2^{31} - 2] \rightsquigarrow \ldots$$

Global-ICP (2)

- ICP with smallest possible bound improvement for i_1:
  $\rightsquigarrow [1, 2^{31} - 1] \rightsquigarrow [2, 2^{31} - 1] \rightsquigarrow [2, 2^{31} - 2] \rightsquigarrow \ldots$
- more than 64 deductions per variable per decision level:
  1. no further deductions for this variable
  2. analyze implication graph, collect involved primitive constraints (the 4 PCs from previous slide)
- analyze primitive constraints semi-symbolically
- conflicting clause which spans more than one PC, e.g.
  $(b \wedge (h_1 \geq 0) \wedge (h_1 \leq 0) \wedge (i_2 \geq 0) \wedge (i_2 \leq 2^{31} - 1)) \Rightarrow (h_4 \leq 0)$

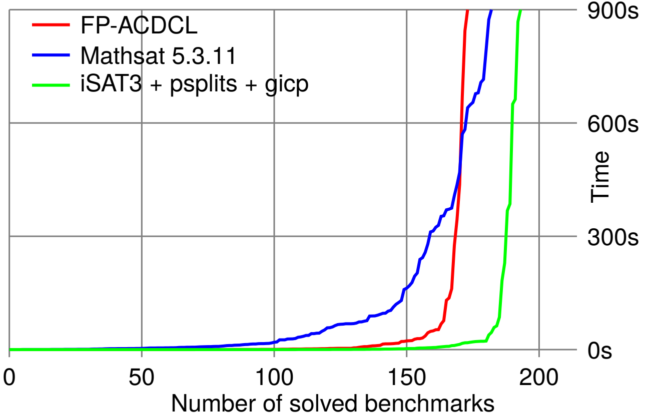
Results

Results (1)

- 213 **pure floating-point** benchmarks from the FP-ACDCL paper
- Comparison between FP-ACDCL (ICP-based), Mathsat (bit-blasting) and iSAT3 (ICP-based)
- Timeout: 900 seconds, Memout: 2 GB

Solver	S+U	SAT	UNSAT	TO	MO
FP-ACDCL	173	97	76	40	0
Mathsat 5.3.11	182	101	81	23	8
iSAT3	164	90	74	47	2
iSAT3 + psplits	186	111	75	27	0
iSAT3 + psplits + gicp	193	111	82	20	0

Results (1)



FP-ACDCL
Mathsat 5.3.11
iSAT3 + psplits + gicp
900s
600s
300s
0s
Time
0
50
100
150
200
Number of solved benchmarks

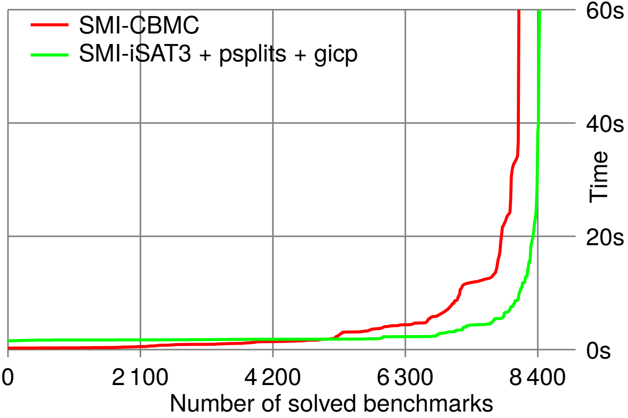
Results (2)

- 8778 BMC benchmarks generated by BTC toolchain, containing **floating-point and bitwise integer operations**
- Comparison between CBMC (bit-blasting, k-induction) and iSAT3 (ICP-based, Craig interpolation)

  both with on-the-fly translation from SMI to their input language
- Timeout: 60 seconds

Solver	S+U	SAT	U51	U∞	TO
SMI-CBMC	8099	7424	44	631	679
SMI-iSAT3	7647	6671	153	823	1131
SMI-iSAT3 + psplits	8169	7192	156	821	609
SMI-iSAT3 + psplits + gicp	8430	7427	172	831	348

Results (2)


SMI-CBMC
SMI-iSAT3 + psplits + gicp
60s
40s
20s
0s
Time
0
2 100
4 200
6 300
8 400
Number of solved benchmarks

Conclusion

Conclusion

- dead-code detection in C programs = accurate floating-point reasoning + bitwise integer operations
- iSAT3: first non-bit-blasting SMT solver supporting the full range of basic data types and operations in C programs
- promising results:
  - outperforms bit-blasting solvers (MathSAT, CBMC)
  - outperforms other ICP-based solver (FP-ACDCL)
- Outlook: also integrate ICP-contractors for floating-point sine, cosine