

CS429: Computer Organization and Architecture

Logic Design

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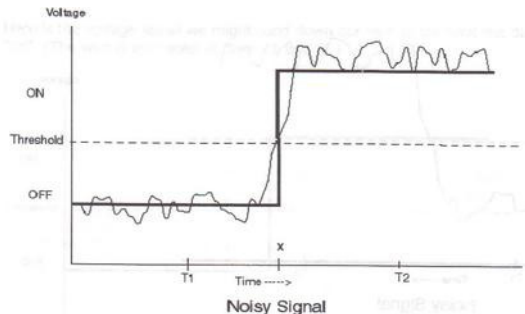
To execute a program we need:

- Communications: getting data from one place to another
- Computation: perform arithmetic or logical operations
- Memory: store the program, variables, results

Everything is expressed in terms of bits (0s and 1s).

- Communication
- Low or high voltage on a wire
- Computation
- Compute boolean functions
- Storage
- Store bits

Digital Signals



- Use voltage thresholds to extract discrete values from a continuous signal: works with light for communication.
- Simplest version: 1-bit signal
 - Either high range (1) or low range (0)
 - With a guard range between them.
- Not strongly affected by noise or low-quality elements; circuits are simple, small and fast.

Truth Tables

And: $A \& B = 1$ when both $A = 1$ and $B = 1$.

0	1		&
0	0		0
0	1		0
1	0		0
1	1		1

Or: $A | B = 1$ when either $A = 1$ or $B = 1$.

0	1		
0	0		0
0	1		1
1	0		1
1	1		1

Not: $\sim A = 1$ when $A = 0$.

0		\sim
0		1
1		0

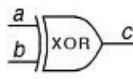
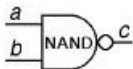
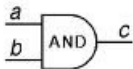
Xor: $A \wedge B = 1$ when either $A = 1$ or $B = 1$, but not both.

0	1		\wedge
0	0		0
0	1		1
1	0		1
1	1		0

Computing with Logic Gates

How are these logic functions actually computed in hardware?

- Logic gates are constructed from transistors.
- The output is a boolean function of inputs.
- The gate responds continuously to changes in input with a small delay.



How many of these do you really need?

It's pretty easy to see that any boolean function can be implemented with AND, OR and NOT. *Why?* We call that a *functionally complete* set of gates.

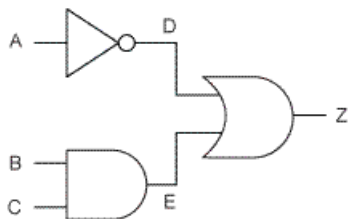
You can get by with fewer gates. *How would you show each of the following?*

- AND and NOT is complete.
- OR and NOT is complete.
- NAND is complete.
- NOR is complete.
- AND alone is not complete.
- OR alone is not complete.

Often circuit designers will restrict themselves to a small subset of gates (e.g., just NAND gates). *Why would they do that?*

A Complex Function

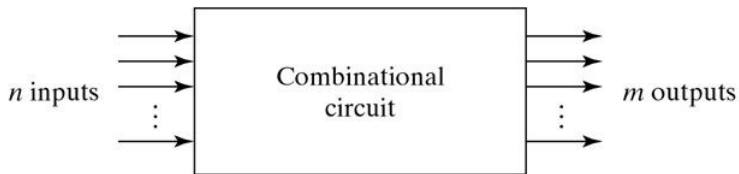
Simple boolean functions are implemented by “logic gates”; more complex functions, by combinations of gates.



A	B	C	Z
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

In C: $Z = !A \ || \ (B \ \&\& \ C);$

Combinational Circuits

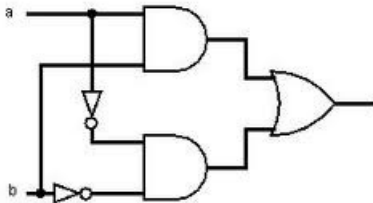


The box contains an acyclic network of logic gates.

- Continuously responds to changes in inputs.
- Outputs become (after a short delay) boolean functions of the inputs.

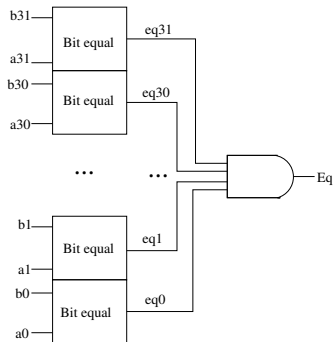
Bit Equality

The following circuit generates a 1 if a and b are equal.

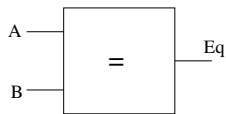


```
In C: int eq = (a&&b) || (!a&&!b);
```

Word Equality

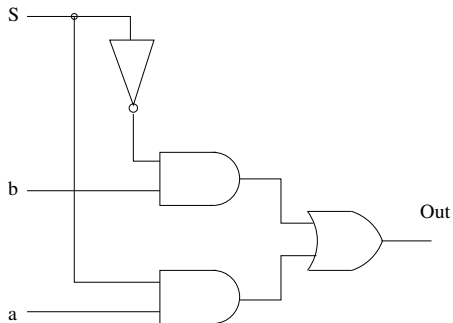


Word-level representation:



- Equality operation
- Generates Boolean value

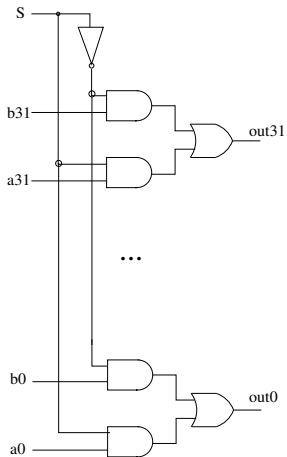
Bit Multiplexor



In C: `int out = (s && a) || (!s && b);`

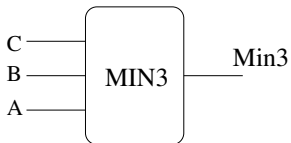
- Control signal `s` selects between two inputs `a` and `b`.
- Output is `a` when `s == 1`, and `b` otherwise.

Word Multiplexor

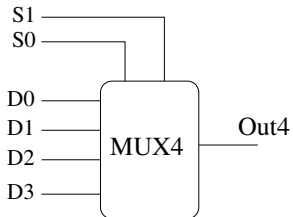


Select input word A or B
depending on control signal S .

Minimum of 3 words



4-way Multiplexor

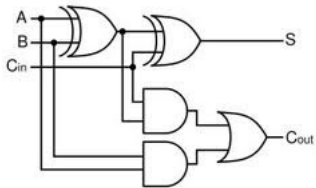


An ALU is an Arithmetic Logic Unit

- Multiple functions: add, subtract, and, xor, others
- Combinational logic to perform functions.
- Control signals select function to be performed.
- Modular: multiple instances of 1-bit ALU

1-Bit Adder

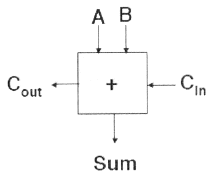
The following circuit is a 1-bit adder:



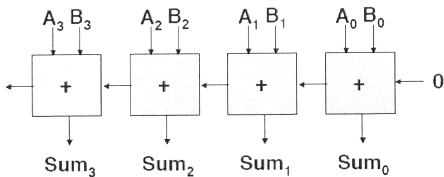
A	B	Cin	S	Cout
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

Adding a Pair of 4-bit Ints

One Bit Adder

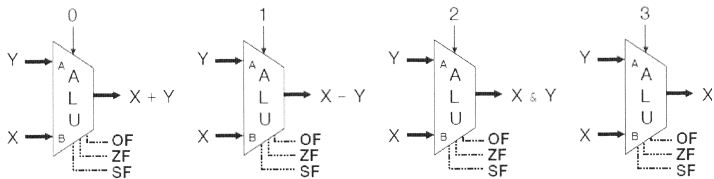


Four Bit Adder



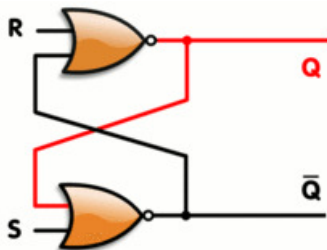
How do you subtract? How do you multiply?

A 4-bit ALU



- Combinational logic: continuously responding to inputs.
- Control signal selects function computed: corresponding to the 4 arithmetic/logical operations in Y86.
- Also computes values of condition codes:
 - OF: overflow flag
 - ZF: zero flag
 - SF: sign flag

SR Flip Flop: Storing a Bit

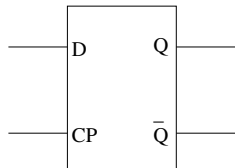
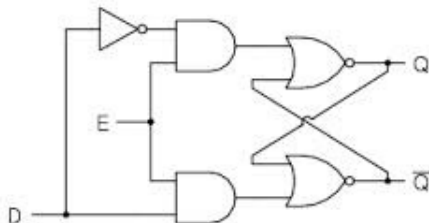


Characteristic table

S	R	Q_{next}	Action
0	0	Q	hold state
0	1	0	reset
1	0	1	set
1	1	X	not allowed

Gated D Latch: Store and Access One Bit

Higher level representation



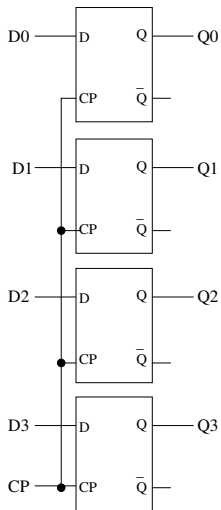
D Latch Truth table

E/C	D	Q	\bar{Q}	Comment
0	X	Q	\bar{Q}	No change
1	0	0	1	Reset
1	1	1	0	Set

A 4-bit Register

4 D latches:

- All share the E (aka WE or Write Enable) input
- D0–D3 are the data input
- Q0–Q3 are the output

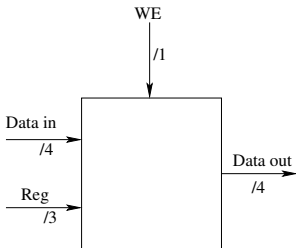


Register File Abstraction

Register file provides the CPU with temporary, fast storage.

- N registers.
- Each of K bits.
- L output ports.

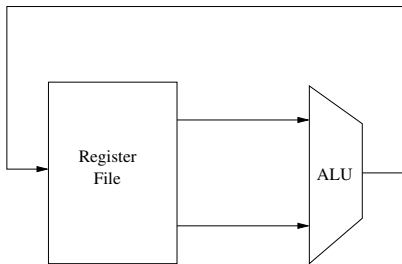
Suppose we want eight 4-bit registers and one output port.



Race-Through Condition with D Latches

Write Enable (WE) must be held at “1” long enough to allow:

- Data to be read;
- Operation (e.g., addition) to be performed;
- Result to be stored in target register.



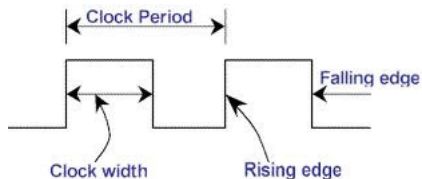
An edge-triggered flip-flop changes states either at the positive edge (rising edge) or at the negative edge (falling edge) of the clock pulse on the control input.

- A register is made up of several flip flops, each providing storage and access for an individual bit.
- A register file is made up of several registers and control logic

Clocking

The clock acts to enforce timing control on the chip.

- An integral part of every synchronous system.
- Can be global



Clock Frequency = $1 / \text{clock period}$

- Measured in cycles per second (Hertz)
- 1 KHz = 1000 cycles / second
- 1ns (10^{-9} seconds) = 1GHz (10^9) clock frequency
- Higher frequency means faster speed.

Stores many words

- Conceptually, a large array where each row is uniquely addressable.
- In reality, much more complex to increase throughput.
- Multiple chips and banks, interleaved, with multi-word operations.

Many implementations

- Dynamic (DRAM) is large, inexpensive, but relatively slow.
 - 1 transistor and 1 capacitor per bit.
 - Reads are destructive.
 - Requires periodic refresh.
 - Access time takes hundreds of CPU cycles.
- Static (SRAM) is fast but expensive.
 - 6 transistors per bit.
 - Streaming orientation.

Computation

- Performed by combinational logic.
- Implements boolean functions.
- Continuously reacts to inputs.

Storage

- Registers: part of the CPU.
 - Each holds a single word.
 - Used for temporary results of computation.
 - Loaded on rising clock.
- Memory is much larger.
- Variety of implementation techniques.