Retiming and Resynthesis with Sweep Are Complete for Sequential Transformations

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Nov. 18, 2009

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Remove or insert registers not observable by output.

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How Powerful are Retiming and Resynthesis?

Are they complete for all sequential transformations?

A Little Bit History

Leiserson & Saxe 83

A circuit transformed by retiming is steady state equivalent to original circuit.

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Asking whether reverse is true, proved that any state re-encoding can be done by RnR.

Malik 90

Proved (wrongly) that any cycle-preserving (CP) transformation can be done by RnR.

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Jiang & Brayton 06

RnR are exactly transformations by a sequence of 1-step merging and splitting.

Main Result

Theorem

Retiming and Resynthesis with Sweep are complete for steady state equivalent sequential transformations

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Retiming and Resynthesis with Sweep are complete for steady state equivalent sequential transformations if one-cycle reachability is allowed in synthesis.

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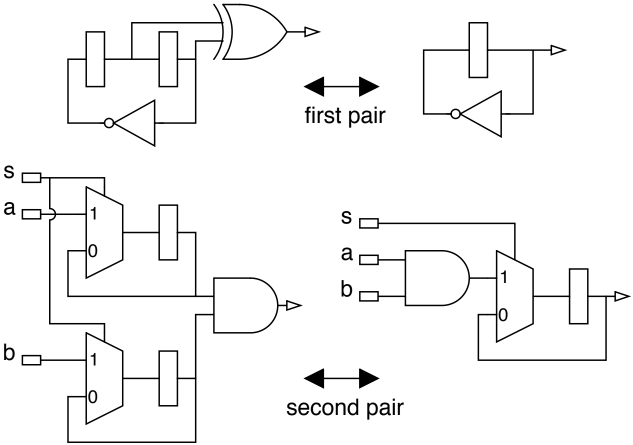
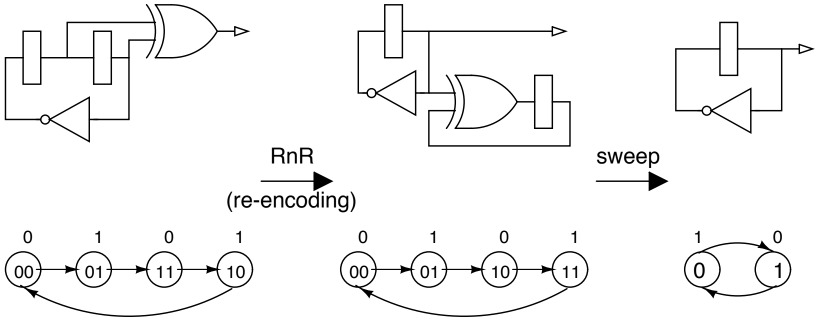
We point out in paper

Re-encoding checking is PSPACE-hard, but the complexity of RnR checking is still open.

Circuits Demonstrating Incompleteness of RnR



Sweep is Necessary



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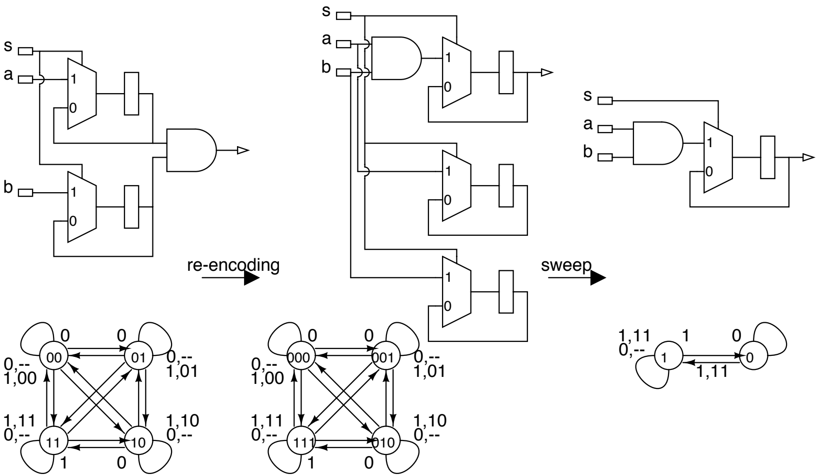

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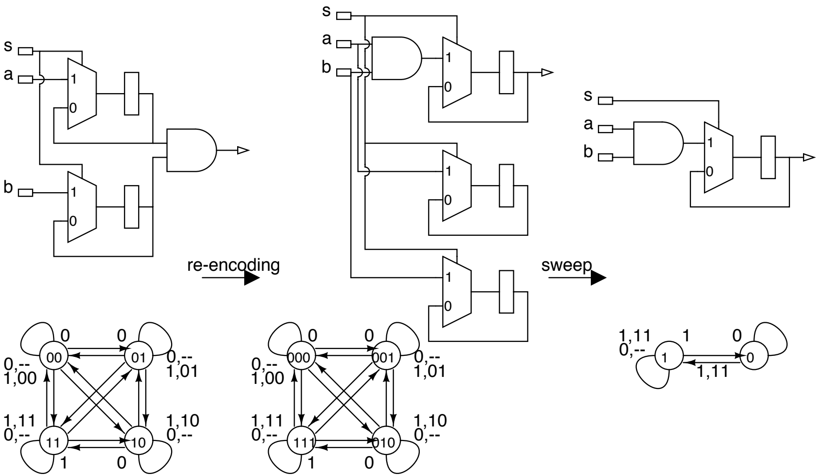

Warning

Re-encoding with different length is needed!

Is RnR Complete for Re-encoding with Different Length?

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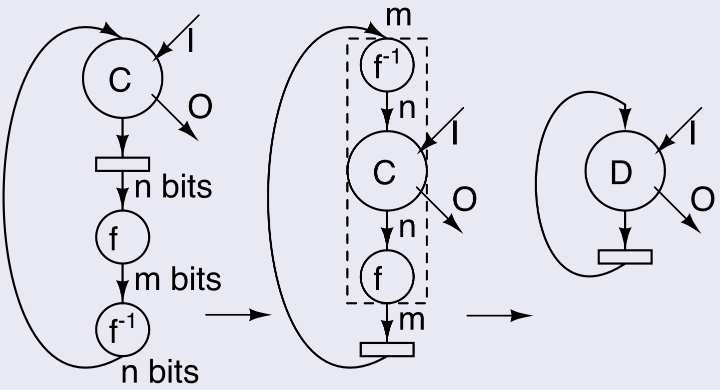
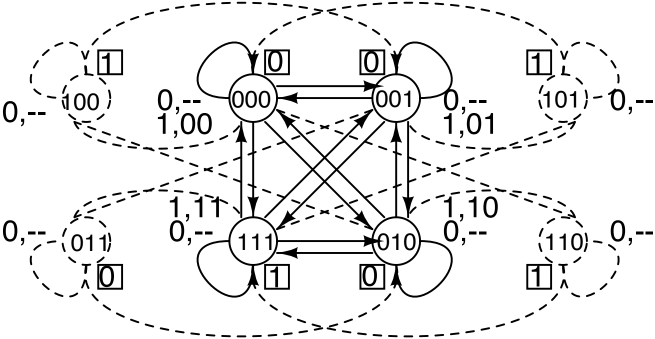
Proof Sketch



Re-encoding with Different Code Length

- Extra shadow states are introduced:



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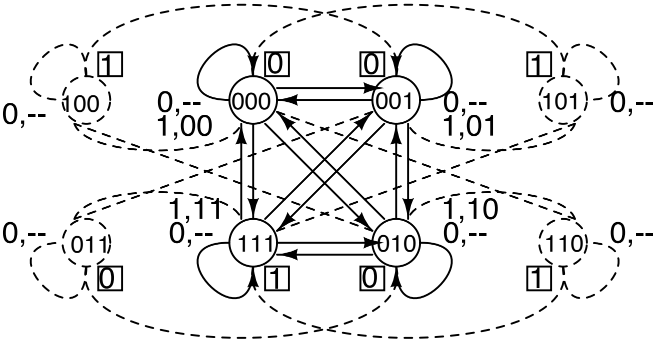

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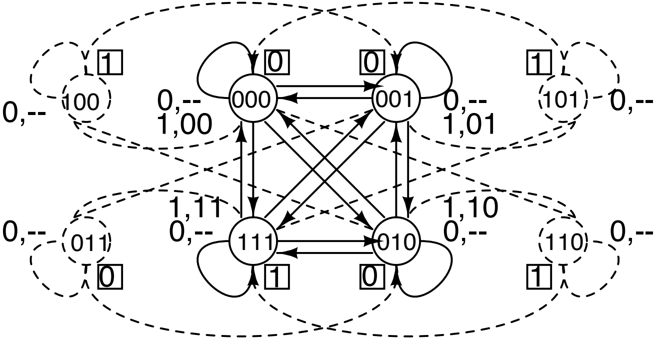

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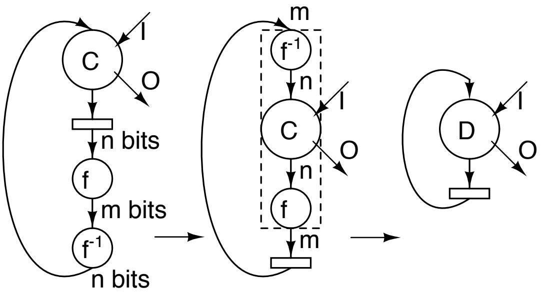
Contradicting w/ Jiang & Brayton 06

What is wrong?

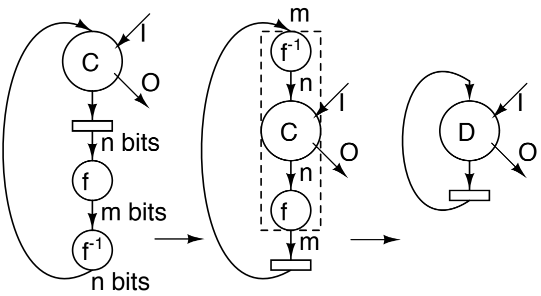
Encoding Representation Is Important



Observation

Treating Boolean functions as abstract discrete functions turns to boast the power of synthesis!
A discrete function may have a range of $2^n + 1$ symbols, but a corresponding Boolean one will have 2^{n+1} values.

Solution



One-Cycle Reachability (OCR)

We need to look into previous cycle to find the domain of f^{-1} which was the range of f!

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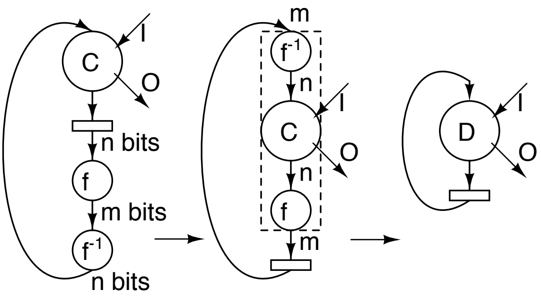

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Lemma

Without OCR, RnR is not complete for transforming between two given circuits that are re-encodings with different code lengths.

Inspiration by Abadi & Lamport 91

The existence of refinement mappings, TCS, 82(2), 1991

Under three general hypotheses about the specifications, if S_1 implements S_2 then one can add auxiliary history and prophecy variables to S_1 to form equivalent specification S_1^{hp} and find a refinement mapping from S_1^{hp} to S_2.

Completeness for Sequential Transformation

Theorem

Retiming and Resynthesis with Sweep are complete for steady state equivalent sequential transformations, if OCR is allowed.

Proof.

1. Circuits C and D are steady state equivalent $\Rightarrow$ every steady state of C maps to at least one D state.
2. Use sweep (inverse) to add registers in C to make an "onto" refinement function F from C states to D states (Abadi & Lamport 91)
3. Bypass signals to make F into a bijection
4. Resynthesis $F^{-1} \circ F$ at the register output of C
5. Retime registers to outputs of F
6. Resynthesis with OCR
7. Sweep to remove unobservable registers to get D $\square$

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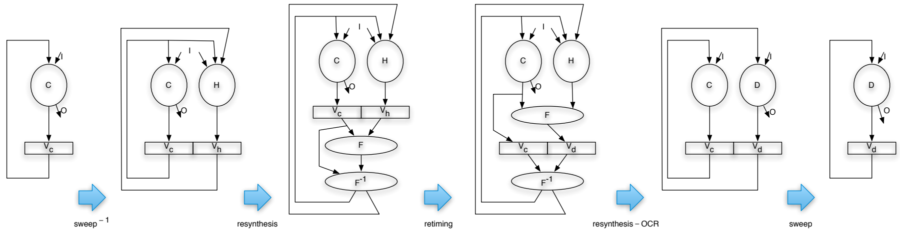

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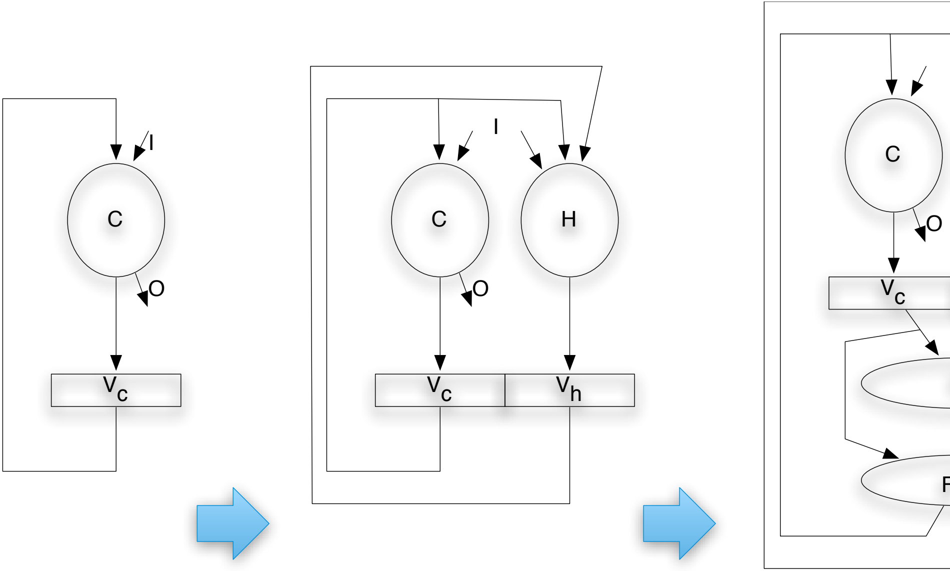

sweep $^{-1}$

resynthesis

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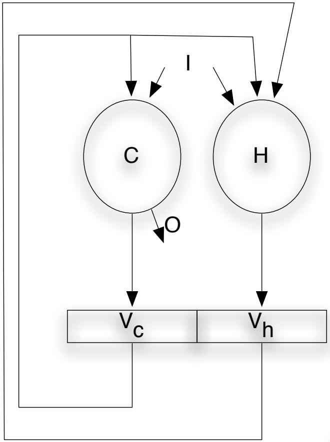

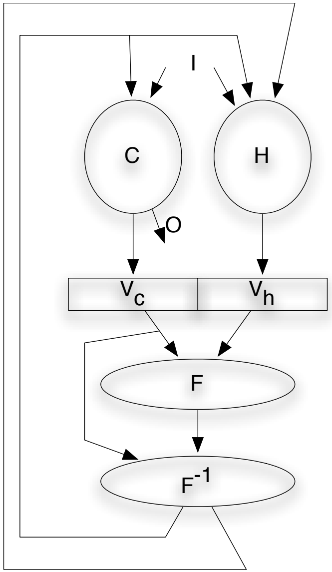

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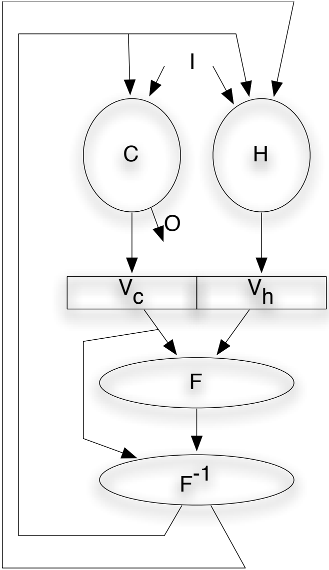

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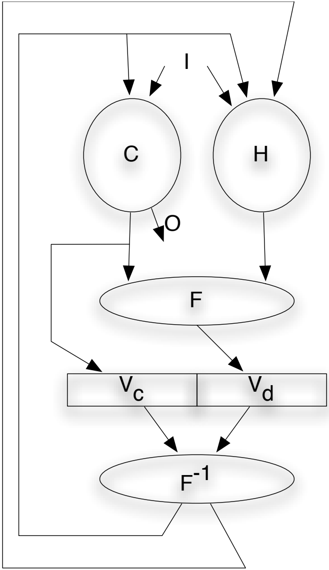
retiming



resynthe

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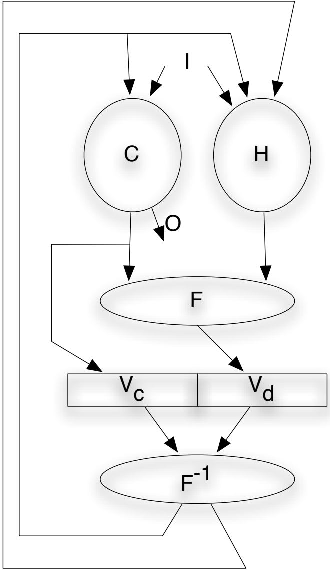

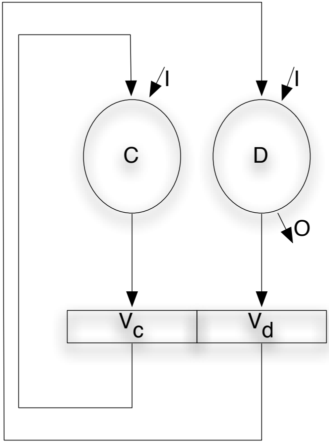

resynthesis – OCR

sweep

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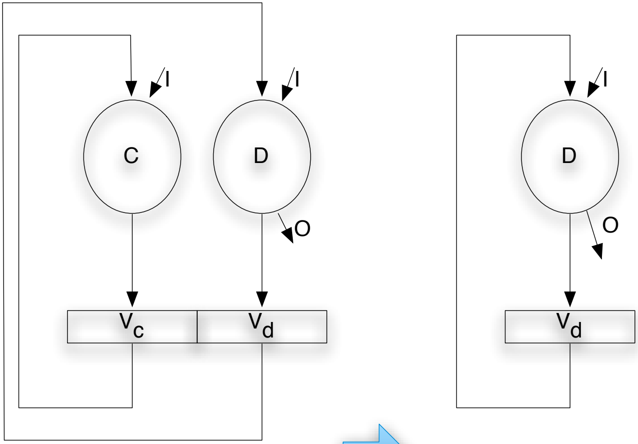

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- OCR needs to be used commonly.
- Efficiently verifiable subset of RnR-Sweep transformations?
- How powerful are RnR-Sweep without OCR?
- What is complexity of RnR equivalence checking?

Q & A