Implementing broadcast and accept

- A process that wants to broadcast m, does so through a series of witnesses
  - Sends m to all
  - Each correct process becomes a witness by relaying m to all
- If a process receives enough witness confirmations, it accepts m

Can we rely on witnesses?

- Only if not too many faulty processes!
- Otherwise, a set of faulty processes could fool a correct process by acting as witnesses of a message that was never broadcast
- How large can be f with respect to n?

Byzantine Generals

- One General G, a set of Lieutenants L_i
- General can order Attack (A) or Retreat (R)
- General may be a traitor; so may be some of the Lieutenants

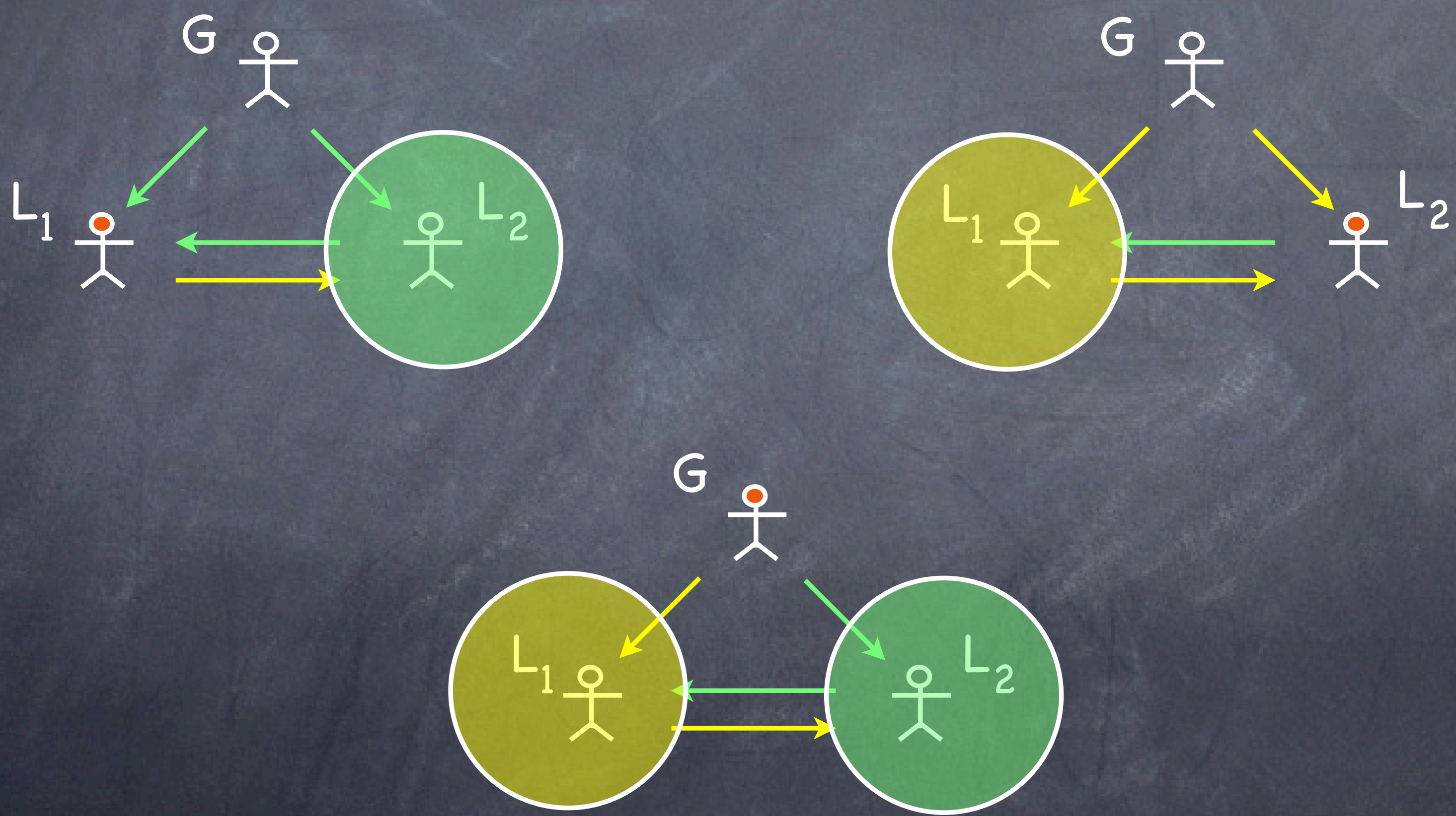
* * *

I. If G is trustworthy, every trustworthy L_i must follow G's orders

II. Every trustworthy L_i must follow same battleplan

The plot thickens...

One traitor



A Lower Bound

Theorem

There is no algorithm that solves TRB for Byzantine failures if $n \leq 3f$

(Lamport, Shostak, and Pease, The Byzantine Generals Problem, ACM TOPLAS, 4 (3), 382-401, 1982)

Back to the protocol...

- To broadcast a message in round r, p sends $(init, p, m, r)$ to all
- A confirmation has the form $(echo, p, m, r)$
- A witness sends $(echo, p, m, r)$ if either:
  - it receives $(init, p, m, r)$ from p directly or
  - it receives confirmations for (p, m, r) from at least $f + 1$ processes (at least one correct witness)
- A process accepts (p, m, r) if it has received $n - f$ confirmations (as many as possible...)
- Protocol proceeds in rounds. Each round has 2 phases

Implementation of broadcast and accept

Phase 2r - 1

p sends (init,p,m,r) to all

Phase 2r

if q received (init,p,m,r) in phase 2r - 1 then
    q sends (echo,p,m,r) to all /* q becomes a witness */
if q receives (echo,p,m,r) from at least n - f distinct processes in phase 2r then
    q accepts (p,m,r)

Phase j > 2r

if q has received (echo,p,m,r) from at least f + 1 distinct processes in phases $(2r, 2r+1, \ldots, j-1)$ then
    q sends (echo,p,m,r) to all processes /* q becomes a witness */
if q has received (echo,p,m,r) from at least n - f processes in phases $(2r, 2r+1, \ldots, j)$ then
    q accepts (p,m,r)

Is termination a problem?

The implementation is correct

Theorem

If $n > 3f$, the given implementation of broadcast(p, m, r) and accept(p, m, r) satisfies Unforgeability, Correctness, and Relay

Assumption

Channels are authenticated

Correctness

If a correct process p executes broadcast(p,m,r) in round r, then all correct processes will execute accept(p,m,r) in round r

If p is correct then

- p sends (init,p,m,r) to all in round r (phase 2r - 1)
- by Validity of the underlying send and receive, every correct process receives (init,p,m,r) in phase 2r - 1
- every correct process becomes a witness
- every correct process sends (echo,p,m,r) in phase 2r
- since there are at least n - f correct processes, every correct process receives at least n - f echoes in phase 2r
- every correct process executes accept(p,m,r) in phase 2r (in round r)

Unforgeability - 1

If a correct process q executes accept(p,m,r) in round j ≥ r, and p is correct, then p did in fact execute broadcast(p,m,r) in round r

- Suppose q executes accept(p,m,r) in round j
- q received (echo,p,m,r) from at least n - f distinct processes by phase k, where $k = 2j - 1$ or $k = 2j$
- Let k′ be the earliest phase in which some correct process q′ becomes a witness to (p,m,r)

Unforgeability - 1

If a correct process q executes accept(p,m,r) in round j ≥ r, and p is correct, then p did in fact execute broadcast(p,m,r) in round r

- Suppose q executes accept(p,m,r) in round j
- q received (echo,p,m,r) from at least n - f distinct processes by phase k, where $k = 2j - 1$ or $k = 2j$
- Let k′ be the earliest phase in which some correct process q′ becomes a witness to (p,m,r)

Case 1: $k' = 2r - 1$

- q′ received (init,p,m,r) from p
- since p is correct, it follows that p did execute broadcast(p,m,r) in round r

Case 2: $k' > 2r - 1$

- q′ has become a witness by receiving (echo,p,m,r) from f + 1 distinct processes
- at most f are faulty; one is correct
- this process was a witness to (p,m,r) before phase k′

CONTRADICTION

The first correct process receives (init,p,m,r) from p!

Unforgeability -2

- For q to accept, some correct process must become witness.
- Earliest correct witness q' becomes so in phase $2r-1$, and only if p did indeed executed broadcast(p, m, r)
- Any correct process that becomes a witness later can only do so if a correct process is already a witness.
- For any correct process to become a witness, p must have executed broadcast(p, m, r)

Relay

If a correct process q executes accept(p, m, r) in round $j \geq r$, then all correct processes will execute accept(p, m, r) by round $j + 1$

Relay

If a correct process q executes accept(p, m, r) in round $j \geq r$, then all correct processes will execute accept(p, m, r) by round $j+1$

- Suppose correct q executes accept(p, m, r) in round j (phase $k = 2j - 1$ or $k = 2j$)
- q received at least $n - f$ $(echo, p, m, r)$ from distinct processes by phase k
- At least $n - 2f$ of them are correct.
- All correct processes received $(echo, p, m, r)$ from at least $n - 2f$ correct processes by phase k
- From $n > 3f$, it follows that $n - 2f \geq f + 1$. Then, all correct processes become witnesses by phase k
- All correct processes send $(echo, p, m, r)$ by phase $k+1$
- Since there are at least n - f correct processes, all correct processes will accept(p, m, r) by phase $k+1$ (round $2j$ or $2j+1$)

Taking a step back...

- Specified Consensus and TRB
- In the synchronous model :
  - solved Consensus and TRB for General Omission failures
  - proved lower bound on rounds required by TRB
  - solved TRB for AFMA
  - proved lower bound on replication for solving TRB with AF
  - solved TRB with AF

What about the asynchronous model?

Theorem

- There is no deterministic protocol that solves Consensus in a message-passing asynchronous system in which at most one process may fail by crashing

(Fisher, Lynch, and Paterson. Impossibility of distributed consensus with one faulty process. JACM, Vol. 32, no. 2, April 1985, pp. 374-382)

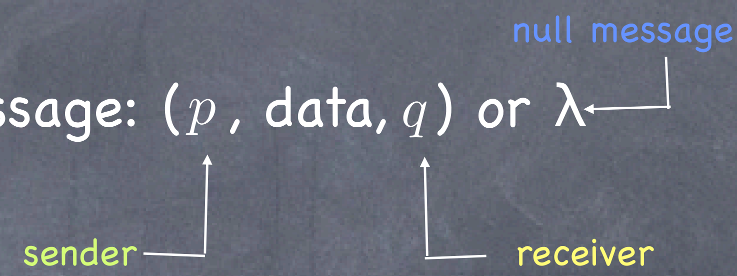
The Intuition

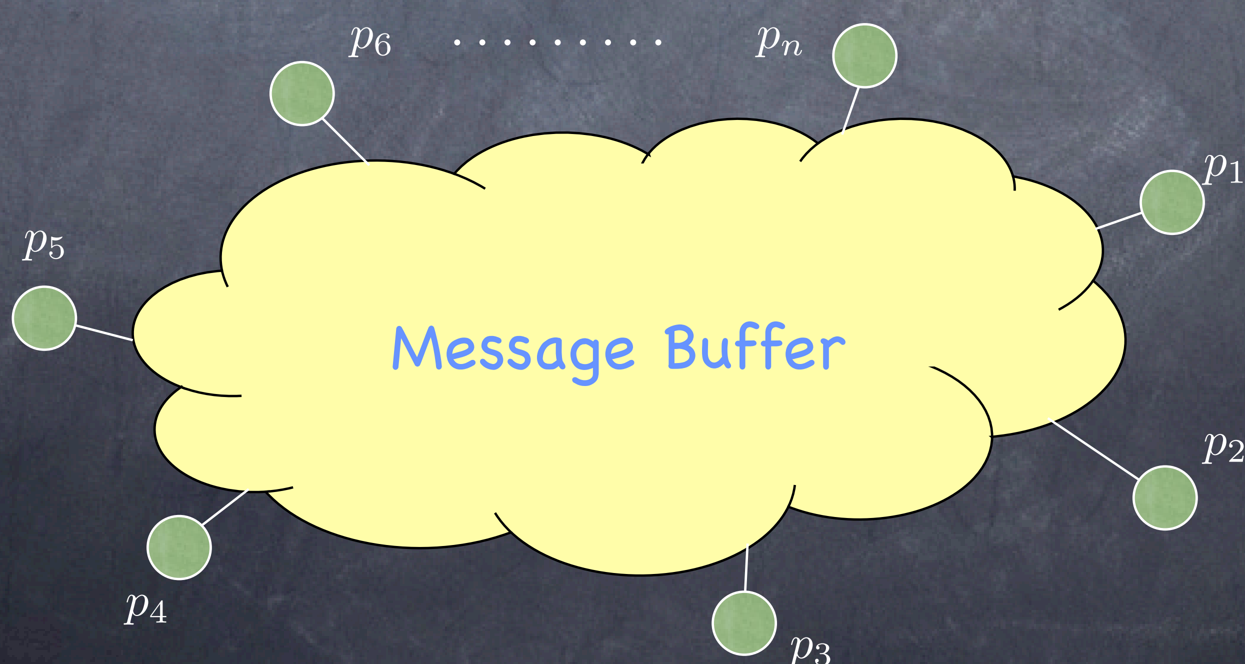
- In an asynchronous system, a process p cannot tell whether a non-responsive process q has crashed or it is just slow
- If p waits, it might do so forever
- If p decides, it may find out later that q came to a different decision

The Model - 1

- n processes
- a message buffer

message: (p , data, q) or λ





The Model - 2

- An algorithm $\mathcal{A}$ is a sequence of steps
- Each step consists of two phases
  - Receive phase - some p removes from buffer (x,data,p) or λ
  - Send phase - p changes its state; adds zero or more messages to buffer
- p can receive λ even if there are messages for p in the buffer

Assumptions

Liveness Assumption:

Every message sent will be eventually received if intended receiver tries infinitely often

- One-time Assumption:

p sends m to q at most once

- WLOG, process p_i can only propose a single bit b_i

Configurations

- A configuration C of $\mathcal{A}$ is a pair (s, M) where:
  - s is a function that maps each p_i to its local state
  - M is the set of messages in the buffer
- A step e $\equiv (p, m, \mathcal{A})$ is applicable to C $= (s, M)$ if and only if $m \in M \cup \{\lambda\}$ Note: $(p, \lambda, \mathcal{A})$ is always applicable to C
- C′ ≡ e(C) is the configuration resulting from applying e to C

Schedules

- A schedule of $\mathcal{A}$ is a finite or infinite sequence of steps S of $\mathcal{A}$
- A schedule S is applicable to a configuration C if and only if either
  - S is the empty schedule $S_\perp$ or
  - $S[1]$ is applicable to C ;
  - $S[2]$ is applicable to $S[1]$(C) ; etc.
- If S is finite, S(C) is the unique configuration obtained by applying S to C

Schedules and configurations

- A configuration C′ is accessible from a configuration C if there exist a schedule S such that C′= S(C)
- C′ is a configuration of S(C) if $\exists S'$ prefix of S such that S′(C) = C′

Runs

- A run of $\mathcal{A}$ is a pair < I, S > where
  - I is an initial configuration
  - S is an infinite schedule of $\mathcal{A}$ applicable to I
- A run is partial if S is a finite schedule of $\mathcal{A}$
- The run is admissible if every process, except possibly one, takes infinitely many steps in S
- The run is unacceptable if every process, except possibly one, takes infinitely many steps in S without deciding

Structure of the proof

- Show that, for any given consensus algorithm $\mathcal{A}$, there always exists an unacceptable run
- In fact, we will show an unacceptable run in which no process crashes!

Classifying Configurations

0-valent: A configuration C is 0-valent if some process has decided 0 in C, or if all configurations accessible from C are 0-valent

1-valent: A configuration C is 1-valent if some process has decided 1 in C, or if all configurations accessible from C are 1-valent

Bivalent: A configuration C is bivalent if some of the configurations accessible from it are 0-valent while others are 1-valent

Bivalent initial configurations happen

Lemma 1

There exists a bivalent initial configuration

Proof

- Suppose $\mathcal{A}$ solves consensus with 1 crash failure
- Let I_j be the initial configuration in which the first j b_i's are 1
- I_0 is 0-valent; I_n is 1-valent
- By contradiction, suppose no bivalent
- Let k be smallest index such that I_k is 1-valent
- Obviously, I_{k-1} is 0-valent
- Suppose p_k crashes before taking any step.
- Since $\mathcal{A}$ solves consensus even with one crash failure, there is a finite schedule S applicable to I_k that has no steps of p_k and such that some process decides in $S(I_k)$
- S is also applicable to I_{k-1}

CONTRADICTION

Commutativity Lemma

Lemma 2

Let S_1 and S_2 be schedules applicable to some configuration C, and suppose that the set of processes taking steps in S_1 is disjoint from the set of processes taking steps in S_2.

Then, $S_1; S_2$ and $S_2; S_1$ are both sequences applicable to C, and they lead to the same configuration.

Procrastination Lemma

Lemma 3

Let C be bivalent, and let e be a step applicable to C.

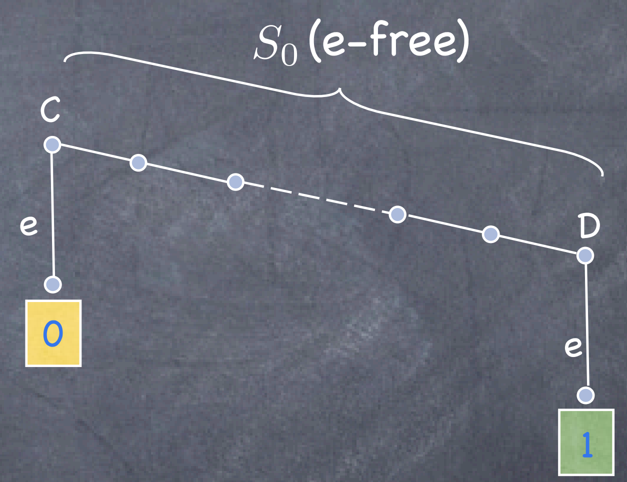
Then, there is a (possibly empty) schedule S not containing e such that $e(S(C))$ is bivalent

Proof Sketch - 1

- By contradiction, assume there is an e for which no such S exists
- Then, $e(C)$ is monovalent; WLOG assume 0-valent
-

Mini Lemma:

There exists an e-free schedule S_0 such that $D = S_0(C)$ and $e(D)$ is 1-valent

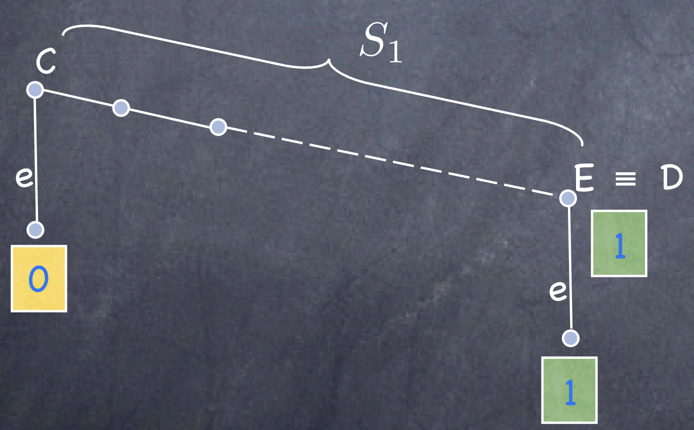


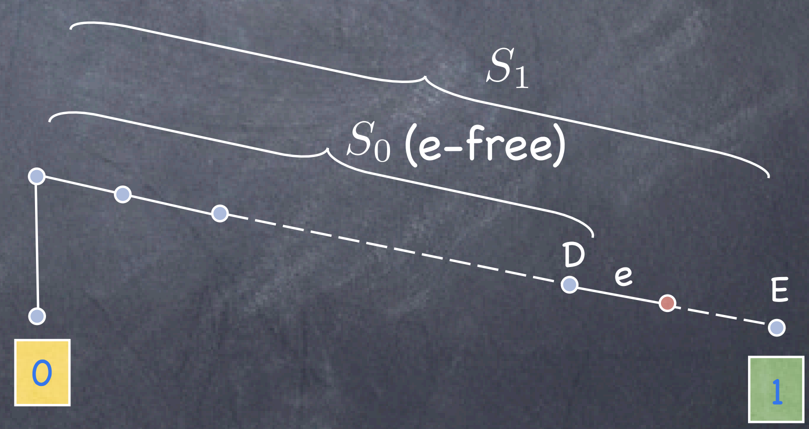
Proof Sketch- 2

Proof of mini Lemma.

Since C is bivalent, there exists a schedule S_1 such that $E = S_1(C)$ is 1-valent

Otherwise, let S_0 be the largest e-free prefix of S_1

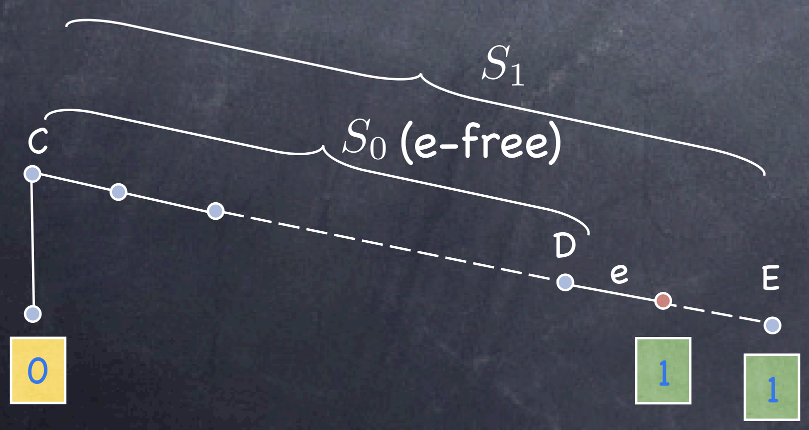




If S_1 is e-free, then D = E

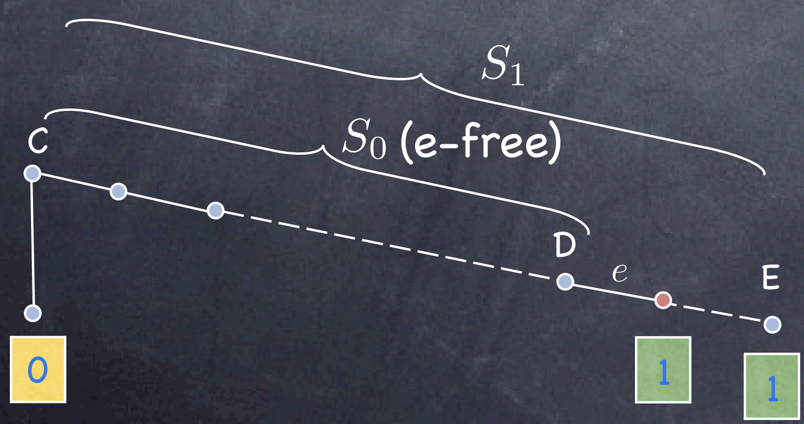
Proof Sketch - 3

- Consider configuration e(D).
- By assumption, e(D) cannot be bivalent (otherwise we would have proved the Procrastination Lemma with S = S_0)
- Since e(D) is monovalent, E is accessible from e(D), and E is 1-valent, then e(D) is 1-valent $\square$



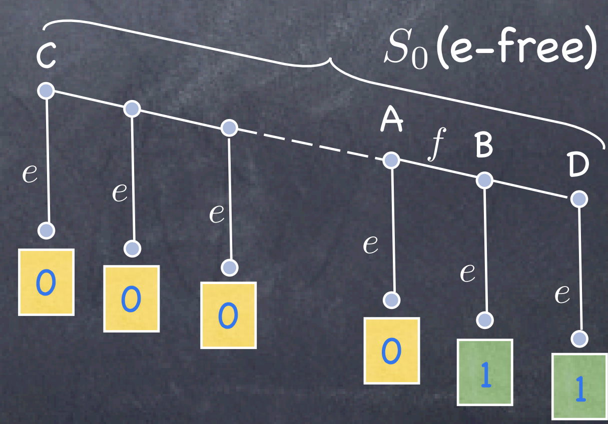
Proof Sketch - 3

- Consider configuration e(D).
- By assumption, e(D) cannot be bivalent (otherwise we would have proved the Procrastination Lemma with S = S_0)
- Since e(D) is monovalent, E is accessible from e(D), and E is 1-valent, then e(D) is 1-valent □



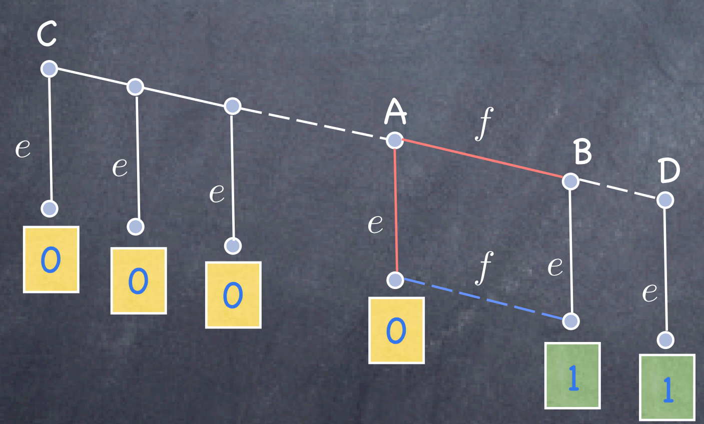
By the mini Lemma, on the "path" from C to D there must be two neighboring configurations A and B and a step f such that

- B = f(A)
- e(A) is 0-valent
- e(B) is 1-valent



Proof Sketch - 4

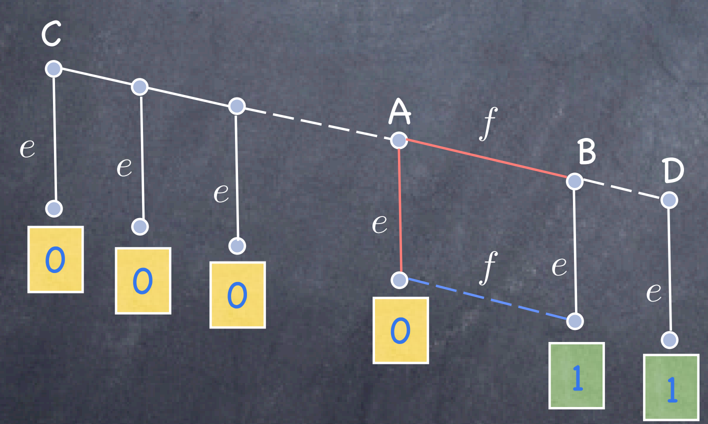
Consider now A and B = f(A)



- Claim: The same processes p must take steps e and f
  - Suppose not
  - By Commutativity lemma, e(B) = e(f(A)) = f(e(A))

Proof Sketch - 4

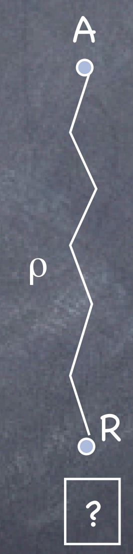
Consider now A and B = f(A)



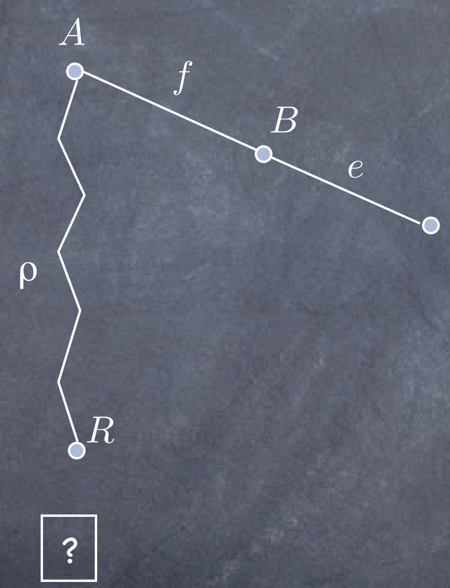
- Claim: The same processes p must take steps e and f
  - Suppose not
  - By Commutativity lemma, e(B) = e(f(A)) = f(e(A))
  - Impossible since e(B) is 1-valent and e(A) is 0-valent

Proof Sketch - 5

- Since our protocol tolerates a failure, there is a schedule ρ applicable to A such that:
  - R = ρ(A)
  - Some process decides in R
  - p does not take any steps in ρ
- We show that the decision value in R can be neither 0 nor 1!



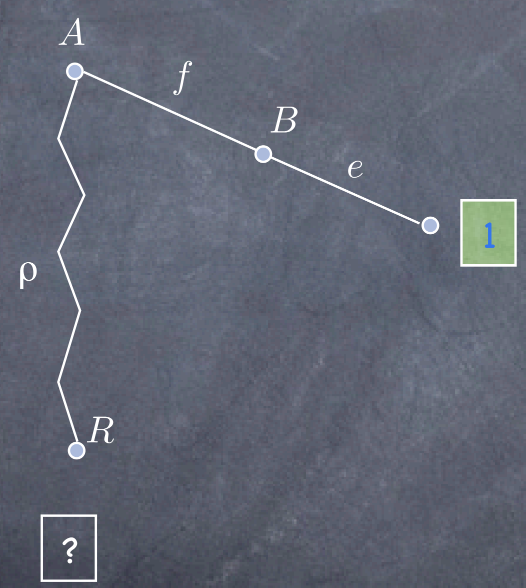
Proof Sketch - 6



Cannot be 0:

- Consider $e(B) = e(f(A))$

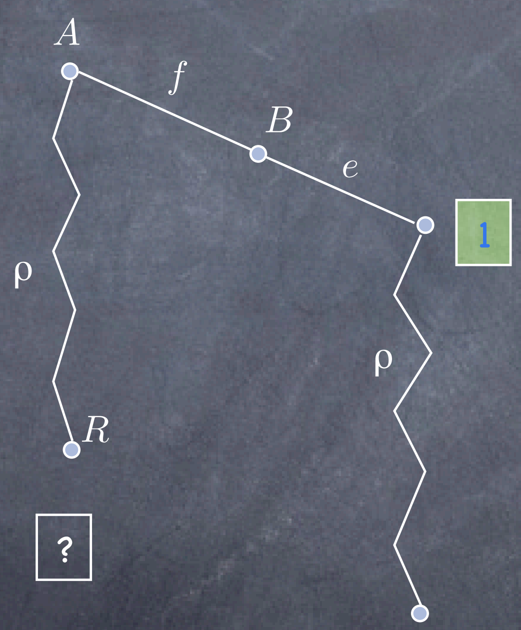
Proof Sketch - 6



Cannot be 0:

- Consider $e(B) = e(f(A))$
- By Mini Lemma, we know it is 1-valent

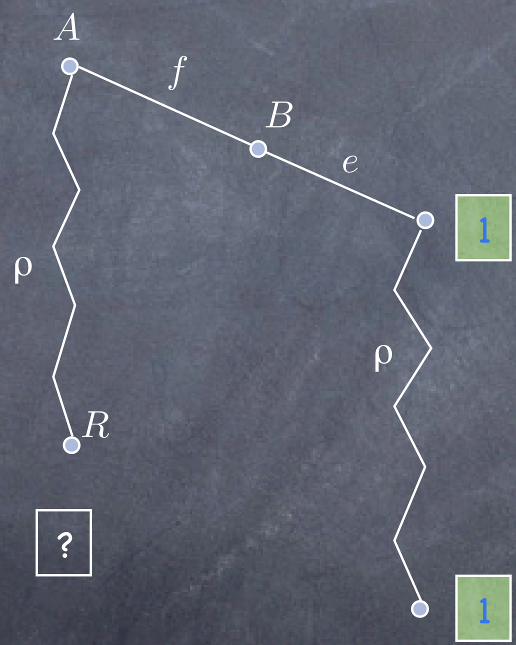
Proof Sketch - 6



Cannot be 0:

- Consider $e(B) = e(f(A))$
- By Mini Lemma, we know it is 1-valent
- Because it contains no steps of p, ρ is applicable to $e(B)$

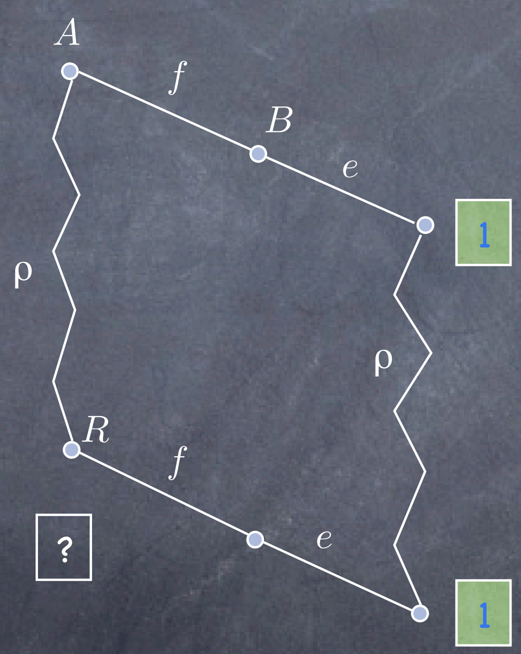
Proof Sketch - 6



Cannot be 0:

- Consider $e(B) = e(f(A))$
- By Mini Lemma, we know it is 1-valent
- Because it contains no steps of p, ρ is applicable to $e(B)$
- The resulting configuration is 1-valent

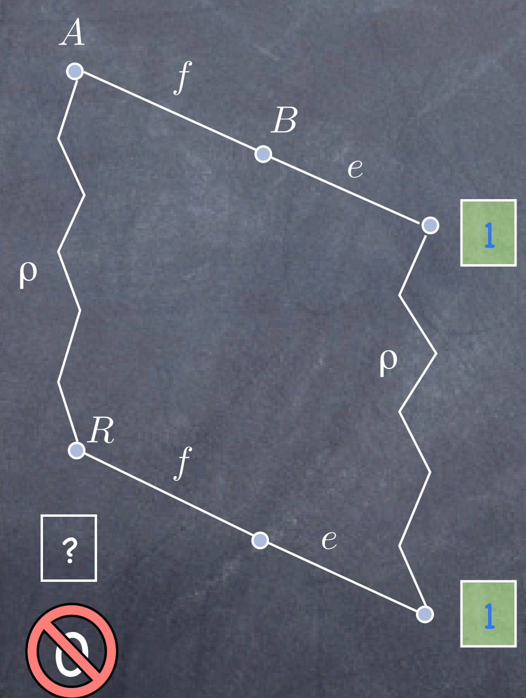
Proof Sketch - 6



Cannot be 0:

- Consider $e(B) = e(f(A))$
- By Mini Lemma, we know it is 1-valent
- Because it contains no steps of p, ρ is applicable to $e(B)$
- The resulting configuration is 1-valent
- By Commutativity Lemma $\rho(e(f(A))) = e(f(\rho(A))) = e(f(R))$

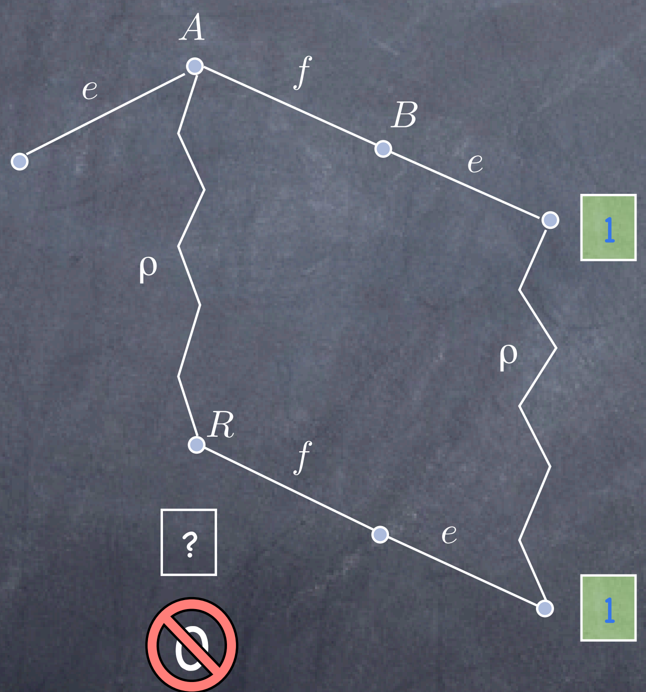
Proof Sketch - 6



Cannot be 0:

- Consider $e(B) = e(f(A))$
- By Mini Lemma, we know it is 1-valent
- Because it contains no steps of p , ρ is applicable to $e(B)$
- The resulting configuration is 1-valent
- By Commutativity Lemma
- $\rho(e(f(A))) = e(f(\rho(A))) = e(f(R))$
- Since $\rho(e(B))$ is accessible from R, and $\rho(e(B))$ is 1-valent, R cannot be 0-valent

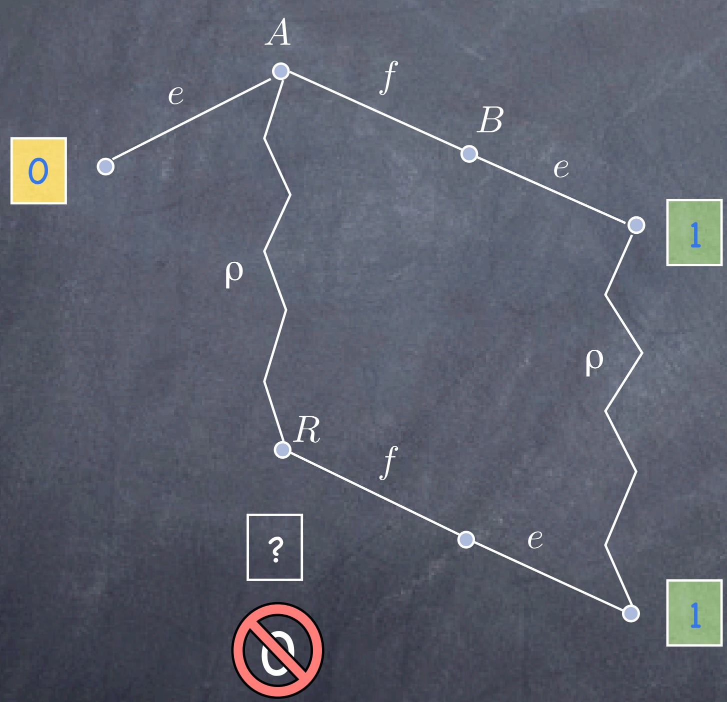
Proof Sketch - 7



Cannot be 1:

- Consider $e(A)$

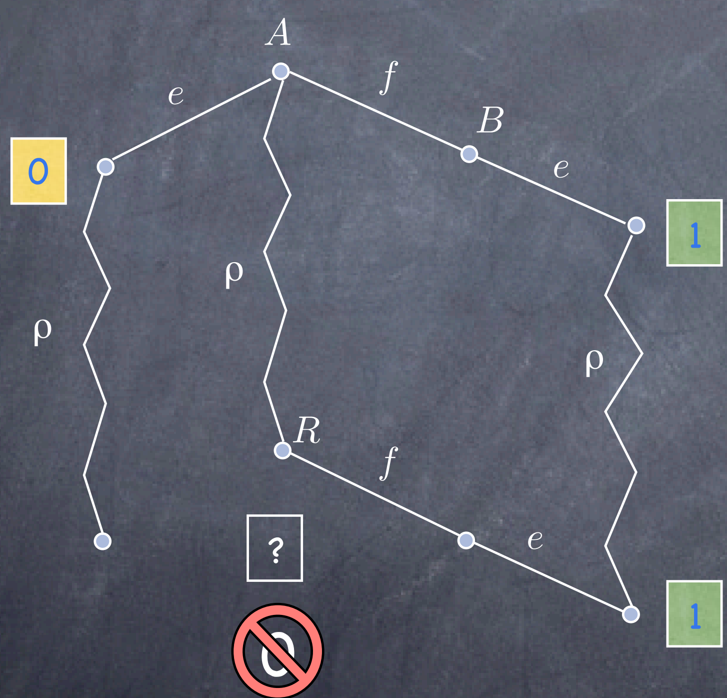
Proof Sketch - 7



Cannot be 1:

- Consider $e(A)$
- By construction, it is 0-valent

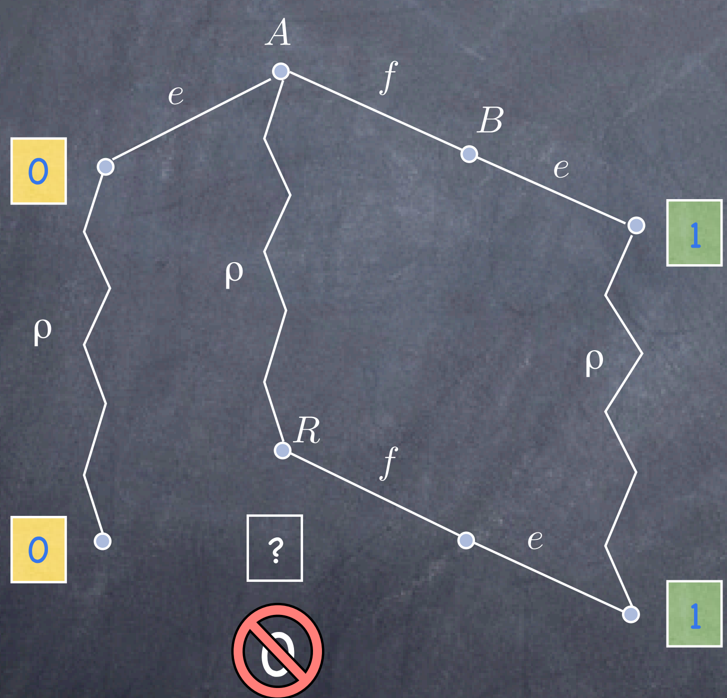
Proof Sketch - 7



Cannot be 1:

- Consider $e(A)$
- By construction, it is 0-valent
- Because it contains no steps of p , ρ is applicable to $e(A)$

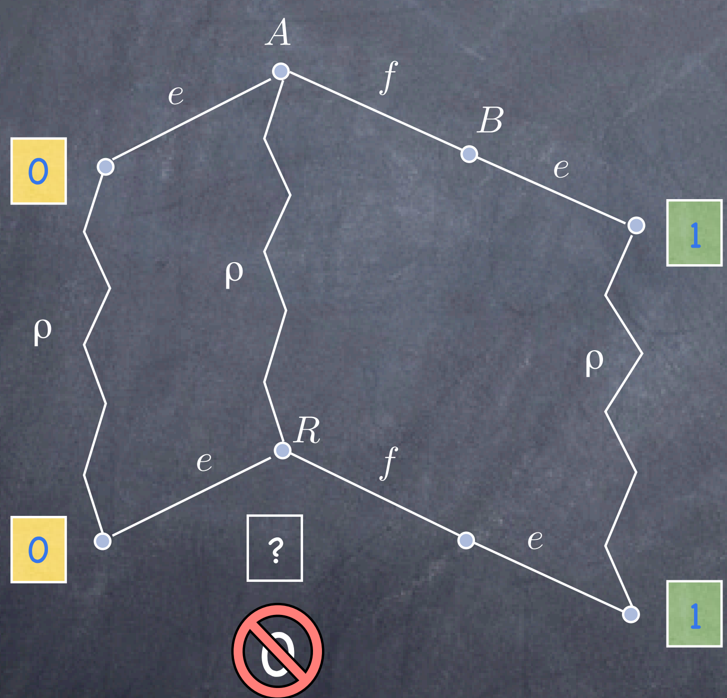
Proof Sketch - 7



Cannot be 1:

- Consider $e(A)$
- By construction, it is 0-valent
- Because it contains no steps of p, ρ is applicable to $e(A)$
- The resulting configuration is 0-valent

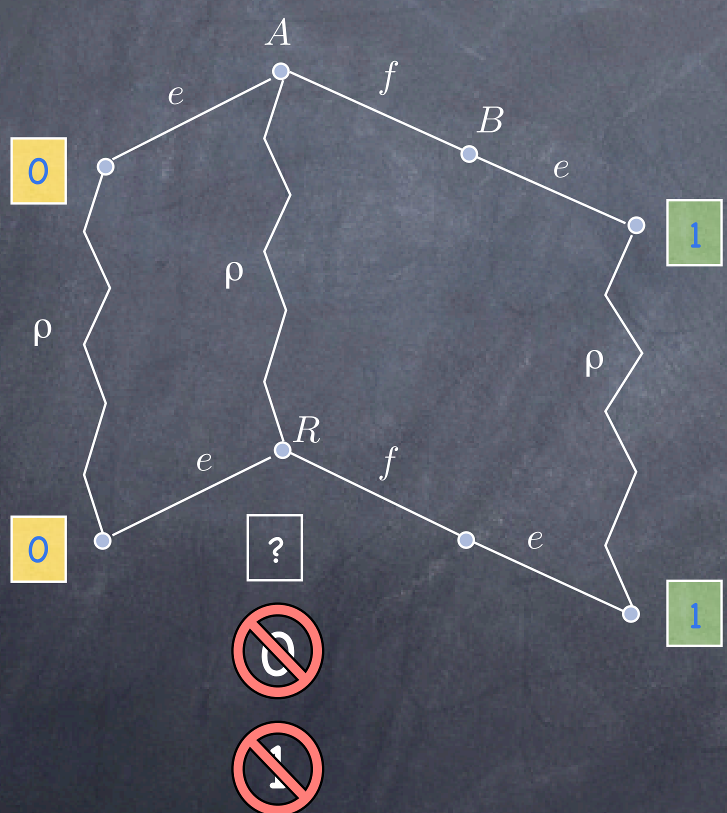
Proof Sketch - 7



Cannot be 1:

- Consider $e(A)$
- By construction, it is 0-valent
- Because it contains no steps of p, ρ is applicable to $e(A)$
- The resulting configuration is 0-valent
- By Commutativity Lemma
- $\rho(e(A)) = e(\rho(A)) = e(R)$

Proof Sketch - 7



Cannot be 1:

- Consider $e(A)$
- By construction, it is 0-valent
- Because it contains no steps of p , ρ is applicable to $e(A)$
- The resulting configuration is 0-valent
- By Commutativity Lemma
- $\rho(e(A)) = e(\rho(A)) = e(R)$
- Since $\rho(e(A))$ is accessible from R, and $\rho(e(A))$ is 0-valent, R cannot be 1-valent

Cannot decide in R: contradiction

Proving the FLP Impossibility Result

Theorem

There is no deterministic protocol that solves Consensus in a message-passing asynchronous system in which at most one process may fail by crashing

- By Lemma 1, there exists an initial bivalent configuration I_{biv}
- Consider any ordering $p_{l_1}, \ldots, p_{l_n}$ of $p_1, \ldots, p_n$
- Pick any applicable step $e_1 = (p_{l_1}, m_1)$
- Apply Procrastination lemma to obtain another bivalent configuration

$$C^1_{biv} = e_1(S_1(I_{biv}))$$

- Pick a step $e_2 = (p_{l_2}, m_2)$ applicable to C^1_{biv}
- Apply Procrastination lemma to obtain another bivalent configuration
- Continue as before in a round-robin fashion. How do we choose a step?
- We have built an unacceptable run!

How can one get around FLP?

Weaken the problem

- Weaken termination
  - use randomization to terminate with probability 1
- Weaken agreement
  - ε - agreement
    - real-valued inputs and outputs
    - agreement within real-valued small positive tolerance ε
  - k-set agreement
    - Agreement: In any execution, there is a subset W of the set of input values, |W| =k, s.t. all decision values are in W
    - Validity: In any execution, any decision value for any process is the input value of some process

How can one get around FLP?

Constrain input values

- Characterize the set of input values for which agreement is possible

Strengthen the system model

- Introduce failure detectors to distinguish between crashed processes and very slow processes