Proofs of Unsatisfiability

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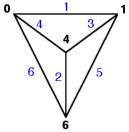
Outline

Introduction

Satisfiability (SAT) solving has many applications

formal verification

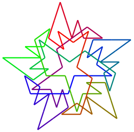


graph theory

bioinformatics

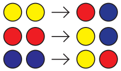
train safety

planning



number theory

cryptography



rewrite termination

$$\begin{tikzcd}
\text{encode} \arrow[r] & \boxed{\text{SAT solver}} \arrow[r] & \text{decode}
\end{tikzcd}$$

A Small Satisfiability (SAT) Problem

$$
\begin{array}{l}
(x_5 \vee x_8 \vee \bar{x}_2) \wedge (x_2 \vee \bar{x}_1 \vee \bar{x}_3) \wedge (\bar{x}_8 \vee \bar{x}_3 \vee \bar{x}_7) \wedge (\bar{x}_5 \vee x_3 \vee x_8) \wedge \\
(\bar{x}_6 \vee \bar{x}_1 \vee \bar{x}_5) \wedge (x_8 \vee \bar{x}_9 \vee x_3) \wedge (x_2 \vee x_1 \vee x_3) \wedge (\bar{x}_1 \vee x_8 \vee x_4) \wedge \\
(\bar{x}_9 \vee \bar{x}_6 \vee x_8) \wedge (x_8 \vee x_3 \vee \bar{x}_9) \wedge (x_9 \vee \bar{x}_3 \vee x_8) \wedge (x_6 \vee \bar{x}_9 \vee x_5) \wedge \\
(x_2 \vee \bar{x}_3 \vee \bar{x}_8) \wedge (x_8 \vee \bar{x}_6 \vee \bar{x}_3) \wedge (x_8 \vee \bar{x}_3 \vee \bar{x}_1) \wedge (\bar{x}_8 \vee x_6 \vee \bar{x}_2) \wedge \\
(x_7 \vee x_9 \vee \bar{x}_2) \wedge (x_8 \vee \bar{x}_9 \vee x_2) \wedge (\bar{x}_1 \vee \bar{x}_9 \vee x_4) \wedge (x_8 \vee x_1 \vee \bar{x}_2) \wedge \\
(x_3 \vee \bar{x}_4 \vee \bar{x}_6) \wedge (\bar{x}_1 \vee \bar{x}_7 \vee x_5) \wedge (\bar{x}_7 \vee x_1 \vee x_6) \wedge (\bar{x}_5 \vee x_4 \vee \bar{x}_6) \wedge \\
(\bar{x}_4 \vee x_9 \vee \bar{x}_8) \wedge (x_2 \vee x_9 \vee x_1) \wedge (x_5 \vee \bar{x}_7 \vee x_1) \wedge (\bar{x}_7 \vee \bar{x}_9 \vee \bar{x}_6) \wedge \\
(x_2 \vee x_5 \vee x_4) \wedge (x_8 \vee \bar{x}_4 \vee x_5) \wedge (x_5 \vee x_9 \vee x_3) \wedge (\bar{x}_5 \vee \bar{x}_7 \vee x_9) \wedge \\
(x_2 \vee \bar{x}_8 \vee x_1) \wedge (\bar{x}_7 \vee x_1 \vee x_5) \wedge (x_1 \vee x_4 \vee x_3) \wedge (x_1 \vee \bar{x}_9 \vee \bar{x}_4) \wedge \\
(x_3 \vee x_5 \vee x_6) \wedge (\bar{x}_6 \vee x_3 \vee \bar{x}_9) \wedge (\bar{x}_7 \vee x_5 \vee x_9) \wedge (x_7 \vee \bar{x}_5 \vee \bar{x}_2) \wedge \\
(x_4 \vee x_7 \vee x_3) \wedge (x_4 \vee \bar{x}_9 \vee \bar{x}_7) \wedge (x_5 \vee \bar{x}_1 \vee x_7) \wedge (x_5 \vee \bar{x}_1 \vee x_7) \wedge \\
(x_6 \vee x_7 \vee \bar{x}_3) \wedge (\bar{x}_8 \vee \bar{x}_6 \vee \bar{x}_7) \wedge (x_6 \vee x_2 \vee x_3) \wedge (\bar{x}_8 \vee x_2 \vee x_5)
\end{array}
$$

Does there exist an assignment satisfying all clauses?

Search for a satisfying assignment (or proof none exists)

$$
\begin{array}{l}
(x_5 \lor x_8 \lor \bar{x}_2) \land (x_2 \lor \bar{x}_1 \lor \bar{x}_3) \land (\bar{x}_8 \lor \bar{x}_3 \lor \bar{x}_7) \land (\bar{x}_5 \lor x_3 \lor x_8) \land \\
(\bar{x}_6 \lor \bar{x}_1 \lor \bar{x}_5) \land (x_8 \lor \bar{x}_9 \lor x_3) \land (x_2 \lor x_1 \lor x_3) \land (\bar{x}_1 \lor x_8 \lor x_4) \land \\
(\bar{x}_9 \lor \bar{x}_6 \lor x_8) \land (x_8 \lor x_3 \lor \bar{x}_9) \land (x_9 \lor \bar{x}_3 \lor x_8) \land (x_6 \lor \bar{x}_9 \lor x_5) \land \\
(x_2 \lor \bar{x}_3 \lor \bar{x}_8) \land (x_8 \lor \bar{x}_6 \lor \bar{x}_3) \land (x_8 \lor \bar{x}_3 \lor \bar{x}_1) \land (\bar{x}_8 \lor x_6 \lor \bar{x}_2) \land \\
(x_7 \lor x_9 \lor \bar{x}_2) \land (x_8 \lor \bar{x}_9 \lor x_2) \land (\bar{x}_1 \lor \bar{x}_9 \lor x_4) \land (x_8 \lor x_1 \lor \bar{x}_2) \land \\
(x_3 \lor \bar{x}_4 \lor \bar{x}_6) \land (\bar{x}_1 \lor \bar{x}_7 \lor x_5) \land (\bar{x}_7 \lor x_1 \lor x_6) \land (\bar{x}_5 \lor x_4 \lor \bar{x}_6) \land \\
(\bar{x}_4 \lor x_9 \lor \bar{x}_8) \land (x_2 \lor x_9 \lor x_1) \land (x_5 \lor \bar{x}_7 \lor x_1) \land (\bar{x}_7 \lor \bar{x}_9 \lor \bar{x}_6) \land \\
(x_2 \lor x_5 \lor x_4) \land (x_8 \lor \bar{x}_4 \lor x_5) \land (x_5 \lor x_9 \lor x_3) \land (\bar{x}_5 \lor \bar{x}_7 \lor x_9) \land \\
(x_2 \lor \bar{x}_8 \lor x_1) \land (\bar{x}_7 \lor x_1 \lor x_5) \land (x_1 \lor x_4 \lor x_3) \land (x_1 \lor \bar{x}_9 \lor \bar{x}_4) \land \\
(x_3 \lor x_5 \lor x_6) \land (\bar{x}_6 \lor x_3 \lor \bar{x}_9) \land (\bar{x}_7 \lor x_5 \lor x_9) \land (x_7 \lor \bar{x}_5 \lor \bar{x}_2) \land \\
(x_4 \lor x_7 \lor x_3) \land (x_4 \lor \bar{x}_9 \lor \bar{x}_7) \land (x_5 \lor \bar{x}_1 \lor x_7) \land (x_5 \lor \bar{x}_1 \lor x_7) \land \\
(x_6 \lor x_7 \lor \bar{x}_3) \land (\bar{x}_8 \lor \bar{x}_6 \lor \bar{x}_7) \land (x_6 \lor x_2 \lor x_3) \land (\bar{x}_8 \lor x_2 \lor x_5)
\end{array}
$$

Solutions are easy to verify, but what about unsatisfiability?

Original motivation for validating unsatisfiability proofs

Satisfiability solvers are used in amazing ways...

- Hardware and software verification (Intel and Microsoft)
- Hard-Combinatorial problems:
  - van der Waerden numbers [Dransfield, Marek, and Truszczynski, 2004; Kouril and Paul, 2008]
  - Gardens of Eden in Conway's Game of Life [Hartman, Heule, Kwekkeboom, and Noels, 2013]
  - Erdős Discrepancy Problem [Konev and Lisitsa, 2014]

..., but satisfiability solvers have errors and only return yes/no.

- Documented bugs in SAT, SMT, and QBF solvers [Brummayer and Biere, 2009; Brummayer et al., 2010]
- Implementation errors often imply conceptual errors
- Mathematical results require a stronger justification than a simple yes/no by a solver. UNSAT must be checkable.

Demo: Validating Solver Output

Proof Checking

Resolution Rule and Resolution Chains

Resolution Rule

$$\frac{(x \lor a_1 \lor \ldots \lor a_i) \qquad (\bar{x} \lor b_1 \lor \ldots \lor b_j)}{(a_1 \lor \ldots \lor a_i \lor b_1 \lor \ldots \lor b_j)}$$

- Many SAT techniques can be simulated by resolution.

Resolution Rule and Resolution Chains

Resolution Rule

$$\frac{(x \lor a_1 \lor \ldots \lor a_i) \quad (\bar{x} \lor b_1 \lor \ldots \lor b_j)}{(a_1 \lor \ldots \lor a_i \lor b_1 \lor \ldots \lor b_j)}$$

- Many SAT techniques can be simulated by resolution.

A resolution chain is a sequence of resolution steps.
The resolution steps are performed from left to right.

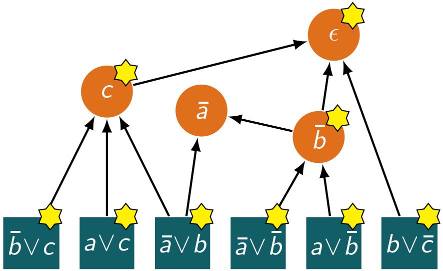
Example

- $(c) := (\bar{a} \lor \bar{b} \lor c) \diamond (\bar{a} \lor b) \diamond (a \lor c)$
- $(\bar{a} \lor c) := (\bar{a} \lor b) \diamond (a \lor c) \diamond (\bar{a} \lor \bar{b} \lor c)$
- The order of the clauses in the chain matter

Resolution Proofs versus Clausal Proofs

Consider the formula $F := (\bar{b} \vee c) \wedge (a \vee c) \wedge (\bar{a} \vee b) \wedge (\bar{a} \vee \bar{b}) \wedge (a \vee \bar{b}) \wedge (b \vee \bar{c})$

A resolution graph of F is:



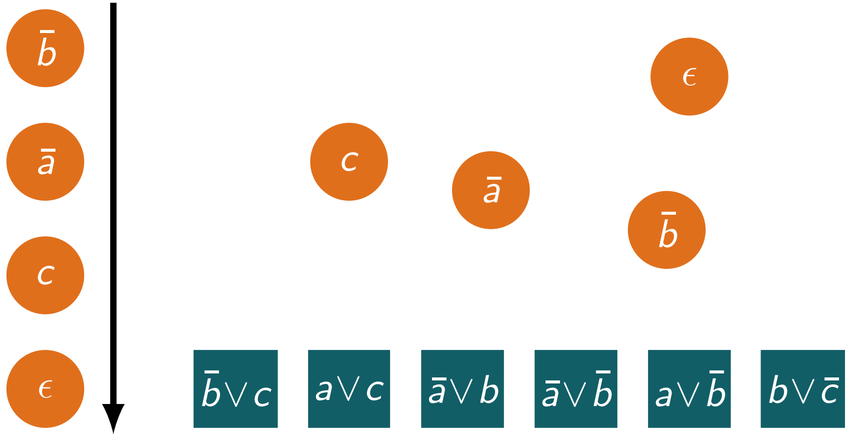
A resolution proof consists of all nodes and edges of the resolution graph

- Graphs from SAT solvers have ~ 400 incoming edges per node
- Resolution proof logging can heavily increase memory usage ($\times 100$)

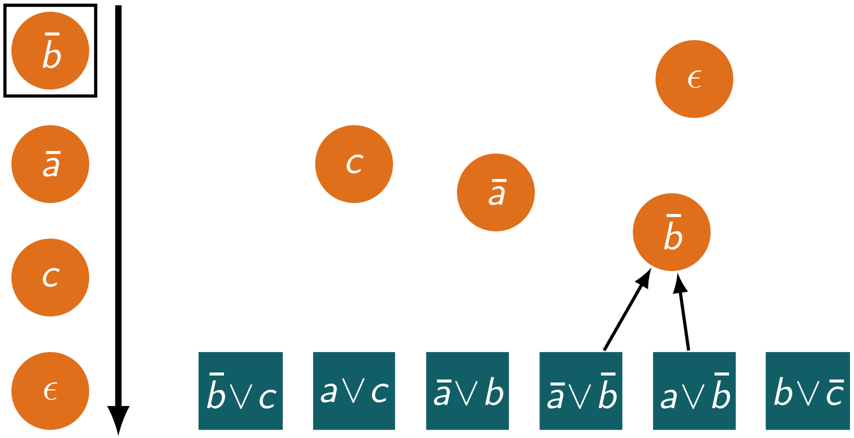
A clausal proof is a list of all nodes sorted by topological order

- Clausal proofs are easy to emit and relatively small
- Clausal proof checking requires to reconstruct the edges (costly)

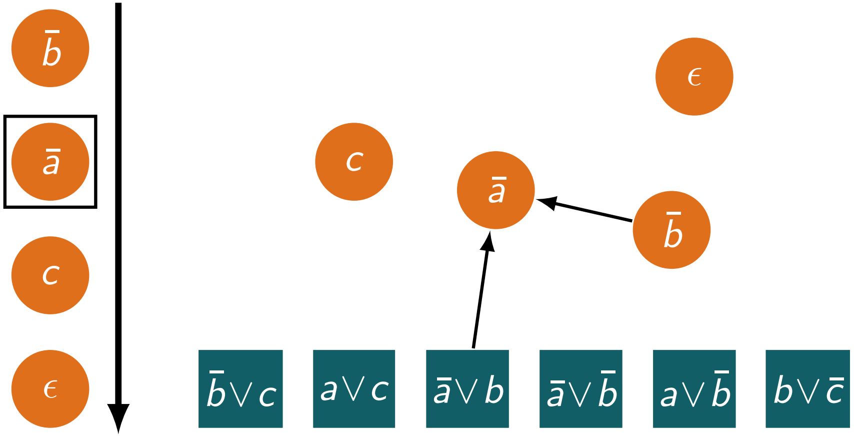
Clausal Proof: Checker has to reconstruct resolution edges



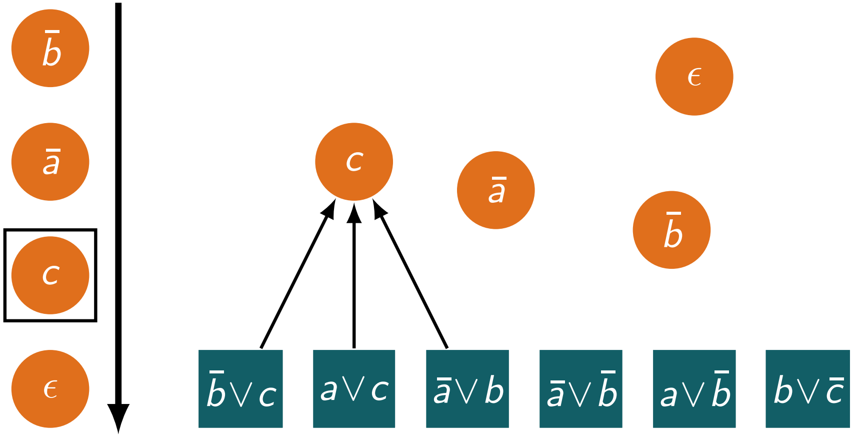
Clausal Proof: Checker has to reconstruct resolution edges



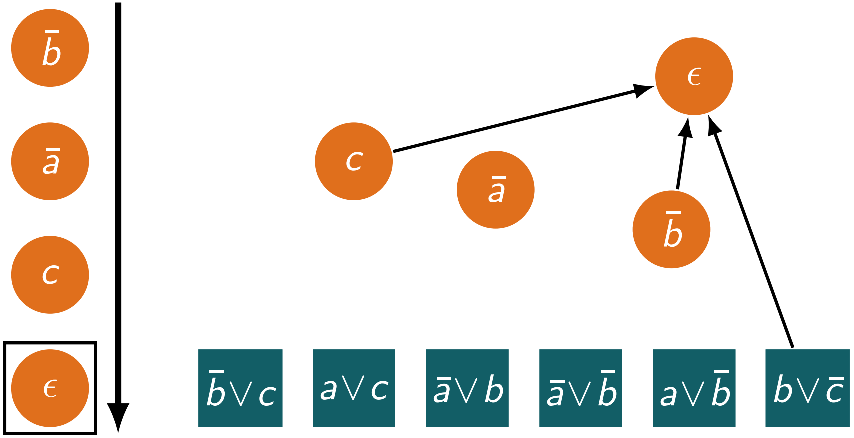
Clausal Proof: Checker has to reconstruct resolution edges



Clausal Proof: Checker has to reconstruct resolution edges



Clausal Proof: Checker has to reconstruct resolution edges



Improvement I: Backwards Checking

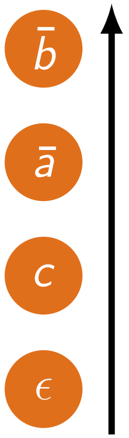
Goldberg and Novikov proposed checking the refutation backwards [DATE 2003]:

- start by validating the empty clause;
- mark all lemmas using conflict analysis;
- only validate marked lemmas.

Advantage: validate fewer lemmas.

Disadvantage: more complex.

We provide a fast open source implementation of this procedure.



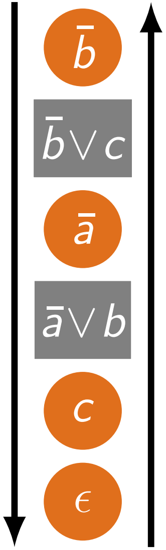
Improvement II: Clause Deletion

We proposed to extend clausal proofs with deletion information [STVR 2014]:

- clause deletion is crucial for efficient solving;
- emit learning and deletion information;
- proof size might double;
- checking speed can be reduced significantly.

Clause deletion can be combined with backwards checking [FMCAD 2013]:

- ignore deleted clauses earlier in the proof;
- optimize clause deletion for trimmed proofs.



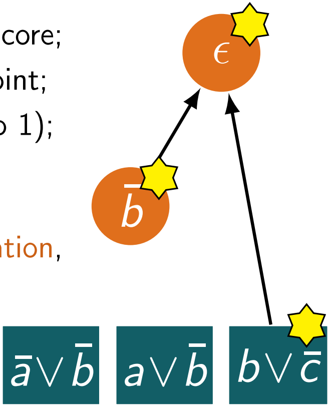
Improvement III: Core-first Unit Propagation

We propose a new unit propagation variant:

1. propagate using clauses already in the core;
2. examine non-core clauses only at fixpoint;
3. if a non-core unit clause is found, goto 1);
4. otherwise terminate.

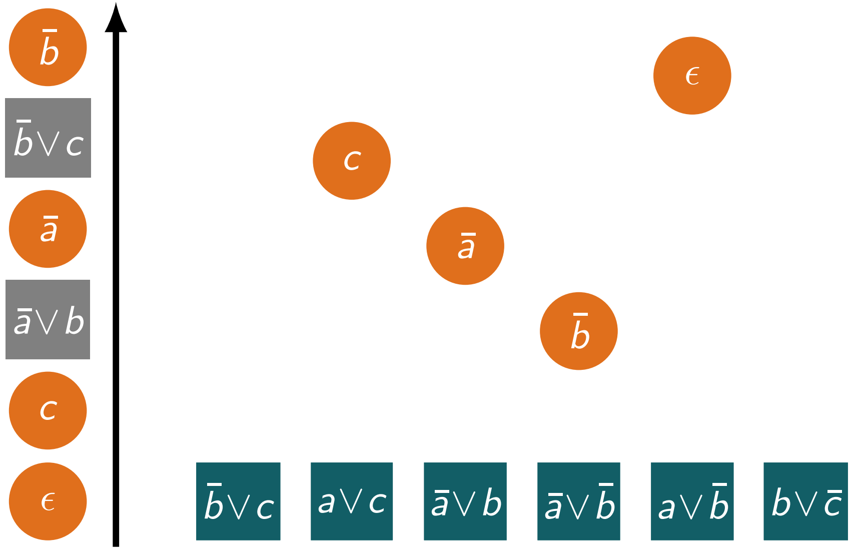
Our variant, called Core-first Unit Propagation, can reduce checking costs considerably.

Fast propagation in a checker is different than fast propagation in a SAT solver.



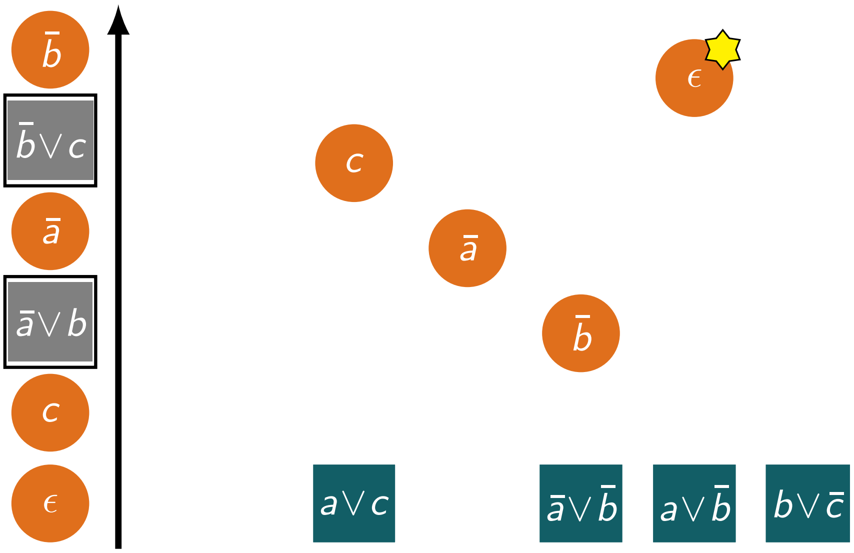
Also, the resulting core and proof are smaller

Checking: Backwards + Core-first + Deletion



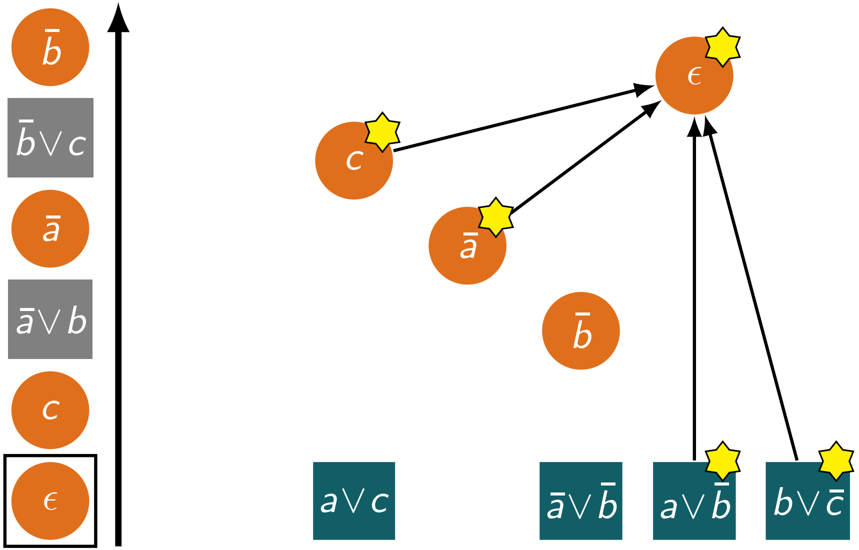
Core-first unit propagation results in smaller cores and proofs

Checking: Backwards + Core-first + Deletion



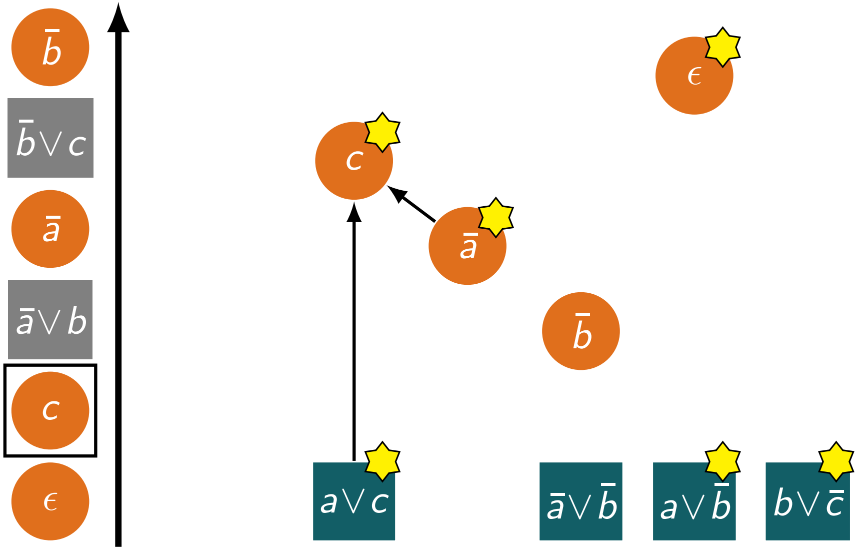
Core-first unit propagation results in smaller cores and proofs

Checking: Backwards + Core-first + Deletion



Core-first unit propagation results in smaller cores and proofs

Checking: Backwards + Core-first + Deletion



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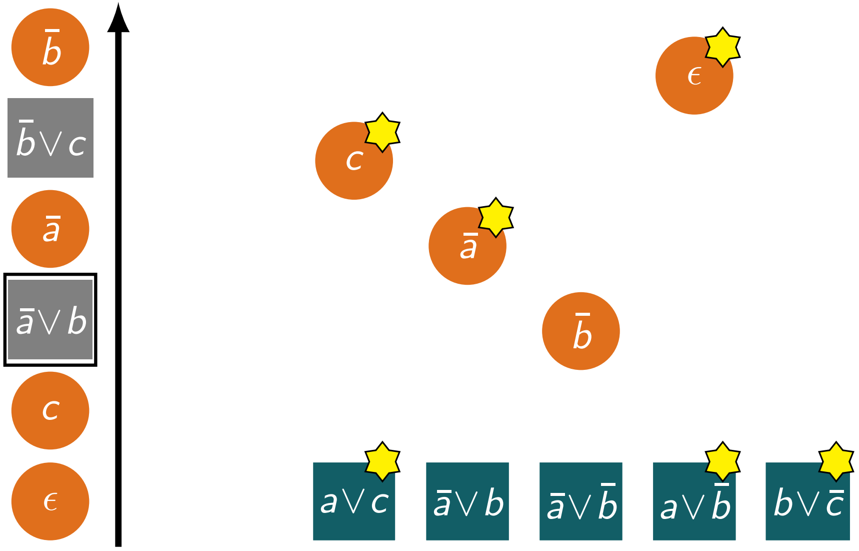

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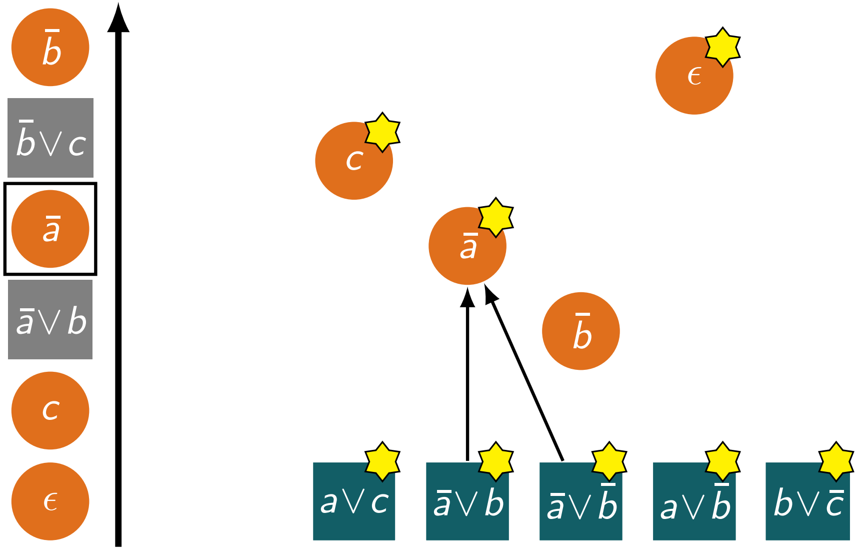
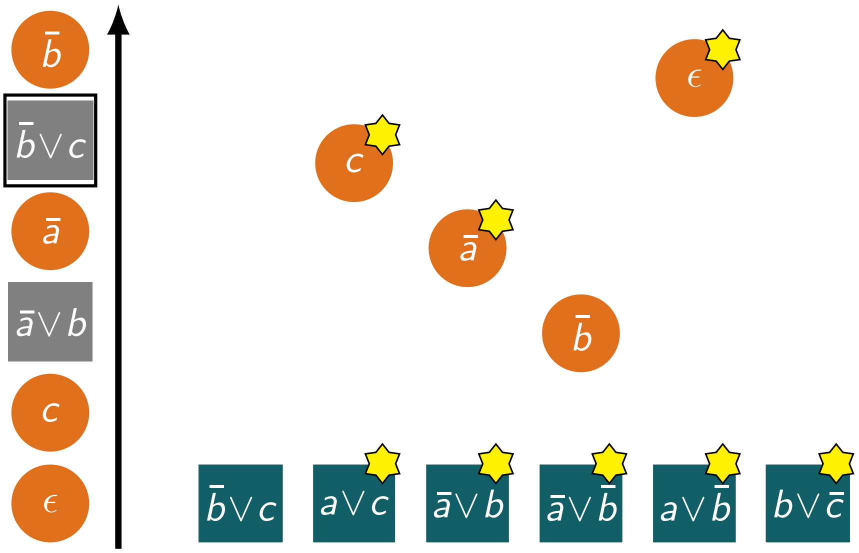
Checking: Backwards + Core-first + Deletion



Core-first unit propagation results in smaller cores and proofs

Checking: Backwards + Core-first + Deletion



Core-first unit propagation results in smaller cores and proofs

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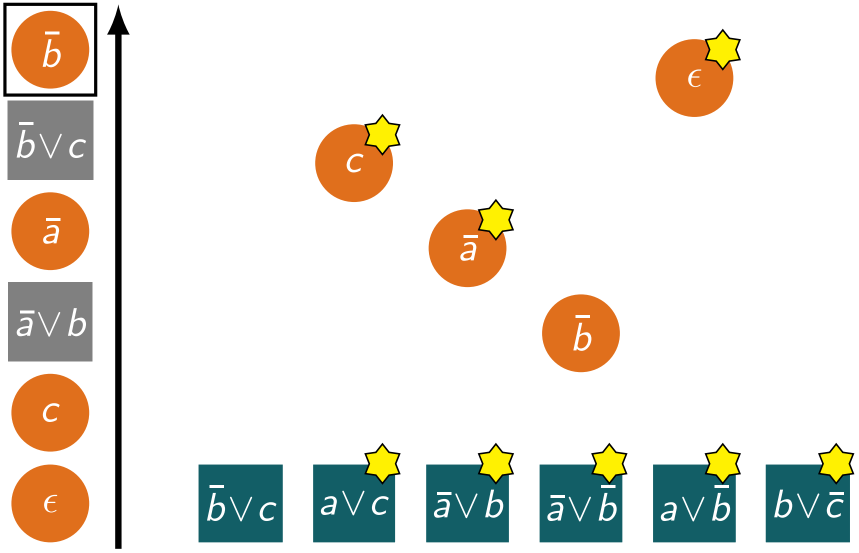
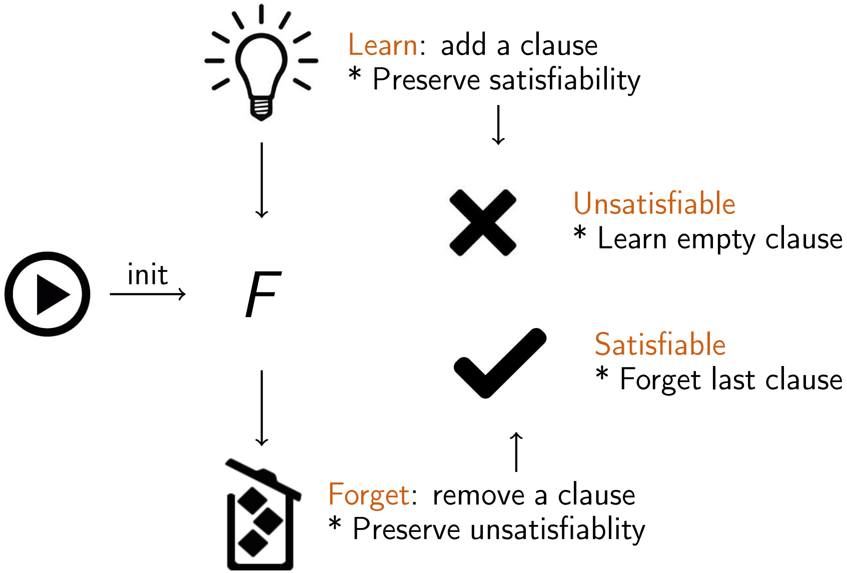


Core-first unit propagation results in smaller cores and proofs

Proof Systems Formats

Clausal Proof System [Järvisalo, Heule, and Biere 2012]



Learn: add a clause
* Preserve satisfiability

Unsatisfiable
* Learn empty clause

init

F

Satisfiable
* Forget last clause

Forget: remove a clause
* Preserve unsatisfiablity

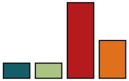
Ideal Properties of a Proof System for SAT Solvers

Easy to Emit

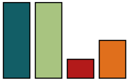
Compact

Checked Efficiently

Expressive



Resolution Proofs
Zhang and Malik, 2003
Van Gelder, 2008; Biere, 2008



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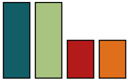
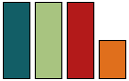
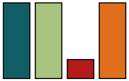


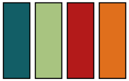
Clausal proofs + clause deletion
Heule, Hunt, Jr., and Wetzler [STVR 2014]



Optimized clausal proof checker
Heule, Hunt, Jr., and Wetzler [FMCAD 2013]



Clausal RAT proofs
Heule, Hunt, Jr., and Wetzler [CADE 2013]



DRAT proofs (RAT + deletion)
Wetzler, Heule, and Hunt, Jr. [SAT 2014]

Ideal Properties of a Proof System for SAT Solvers

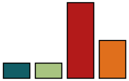
Easy to Emit

Compact

Checked Efficiently

Expressive

Verified



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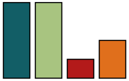

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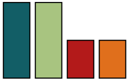

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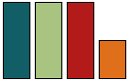

Optimized clausal proof checker
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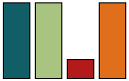
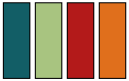


Clausal RAT proofs
Heule, Hunt, Jr., and Wetzler [CADE 2013]



DRAT proofs (RAT + deletion)
Wetzler, Heule, and Hunt, Jr. [SAT 2014]

Proof Formats: The Input Format DIMACS

$$E := (\bar{b} \lor c) \land (a \lor c) \land (\bar{a} \lor b) \land (\bar{a} \lor \bar{b}) \land (a \lor \bar{b}) \land (b \lor \bar{c})$$

The input format of SAT solvers is known as DIMACS

- header starts with `p cnf` followed by the number of variables (n) and the number of clauses (m)
- the next m lines represent the clauses
- positive literals are positive numbers
- negative literals are negative numbers
- clauses are terminated with a `0`

```
p cnf 3 6
-2  3 0
 1  3 0
-1  2 0
-1 -2 0
 1 -2 0
 2 -3 0
```

Most proof formats use a similar syntax.

Proof Formats: TraceCheck Overview

TraceCheck is the most popular resolution-style format.

$$E := (\bar{b} \vee c) \wedge (a \vee c) \wedge (\bar{a} \vee b) \wedge (\bar{a} \vee \bar{b}) \wedge (a \vee \bar{b}) \wedge (b \vee \bar{c})$$

TraceCheck is readable and resolution chains make it relatively compact

$$\begin{aligned}
\langle \text{trace} \rangle &= \{\langle \text{clause} \rangle\} \\
\langle \text{clause} \rangle &= \langle \text{pos} \rangle \langle \text{literals} \rangle \langle \text{antecedents} \rangle \\
\langle \text{literals} \rangle &= \text{“}*\text{”} \mid \{\langle \text{lit} \rangle\}\text{“0”} \\
\langle \text{antecedents} \rangle &= \{\langle \text{pos} \rangle\}\text{“0”} \\
\langle \text{lit} \rangle &= \langle \text{pos} \rangle \mid \langle \text{neg} \rangle \\
\langle \text{pos} \rangle &= \text{“1”} \mid \text{“2”} \mid \cdots \mid \langle \text{max}-\text{idx} \rangle \\
\langle \text{neg} \rangle &= \text{“}-\text{”}\langle \text{pos} \rangle
\end{aligned}$$

```
1  -2   3  0  0
2   1   3  0  0
3  -1   2  0  0
4  -1  -2  0  0
5   1  -2  0  0
6   2  -3  0  0
7  -2   0  4  5  0
8   3   0  1  2  3  0
9   0   6  7  8  0
```

Proof Formats: TraceCheck Examples

TraceCheck is the most popular resolution-style format.

$$E := (\bar{b} \vee c) \wedge (a \vee c) \wedge (\bar{a} \vee b) \wedge (\bar{a} \vee \bar{b}) \wedge (a \vee \bar{b}) \wedge (b \vee \bar{c})$$

TraceCheck is readable and resolution chains make it relatively compact

The clauses **1** to **6** are input clauses

Clause **7** is the resolvent **4** and **5**:

- $(\bar{b}) := (\bar{a} \vee \bar{b}) \diamond (a \vee \bar{b})$

Clause **8** is the resolvent **1**, **2** and **3**:

- $(c) := (\bar{b} \vee c) \diamond (\bar{a} \vee b) \diamond (a \vee c)$
- NB: the antecedents are swapped!

Clause **9** is the resolvent **6**, **7** and **8**:

- $\epsilon := (b \vee \bar{c}) \diamond (\bar{b}) \diamond (c)$

```
1  -2   3  0  0
2   1   3  0  0
3  -1   2  0  0
4  -1  -2  0  0
5   1  -2  0  0
6   2  -3  0  0
7  -2   0  4  5  0
8   3   0  1  2  3  0
9   0   6  7  8  0
```

Proof Formats: TraceCheck Don't Cares

Support for unsorted clauses, unsorted antecedents and omitted literals.

- Clauses are not required to be sorted based on the clause index

```
8  3  0 1 2 3 0        7 -2  0 4 5 0
7 -2  0 4 5 0     ≡    8  3  0 1 2 3 0
```

- The antecedents of a clause can be in arbitrary order

```
7 -2  0 5 4 0          7 -2  0 4 5 0
8  3  0 3 1 2 0   ≡    8  3  0 1 2 3 0
```

- For learned clauses, the literals can be omitted using *

```
7 * 5 4 0              7 -2  0 4 5 0
8 * 3 1 2 0       ≡    8  3  0 1 2 3 0
```

Demo: Clausal Proof to TraceCheck

Proof Formats: Reverse Unit Propagation (RUP)

Unit Propagation

Given an assignment φ, extend it by making unit clauses true — until fixpoint or a clause becomes false

Reverse Unit Propagation (RUP)

A clause $C = (l_1 \vee l_2 \vee \cdots \vee l_k)$ has reverse unit propagation w.r.t. formula F if unit propagation of the assignment $\varphi = \bar{C} = (\bar{l}_1 \wedge \bar{l}_2 \wedge \ldots \wedge \bar{l}_k)$ on F results in a conflict.
We write: $F \wedge \bar{C} \vdash_1 \epsilon$

A clause sequence $C_1, \ldots, C_m$ is a RUP proof for formula F

- $F \wedge C_1 \wedge \cdots \wedge C_{i-1} \wedge \bar{C}_i \vdash_1 \epsilon$
- $C_m = \epsilon$

Proof Formats: RUP, DRUP, RAT, and DRAT

RUP and extensions is the most popular clausal-style format.

$$E := (\bar{b} \vee c) \wedge (a \vee c) \wedge (\bar{a} \vee b) \wedge (\bar{a} \vee \bar{b}) \wedge (a \vee \bar{b}) \wedge (b \vee \bar{c})$$

RUP is much more compact than TraceCheck because it does not includes the resolution steps.

$$\begin{array}{rcl}
\langle\text{proof}\rangle & = & \{\langle\text{lemma}\rangle\} \\
\langle\text{lemma}\rangle & = & \langle\text{delete}\rangle\{\langle\text{lit}\rangle\}\,\text{“0”} \\
\langle\text{delete}\rangle & = & \text{“”} \mid \text{“d”} \\
\langle\text{lit}\rangle & = & \langle\text{pos}\rangle \mid \langle\text{neg}\rangle \\
\langle\text{pos}\rangle & = & \text{“1”} \mid \text{“2”} \mid \cdots \mid \langle\text{max}-\text{idx}\rangle \\
\langle\text{neg}\rangle & = & \text{“}-\text{”}\langle\text{pos}\rangle
\end{array}$$

```
  −2   0
   3   0
   0
```

$$E \wedge (b) \vdash_1 \epsilon$$
$$E \wedge (\bar{b}) \wedge (\bar{c}) \vdash_1 \epsilon$$
$$E \wedge (\bar{b}) \wedge (c) \vdash_1 \epsilon$$

Proof Formats: Open Issues and Challenges

How get useful information from a proof?

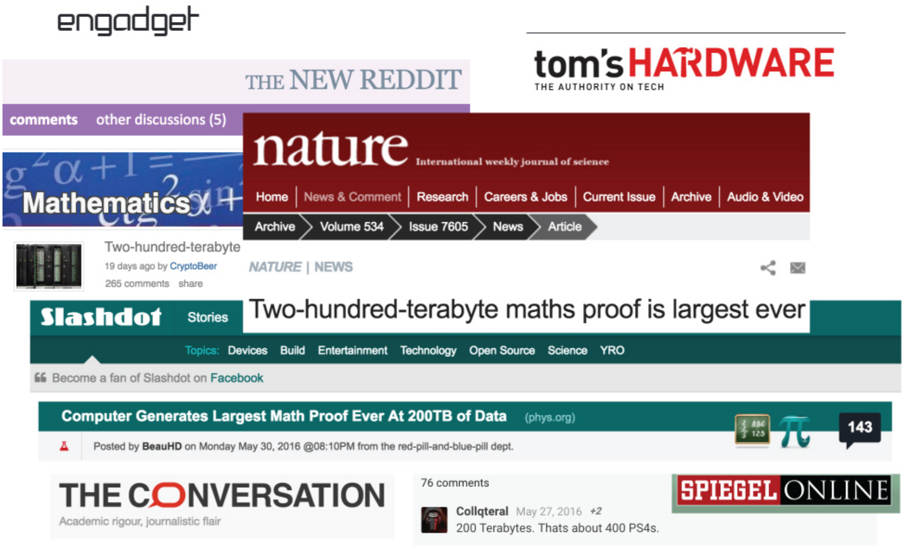
- Clausal or variable core
- Resolution proof from a clausal proof
- Interpolant
- Proof minimization
- Inside the SAT solver or using an external tool?
- What would be a good API to manipulate proofs?

How to store proofs compactly?

- Question is important for resolution and clausal proofs
- Current formats are "readable" and hence large
- Recently we proposed a binary format, reducing size by a factor of three.

Media and Applications

Media: The Largest Math Proof Ever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Posted by **BeauHD** on Monday May 30, 2016 @08:10PM from the red-pill-and-blue-pill dept.

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76 comments

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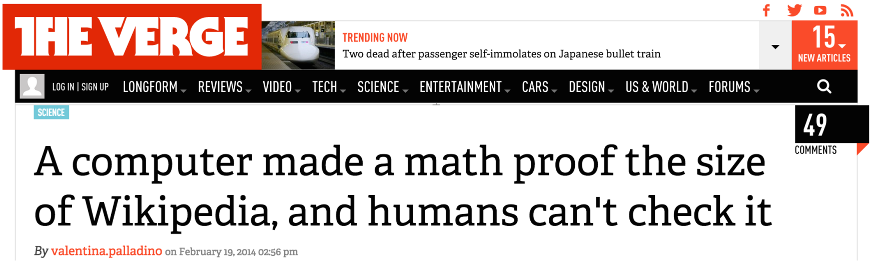
200 Terabytes. Thats about 400 PS4s.

SPIEGEL ONLINE

Applications: Erdős Discrepancy Conjecture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Two dead after passenger self-immolates on Japanese bullet train

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SCIENCE

49 COMMENTS

A computer made a math proof the size of Wikipedia, and humans can't check it

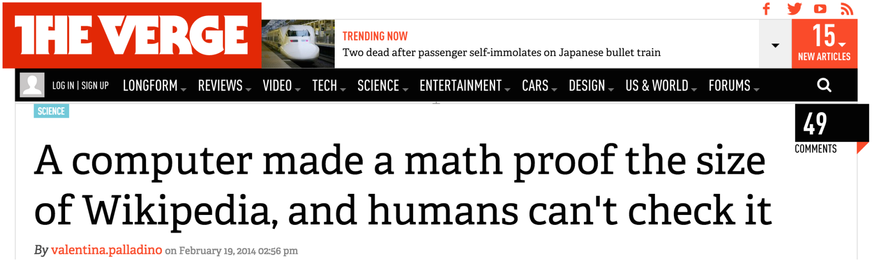
By valentina.palladino on February 19, 2014 02:56 pm

Erdős Discrepancy Conjecture was recently solved using SAT.

The conjecture states that there exists no infinite sequence of -1, +1 such that for all d, k holds that ($x_i \in \{-1, +1\}$):

$$\left|\sum_{i=1}^{k} x_{id}\right| \le 2$$

Applications: Erdős Discrepancy Conjecture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The DRAT proof was 13Gb and checked with our tool DRAT-trim [SAT14]

Applications: SAT Competitions (mandatory proof logging)

DRAT proof logging supported by all the top-tier solvers:

- e.g. Lingeling, MiniSAT, Glucose, and CryptoMiniSAT

DRAT-trim validates proofs in a time similar to solving time.

- computes also unsatisfiable core;
- optimizes the proof for possible later validations; and
- can emit a resolution proof (typically huge).

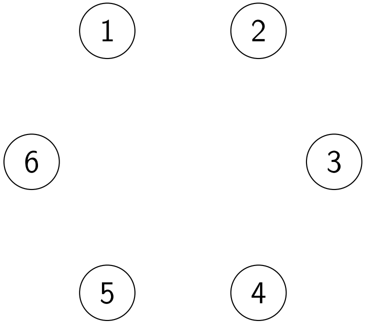
Example run of DRAT-trim on Erdős Discrepancy Proof

```
fud$ ./DRAT-trim EDP2_1161.cnf EDP2_1161.drat
c finished parsing
c detected empty clause; start verification via backward checking
c 23090 of 25142 clauses in core
c 5757105 of 6812396 lemmas in core using 469808891 resolution steps
c 16023 RAT lemmas in core; 5267754 redundant literals in core lemmas
s VERIFIED
```

Applications: Ramsey Numbers

Ramsey Number $R(k)$: What is the smallest n such that any graph with n vertices has either a clique or a co-clique of size k?

$$\begin{aligned} & R(3) = 6 \\ & R(4) = 18 \\ 43 \leq\ & R(5) \leq 49 \end{aligned}$$



SAT solvers can determine that $R(4) = 18$ in 1 second using symmetry breaking; w/o symmetry breaking it requires weeks.

Symmetry breaking can be validated using DRAT [CADE'15]

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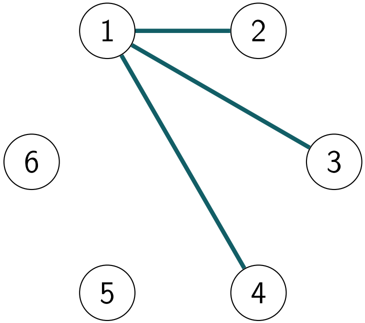

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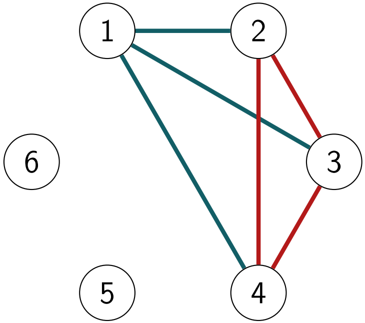

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Symmetry breaking can be validated using DRAT [CADE'15]

Conclusions

Conclusions

Proofs of unsatisfiability useful for several applications:

- Validate results of SAT solvers;
- Extracting minimal unsatisfiable cores;
- Computing Interpolants;
- Tools that use SAT solvers, such as theorem provers.

Challenges:

- Reduce size of proofs on disk and in memory;
- Reduce the cost to validate clausal proofs;
- How to deal with Gaussian elimination, cardinality resolution, and pseudo-Boolean reasoning?

Thanks!