

# Number System

## 1. Positional Number System

$$7358 = 7 \times 10^3 + 3 \times 10^2 + 5 \times 10^1 + 8$$

Decimal number system: 10 digits: 0 to 9.

Better number system: base 12.

- time and angle measure.

## 2. Binary Number System: 0 or 1.

Two ways of converting a decimal number to binary

- repeated division by 2.

- use powers of 2

## 3. Repeated division: Convert 43 to binary

$$\begin{array}{r|l} 2 & 43 \\ \hline 2 & 21 \quad 1 \\ \hline 2 & 10 \quad 1 \\ \hline 2 & 5 \quad 0 \\ \hline 2 & 2 \quad 1 \quad \uparrow \\ \hline 2 & 1 \quad 0 \\ \hline & 0 \quad 1 \end{array}$$

$$43_{10} = 101011_2$$

## 4. Convert 43 to 8 bit binary

	$2^7$	$2^6$	$2^5$	$2^4$	$2^3$	$2^2$	$2^1$	$2^0$
	128	64	32	16	8	4	2	1
$43_{10} =$	0	0	1	0	1	0	1	1

5. Convert fractional numbers into

$$2^3 \ 2^2 \ 2^1 \ 2^0 \cdot 2^{-1} \ 2^{-2} \ 2^{-3} \ 2^{-4}$$

- Convert 37.82 into binary
- Separate whole number from fractional part
- $37_{10} = 100101$ .
- Now the fractional part

$$\begin{array}{r} .82 \\ \times 2 \\ \hline .64 \\ \times 2 \\ \hline .28 \\ \times 2 \\ \hline .56 \\ \times 2 \\ \hline .12 \\ \times 2 \\ \hline .24 \\ \times 2 \\ \hline .48 \\ \times 2 \\ \hline .96 \\ \times 2 \\ \hline .92 \end{array} \quad \begin{array}{l} = .11010001 \\ 1 \downarrow \\ 1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \end{array}$$

## 6. Representation of Negative Numbers.

- 2's Complement relies on the simple

+ fact  $x + (-x) = 0$ .

- Rule :- start with the representation of the positive number.

- flip the bits and add one.

- Convert  $-45$  to binary.

Start with  $+45$  and represent it in 8-bit binary.

$$45_{10} = 00101101$$

Flip the bits  $11010010$

Add 1  $\quad\quad\quad + 1$

$$\underline{11010011} = -45_{10}$$

Verify

$$45_{10} = 00101101$$

$$-45_{10} = 11010011$$

$$\underline{100000000}$$

↓  
0

7. Given a 2's complement number find the decimal equivalent

$$11010110 = ? \text{ in decimal.}$$

Rule: Flip the bits and add one (go figure).

$$\begin{array}{r} \text{Flip} \\ \text{the bits} \\ 11010110 \\ 00101001 \\ \hline \text{Add 1} \quad \quad \quad + 1 \\ 00101010 = 42_{10} \end{array}$$

$$0011010110 = -42_{10}$$

8. Hexadecimal Representation.

0	0000	8	1000	
1	0001	9	1001	
2	0010	10	1010	A
3	0011	11	1011	B
4	0100	12	1100	C
5	0101	13	1101	D
6	0110	14	1110	E
7	0111	15	1111	F

9. Conversion of Binary into Hex and vice versa.

- Group by four from right to left

$$\begin{array}{cccc} \underline{1001} & \underline{1100} & \underline{0111} & \underline{1101} & = & 9C7D_{16} \\ 9 & C & 7 & D & & \end{array}$$

- Convert Hex to Binary

$$AF1D_{16} = 1010\ 1111\ 0001\ 1101_2$$

10. Conversion of Binary into Octal and vice versa.

- Group by three from right to left

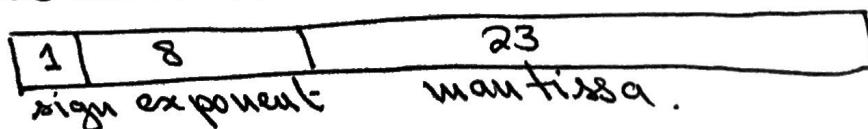
$$\begin{array}{cccc} \underline{110} & \underline{010} & \underline{111} & \underline{001} & = & 6271_8 \\ 6 & 2 & 7 & 1 & & \end{array}$$

- Convert from Octal to Binary

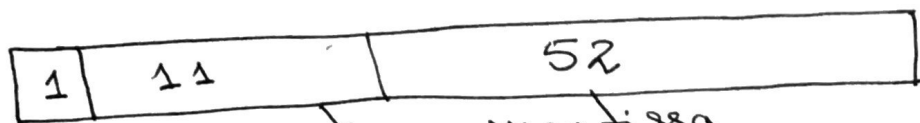
$$5432_8 = \underline{101}\ \underline{100}\ \underline{011}\ \underline{010}_2$$

11. IEEE 754 format for representing floating point numbers

- Consider 32 bit



- Consider 64 bit.



Sign      exponent      mantissa.

12. Conversion of floating point number into IEEE 754 format

- Convert floating point number into scientific notation in powers of 2.

- Convert -0.09375 to IEEE 754 format

- Since negative number first bit is 1.

0.09375

$\times 2$   
0.1875

$\times 2$   
0.3750

$\times 2$   
0.75

$\times 2$   
1.50

$\times 2$   
3.00

0 ↓

0

0

1

1

$$0.09375_{10} = 0.00011_2$$

$$= 1.1 \times 2^{-4}$$

Mantissa = 1

Exponent = exponent bias + power

exponent bias =  $2^{k-1} - 1$

For 8-bit exponent  $k = 8$ , bias = 127

Exponent = bias + power =  $127 - 4 = 123$

$123_{10} = 01111011$

So -  $0.09375 = 1 \ 01111011 \ 100000000000000000000000$

In Hex BDC00000<sub>16</sub>

13. Bit Operators:  $\sim$  &  $|$   $\wedge$ .

A	$\sim A$
0	1
1	0

A	B	$A \& B$
0	0	0
0	1	0
1	0	0
1	1	1

A	B	$A   B$
0	0	0
0	1	1
1	0	1
1	1	1

A	B	$A \wedge B$
0	0	0
0	1	1
1	0	1
1	1	0

$a = 47 = 0010\ 1111$   
 $b = 55 = 0011\ 0111$

Find a)  $a \& b$ , b)  $a | b$   
 c)  $a \wedge b$ .

$$\begin{array}{r} a = 0010\ 1111 \\ b = 0011\ 0111 \\ \hline a \& b = 0010\ 0111 \\ = 39 \end{array}$$

$$\begin{array}{r} a = 0010\ 1111 \\ b = 0011\ 0111 \\ \hline a | b = 0011\ 1111 \\ = 63 \end{array}$$

$$\begin{array}{r} a = 0010\ 1111 \\ b = 0011\ 0111 \\ \hline a \wedge b = 0001\ 1000 \\ = 24 \end{array}$$