Number System

1. Positional Number System

$7358 = 7 \times 10^3 + 3 \times 10^2 + 5 \times 10^1 + 8$

Decimal number system : 10 digits : 0 to 9.

Better number system : base 12.
- time and angle measure.

2. Binary Number System : 0 or 1.

Two ways of converting a decimal number to binary
- repeated division by 2.
- use powers of 2

3. Repeated division : Convert 43 to binary

2	43	
2	21	1
2	10	1
2	5	0
2	2	1 ↑
2	1	0
	0	1

$43_{10} = 101011_2$

4. Convert 43 to 8 bit binary

	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
	128	64	32	16	8	4	2	1
43_{10} =	0	0	1	0	1	0	1	1

5. Convert fractional numbers into

$2^3 \; 2^2 \; 2^1 \; 2^0 \,.\, 2^{-1} \; 2^{-2} \; 2^{-3} \; 2^{-4}$

- Convert 37.82 into binary
- Separate whole number from fractional part
- $37_{10} = 100101$.
- Now the fractional part

$.82 = .11010001$

.82 × 2		
.64	1	↓
× 2		
.28	1	
× 2		
.56	0	
× 2		
.12	1	
× 2		
.24	0	
× 2		
.48	0	
× 2		
.96	0	
× 2		
.92	1	

6. Representation of Negative Numbers.

- 2's Complement relies on the simple
- fact $x + (-x) = 0$.
- Rule :- start with the representation of the positive number.
  - flip the bits and add one.
- Convert -45 to binary.

Start with $+45$ and represent it in 8-bit binary.

45_{10} = 0 0 1 0 1 1 0 1

Flip the bits: 1 1 0 1 0 0 1 0

Add 1: + 1

1 1 0 1 0 0 1 1. = -45_{10}

Verify
45_{10} = 0010 1101.
-45_{10} = 1101 0011

1 00000000 ⇒ 0

7. Given a 2's complement number find the decimal equivalent

11010110 = ? in decimal.

Rule: Flip the bits and add one (go figure).

	11010110
Flip the bits	00101001
Add 1	+ 1
	0010 1010 = 42_{10}

$\therefore$ 11010110 = -42_{10}

8. Hexadecimal Representation.

0	0000	8	1000	
1	0001	9	1001	
2	0010	10	1010	A
3	0011	11	1011	B
4	0100	12	1100	C
5	0101	13	1101	D
6	0110	14	1110	E
7	0111	15	1111	F

9. Conversion of Binary into Hex and vice versa.

- Group by four from right to left

$\underbrace{1001}_{9}\ \underbrace{1100}_{C}\ \underbrace{0111}_{7}\ \underbrace{1101}_{D}\,{}_2 = 9C7D_{16}$

- Convert Hex to Binary

$AF1D_{16} = 1010\ 1111\ 0001\ 1101_2$

10. Conversion of Binary into Octal and vice versa.

- Group by three from right to left

$\underbrace{110}_{6}\ \underbrace{010}_{2}\ \underbrace{111}_{7}\ \underbrace{001}_{1} = 6271_8$

- Convert from Octal to Binary

$5432_8 = \underbrace{101}\ \underbrace{100}\ \underbrace{011}\ \underbrace{010}_2$

11. IEEE 754 format for representing floating point numbers

- Consider 32 bit

1	8	23
sign	exponent	mantissa.

- Consider 64 bit.

Sign	exponent	mantissa
1	11	52

12. Conversion of floating point number into IEEE 754 format

- Convert floating point number into scientific notation in powers of 2.
- Convert -0.09375 to IEEE 754 format
- Since negative number first bit is 1.

$$
\begin{array}{rl}
0.09375 & \\
\times 2 & \\
\hline
0.1875 & 0 \\
\times 2 & \\
\hline
0.3750 & 0 \\
\times 2 & \\
\hline
0.75 & 0 \\
\times 2 & \\
\hline
0.50 & 1 \\
\times 2 & \\
\hline
0.00 & 1
\end{array}
\downarrow
$$

$0.09375_{10} = .00011_2$

$= 1.\underline{1} \times 2^{-4}$

Mantissa = 1

Exponent = exponent bias + power

exponent bias $= 2^{k-1} - 1$

For 8-bit exponent $k = 8$, bias = 127

Exponent = bias + power = $127 - 4 = 123$

$123_{10} = 01111011$.

So $-0.09375 = 1\ 01111011\ 10000000000000000000000$

In Hex $BDC00000_{16}$

13. Bit Operators: ~ & | ^.

A	~A
0	1
1	0

A	B	A&B
0	0	0
0	1	0
1	0	0
1	1	1

A	B	A|B
0	0	0
0	1	1
1	0	1
1	1	1

A	B	A^B
0	0	0
0	1	1
1	0	1
1	1	0

a = 47 = 0010 1111

b = 55 = 0011 0111.

Find a) a&b, b) a|b

c) a^b.

a = 0010 1111.
b = 0011 0111
a&b = 0010 0111
= 39

a = 0010 1111
b = 0011 0111
a|b = 0011 1111
= 63

a = 0010 1111.
b = 0011 0111
a^b = 0001 1000
= 24