CASE SPLITTING TRANSFORMATION

Alessandro Coglio

Kestrel Institute

2019

Rephrase Function by Cases

old function : $f : \mathcal{U} \to \mathcal{U}$

cases : $c_1, \ldots, c_p \subseteq \mathcal{U}$, $p \geq 1$

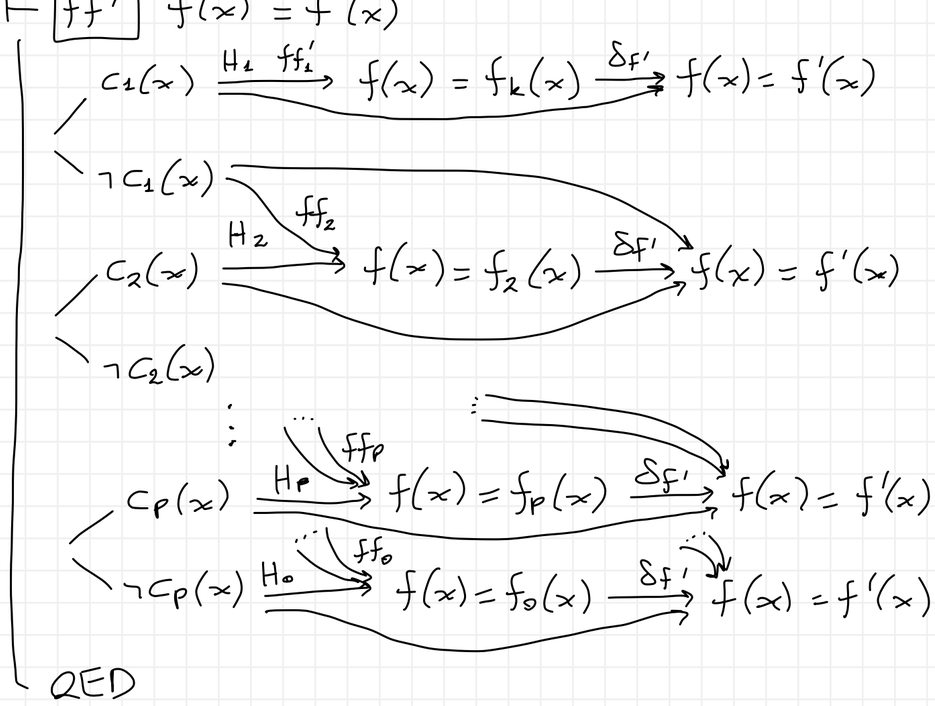
existing theorems : $\vdash$ [ff_k] $h_k(x) \Rightarrow f(x) = f_k(x)$, $0 \leq k \leq p$ — h_k may be absent

[H_k] $\left[\bigwedge_{1 \leq k' < k} \neg c_{k'}(x)\right] \wedge c_k(x) \Rightarrow h_k(x)$, $1 \leq k \leq p$

[H_0] $\left[\bigwedge_{1 \leq k \leq p} \neg c_k(x)\right] \Rightarrow h_0(x)$

new function : $f'(x) \triangleq$ <u>if</u> $c_1(x)$ <u>then</u> $f_1(x)$ <u>else</u> ... <u>if</u> $c_p(x)$ <u>then</u> $f_p(x)$ <u>else</u> $f_0(x)$ — non-recursive

$\vdash$ [ff'] $f(x) = f'(x)$



QED

$x \rightsquigarrow x_1, \ldots, x_n$ — generalizes to more parameters

Guards

$\boxed{\forall f}$ $\gamma_{\gamma_f}(x) \wedge \dots$

$\gamma_{f'}(x) \triangleq \gamma_f(x)$

$\boxed{GC_k}$ $\gamma_f(x) \wedge \left[\bigwedge_{1 \le k' < k} \neg c_{k'}(x)\right] \Rightarrow \gamma_{c_k}(x)$, $1 \le k \le p$

$\boxed{Gf_k}$ $\gamma_f(x) \wedge \left[\bigwedge_{1 \le k' < k} \neg c_{k'}(x)\right] \wedge c_k(x) \Rightarrow \gamma_{f_k}(x)$, $1 \le k \le p$

$\boxed{Gf_0}$ $\gamma_f(x) \wedge \left[\bigwedge_{1 \le k \le p} \neg c_k(x)\right] \Rightarrow \gamma_{f_0}(x)$

$\vdash \boxed{\forall f'}$

$\omega_{f'}(x) = \cancel{\gamma_{\gamma_f}(x) \wedge}$ ($\forall f$)

$\cancel{[\gamma_f(x) \Rightarrow \gamma_{c_1}(x)] \wedge}$ (GC_1)

$\cancel{[\gamma_f(x) \wedge c_1(x) \Rightarrow \gamma_{f_1}(x)] \wedge}$ (Gf_1)

$\cancel{[\gamma_f(x) \wedge \neg c_1(x) \Rightarrow \gamma_{c_2}(x)] \wedge}$ (GC_2)

$\cancel{[\gamma_f(x) \wedge \neg c_1(x) \wedge c_2(x) \Rightarrow \gamma_{f_2}(x)] \wedge}$ (Gf_2)

$\vdots$

$[\gamma_f(x) \wedge \neg c_1(x) \wedge \dots \wedge \neg c_p(x) \Rightarrow \gamma_{f_0}(x)]$ (Gf_0)

QED

generalizes to more parameters, as before