PARTIAL EVALUATION TRANSFORMATION

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Non-Recursive Function

$$\left.\begin{array}{ll} \text{old function :} & f(x,y) \triangleq e(x,y) \\ y \triangleq \tilde{y} & \text{— } \tilde{y} \text{ ground term} \end{array}\right\} \; y \text{ static , } x \text{ dynamic}$$

new function : $f'(x) \triangleq e(x,\tilde{y})$

$\vdash \boxed{ff'}$ $y=\tilde{y} \Rightarrow f(x,\tilde{y}) = f'(x)$ — trivial , by δ_f and $\delta_{f'}$

then optimize f' via further transformations

$$\left.\begin{array}{l}
\boxed{\checkmark f} \;\; \gamma_{\gamma_f}(x,y) \wedge [\gamma_f(x,y) \Rightarrow \gamma_e(x,y)] \\
\gamma_{f'}(x) \triangleq \gamma_f(x,\tilde{y}) \\
\vdash \boxed{\checkmark f'} \;\; \omega_{f'}(x) \\
\quad \omega_{f'}(x) = \underbrace{\cancel{\gamma_{\gamma_f}(x,\tilde{y})}}_{\checkmark f} \wedge [\gamma_f(x,\tilde{y}) \xrightarrow{\checkmark f} \cancel{\gamma_e(x,\tilde{y})}] \\
\text{QED}
\end{array}\right\} \text{guards}$$

$$\left.\begin{array}{l} x \rightarrow x_1,\dots,x_n \\ y \rightarrow y_1,\dots,y_m \\ \tilde{y} \rightarrow \tilde{y}_1,\dots,\tilde{y}_m \end{array}\right\} \text{generalizes to more parameters } (m \neq 0)$$

Recursive Function – Default Treatment

old function : $f(x,y) \triangleq \ldots f \ldots$

$y \triangleq \tilde{y}$ — $\tilde{y}$ ground term

$\Big\}$ y static , x dynamic

new function : $f'(x) \triangleq f(x,\tilde{y})$ — non-recursive — preliminary simple approach

$\vdash \boxed{ff'}\ \ y=\tilde{y} \Rightarrow f(x,\tilde{y}) = f'(x)$ — trivial , by $\delta_{f'}$

optimize f' via successive transformations , which may unfold the recursion completely if driven by y

$\boxed{\surd f}\ \ \gamma_{\gamma_f}(x,y) \land \ldots$

$\gamma_{f'}(x) \triangleq \gamma_f(x,\tilde{y})$

$\vdash \boxed{\surd f'}\ \ \omega_{f'}(x)$

$\omega_{f'}(x) = \underset{\surd f}{\cancel{\gamma_{\gamma_f}(x,\tilde{y})}} \land \left[\gamma_f(x,\tilde{y}) \Rightarrow \cancel{\gamma_f(x,\tilde{y})}\right]$

QED

$\Big\}$ guards

generalizes to more parameters as in non-recursive case

Recursive Function with Unchanging Static Argument

old function: $f(x,y) \triangleq \underline{\text{if}}\ a(x,y)\ \underline{\text{then}}\ b(x,y)\ \underline{\text{else}}\ c(x,y,f(d(x,y),y))$

$\boxed{\tau_f}\quad \neg a(x,y) \Rightarrow \mu_f(d(x,y)) \prec_f \mu_f(x)$ — x measured argument, y not measured argument

$y \triangleq \tilde{y}$ — $\tilde{y}$ ground term

new function: $f'(x) \triangleq \underline{\text{if}}\ a(x,\tilde{y})\ \underline{\text{then}}\ b(x,\tilde{y})\ \underline{\text{else}}\ c(x,\tilde{y},f'(d(x,\tilde{y})))$

$\mu_{f'}(x) \triangleq \mu_f(x) \qquad \prec_{f'} \triangleq \prec_f$

$\vdash \boxed{\tau_{f'}}\quad \neg a(x,\tilde{y}) \Rightarrow \mu_{f'}(d(x,\tilde{y})) \prec_{f'} \mu_{f'}(x)$

$$\neg a(x,\tilde{y}) \xrightarrow[y:=\tilde{y}]{\tau_f} \mu_f(d(x,\tilde{y})) \prec_f \mu_f(x)$$

$$\delta_{\mu_{f'}} \| \qquad \delta_{\prec_{f'}} \| \qquad \| \delta_{\mu_{f'}}$$

$$\mu_{f'}(d(x,\tilde{y})) \prec_{f'} \mu_{f'}(x)$$

QED

$\vdash \boxed{ff'}\quad y=\tilde{y} \Rightarrow f(x,y) = f'(x)$

induct f'

$$a(x,\tilde{y}) \xrightarrow{\delta_{f'}} f'(x) = b(x,\tilde{y})$$
$$a(x,\tilde{y}) \xrightarrow[y:=\tilde{y}]{\delta_f} f(x,\tilde{y}) = b(x,\tilde{y})$$
$$\Rightarrow f'(x) = f(x,\tilde{y})$$

$$\neg a(x,\tilde{y}) \xrightarrow{\delta_{f'}} f'(x) = c(x,\tilde{y},f'(d(x,\tilde{y})))$$
$$\neg a(x,\tilde{y}) \xrightarrow[y:=\tilde{y}]{\delta_f} f(x,y) = c(x,\tilde{y},f(d(x,\tilde{y}),\tilde{y})) \overset{IH}{=} c(x,\tilde{y},f'(d(x,\tilde{y})))$$
$$\Rightarrow f'(x) = f(x,\tilde{y})$$

QED

$\boxed{\gamma f}$ $\gamma_{\gamma_f}(x,y) \wedge [\gamma_f(x,y) \Rightarrow \gamma_a(x,y) \wedge [a(x,y) \Rightarrow \gamma_b(x,y)] \wedge [\neg a(x,y) \Rightarrow \gamma_d(x,y) \wedge \gamma_f(d(x,y),y) \wedge \gamma_c(x,y,f(d(x,y),y))]]$

$\gamma_{f'}(\bar{x}) \triangleq \gamma_f(x,\tilde{y})$

$\vdash \boxed{\gamma f'}$

$\omega_{f'}(x) = [\gamma_{f'}(\bar{x}) \Rightarrow \gamma_a(x,\tilde{y}) \wedge [a(x,\tilde{y}) \Rightarrow \gamma_b(x,\tilde{y})] \wedge [\neg a(x,\tilde{y}) \Rightarrow \gamma_d(x,\tilde{y}) \wedge \gamma_{f'}(d(x,\tilde{y})) \wedge \gamma_c(x,\tilde{y},f'(d(x,\tilde{y})))]]$

$\gamma f,\ y := \tilde{y}$

QED

generalizes to more parameters as in non-recursive case — $y_1, \cdots, y_m$ all unchanging