

DOMAIN RESTRICTION TRANSFORMATION

Alessandro Coglio

Kestrel Institute

2017

Restrict Domain of Non-Recursive Function

old function: $f(\bar{x}) \triangleq e(\bar{x})$ $\bar{x} = (x_1, \dots, x_n)$ $n > 0$

restricting predicate: $R \subseteq \mathcal{U}^n$

new function: $f'(\bar{x}) \triangleq$ if $R(\bar{x})$ then $e(\bar{x})$ else ...
 ↙ any value (irrelevant)

⊢ ff' $R(\bar{x}) \Rightarrow f(\bar{x}) = f'(\bar{x})$ - relation between f and f'
 $\left\{ \begin{array}{l} R(\bar{x}) \xrightarrow{\delta_{f'}} f'(\bar{x}) = e(\bar{x}) \\ \delta_f f(\bar{x}) \end{array} \right.$
 QED

Restrict Domain of Recursive Function

old function: $f(\bar{x}) \triangleq \underline{\text{if } a(\bar{x}) \text{ then } b(\bar{x}) \text{ else } c(\bar{x}, f(\bar{d}(\bar{x})))}$ $\bar{x} = (x_1, \dots, x_n)$ $\bar{d}(\bar{x}) = (d_1(\bar{x}), \dots, d_n(\bar{x}))$ $n > 0$

$$\boxed{\tau_f} \quad \neg a(\bar{x}) \Rightarrow \mu_f(\bar{d}(\bar{x})) <_f \mu_f(\bar{x})$$

restricting predicate: $R \in \mathcal{U}^n$

condition: $\boxed{R_d} \quad R(\bar{x}) \wedge \neg a(\bar{x}) \Rightarrow R(\bar{d}(\bar{x}))$ — preservation of R across recursive calls

new function: $f'(\bar{x}) \triangleq \underline{\text{if } R(\bar{x}) \text{ then } [\text{if } a(\bar{x}) \text{ then } b(\bar{x}) \text{ else } c(\bar{x}, f'(\bar{d}(\bar{x})))]} \text{ else } \dots$

$$\mu_{f'}(\bar{x}) \triangleq \mu_f(\bar{x}) \quad <_{f'} \triangleq <_f$$

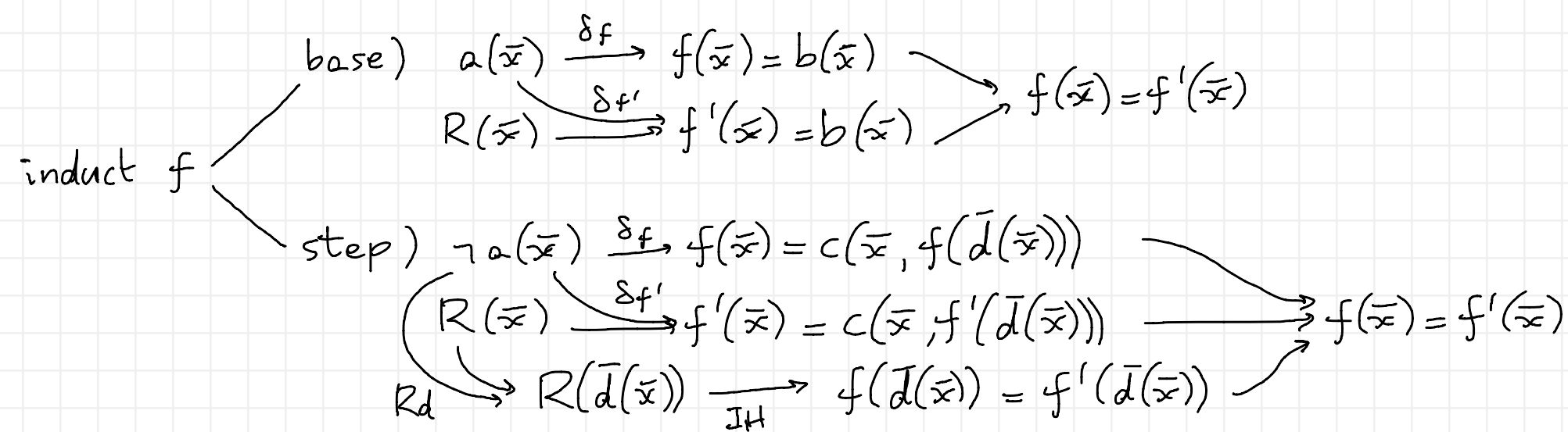
any value (irrelevant)

$\vdash \boxed{\tau_{f'}} \quad R(\bar{x}) \wedge \neg a(\bar{x}) \Rightarrow \mu_{f'}(\bar{d}(\bar{x})) <_{f'} \mu_{f'}(\bar{x})$ — f' terminates

$$\neg a(\bar{x}) \xrightarrow{\tau_f} \begin{array}{ccc} \mu_f(\bar{d}(\bar{x})) <_f \mu_f(\bar{x}) \\ \delta_{\mu_{f'}} \parallel & \parallel \delta_{f'} & \parallel \delta_{\mu_{f'}} \\ \mu_{f'}(\bar{d}(\bar{x})) <_{f'} \mu_{f'}(\bar{x}) \end{array}$$

QED

$\vdash \boxed{ff'} \quad R(\bar{x}) \Rightarrow f(\bar{x}) = f'(\bar{x})$ — relation between f and f'



QED

Guards for the Non-Recursive Case

old function: $f(\bar{x}) \triangleq e(\bar{x})$

$$\boxed{\sqrt{f}} \quad \gamma_{f'}(\bar{x}) \wedge [\gamma_f(\bar{x}) \Rightarrow \gamma_e(\bar{x})]$$

$\boxed{\sqrt{R}}$

condition: $\boxed{GR} \quad \gamma_f(\bar{x}) \Rightarrow \gamma_R(\bar{x})$ — R well-defined at least over the guard of f

new function: $f'(\bar{x}) \triangleq$ if $R(\bar{x})$ then $e(\bar{x})$ else ...

$$\gamma_{f'}(\bar{x}) \triangleq \gamma_f(\bar{x}) \wedge R(\bar{x})$$

$\vdash \boxed{\sqrt{f'}}$

$$\omega_{f'}(\bar{x}) = \cancel{\gamma_{f'}(\bar{x})} \wedge \cancel{[\gamma_f(\bar{x}) \Rightarrow \gamma_e(\bar{x})]} \wedge \cancel{[\gamma_f(\bar{x}) \wedge R(\bar{x}) \Rightarrow \gamma_R(\bar{x}) \wedge [R(\bar{x}) \Rightarrow \gamma_e(\bar{x})]} \wedge \cancel{[\neg R(\bar{x}) \Rightarrow \dots]}]$$

(Note: The above expression is a complex logical derivation with annotations. Arrows labeled 'GR' and '√f' point from the condition and guard to the corresponding parts of the expression.)

QED

Guards for the Recursive Case

old function: $f(\bar{x}) \triangleq \underline{\text{if}}\ a(\bar{x})\ \underline{\text{then}}\ b(\bar{x})\ \underline{\text{else}}\ c(\bar{x}, f(\bar{d}(\bar{x})))$

$$\gamma_{\bar{d}}(\bar{x}) = \gamma_{d_1}(\bar{x}) \wedge \dots \wedge \gamma_{d_n}(\bar{x})$$

$$\boxed{\forall f} \quad \gamma_{\gamma_f}(\bar{x}) \wedge \left[\gamma_f(\bar{x}) \Rightarrow \gamma_a(\bar{x}) \wedge [a(\bar{x}) \Rightarrow \gamma_b(\bar{x})] \wedge [\neg a(\bar{x}) \Rightarrow \gamma_{\bar{d}}(\bar{x}) \wedge \gamma_f(\bar{d}(\bar{x})) \wedge \gamma_c(\bar{x}, f(\bar{d}(\bar{x})))] \right]$$

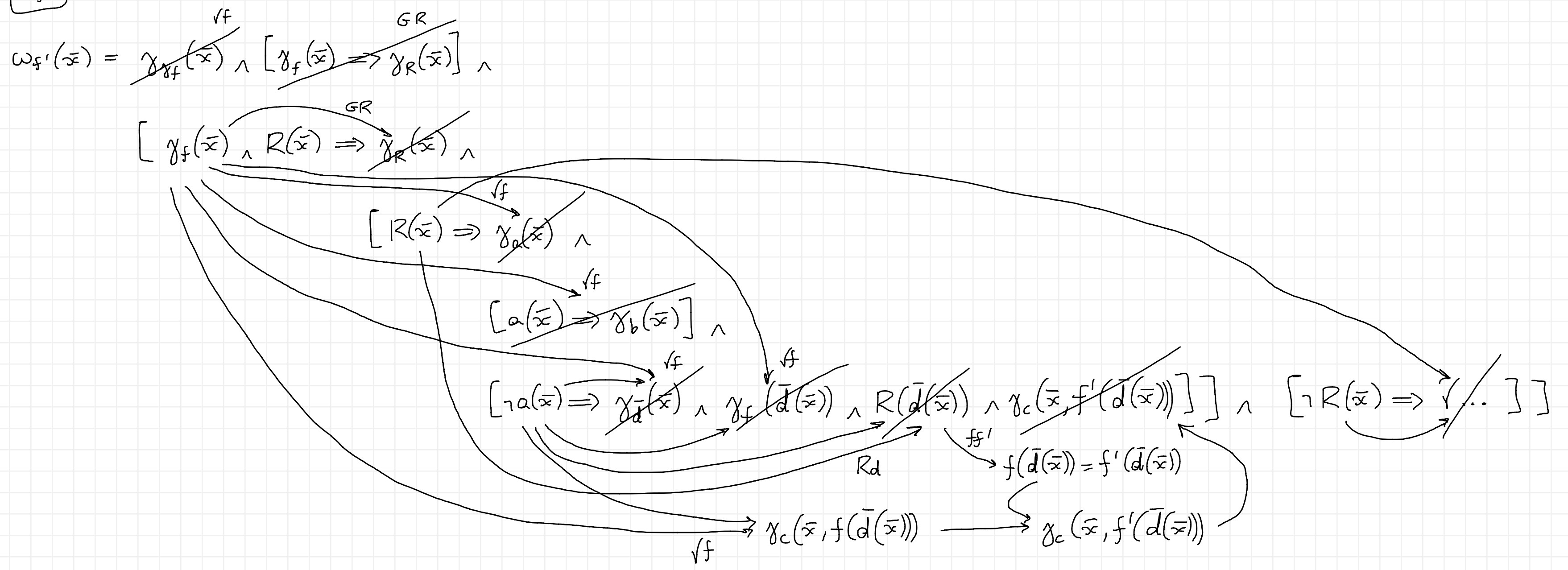
$\boxed{\forall R}$

condition: $\boxed{GR} \quad \gamma_f(\bar{x}) \Rightarrow \gamma_R(\bar{x})$ — as in the non-recursive case

new function: $f'(\bar{x}) \triangleq \underline{\text{if}}\ R(\bar{x})\ \underline{\text{then}}\ [\underline{\text{if}}\ a(\bar{x})\ \underline{\text{then}}\ b(\bar{x})\ \underline{\text{else}}\ c(\bar{x}, f'(\bar{d}(\bar{x})))]$ else ...

$\gamma_{f'}(\bar{x}) \triangleq \gamma_f(\bar{x}) \wedge R(\bar{x})$ — as in the non-recursive case

$\vdash \boxed{\forall f'}$



QED