SOLVING TRANSFORMATION

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Solution by User

old specification: $S(f) \triangleq \ldots$, $S \subseteq \mathcal{U}^n \to \mathcal{U}^m$

user-supplied solution: $\hat{f} \triangleq \ldots$, $\hat{f} : \mathcal{U}^n \to \mathcal{U}^m$

condition: $\boxed{\text{SOL}}$ $S(\hat{f})$ — solution solves

$S'(f) \triangleq [f = \hat{f}]$ — new specification

$\boxed{SS'}$ $\vdash S'(f) \Rightarrow S(f)$ — correctness theorem

$$\begin{tikzcd} S'(f) \arrow[r, "\delta_{S'}"] & f = \hat{f} \arrow[dr] & \\ & & S(f) \\ \text{SOL} \arrow[r] & S(\hat{f}) \arrow[ur] & \end{tikzcd}$$

QED

Solution by Rewriting

old specification: $S(f) \triangleq [\forall \bar{x}.\, R(\bar{x}, f(\bar{x}))]$, $S \subseteq \mathcal{U}^n \to \mathcal{U}^m$, $R \subseteq \mathcal{U}^{n+m}$
(form $\boxed{Rf}$ in 'Specifications & Refinements' notes)

$R(\bar{x}, f(\bar{x})) \xrightarrow{\text{rewriting}} \Phi[\bar{x}]$ — matrix $R(\bar{x}, f(\bar{x}))$ rewrites to term $\Phi[\bar{x}]$

① $\Phi[\bar{x}] = \top$ — matrix rewrites to true $\Longrightarrow$ no constraints

$\boxed{RW}$ $\vdash R(\bar{x}, f(\bar{x})) = \top$ — rewriting theorem

$\hat{f}(\bar{x}) \triangleq \ldots$ — determined solution (can be anything)

$\boxed{SOL}$ $\vdash S(\hat{f})$ — solution solves

RW $\xrightarrow[f:=\hat{f}]{} \forall \bar{x}.\, R(\bar{x}, \hat{f}(\bar{x})) \xrightarrow{\delta_S} S(\hat{f})$

QED

② $\Phi[\bar{x}] = [f(\bar{x}) = \Psi[\bar{x}]]$ — matrix rewrites to equality of f to something

$\boxed{RW}$ $\vdash R(\bar{x}, f(\bar{x})) = [f(\bar{x}) = \Psi[\bar{x}]]$ — rewriting theorem

$\hat{f}(\bar{x}) \triangleq \Psi[\bar{x}]$ — determined solution

$\boxed{SOL}$ $\vdash S(\hat{f})$

RW $\xrightarrow[f:=\hat{f}]{} \forall \bar{x}.\, [R(\bar{x}, \hat{f}(\bar{x})) = [\hat{f}(\bar{x}) = \Psi[\bar{x}]]] \xrightarrow{\delta_{\hat{f}}} \forall \bar{x}.\, R(\bar{x}, \hat{f}(\bar{x})) \xrightarrow{\delta_S} S(f)$

QED

3 $\Phi[\bar{x}] = \underline{\text{if}} \ldots \underline{\text{then}}\ T\ \underline{\text{else}}\ [f(\bar{x}) = \Psi[\bar{x}]]$ — matrix rewrites to $\underline{\text{if}}$ tree (representative shown)

one leaf is equality
other leafs are T $\Big\} \Rightarrow$ same as 2 — T leaves do not contribute constraints

$S'(f) \triangleq [f = \hat{f}]$
$\boxed{SS'} \vdash S'(f) \Rightarrow S(f)$ $\Big\}$ as in solution by user, in all cases 1, 2, 3