

SOLVING TRANSFORMATION

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Solution by User

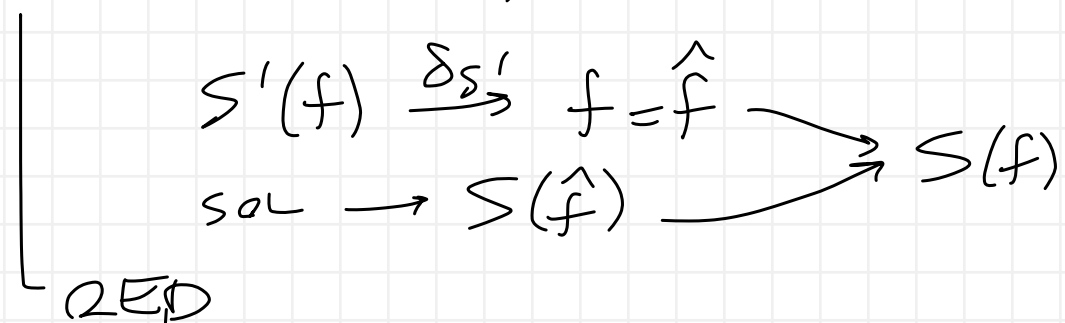
old specification: $S(f) \triangleq \dots$, $S \subseteq U^n \rightarrow U^m$

user-supplied solution: $\hat{f} \triangleq \dots$, $\hat{f}: U^n \rightarrow U^m$

condition: $\boxed{\text{sol}}$ $S(\hat{f})$ - solution solves

$S'(f) \triangleq [f = \hat{f}]$ - new specification

$\boxed{SS'}$ $\vdash S'(f) \Rightarrow S(f)$ - correctness theorem



Solution by Rewriting

old specification: $S(f) \triangleq [\forall \bar{x}. R(\bar{x}, f(\bar{x}))]$, $S \subseteq U^n \rightarrow U^m$, $R \subseteq U^{n+m}$
(form \boxed{Rf} in 'Specifications & Refinements' notes)

$R(\bar{x}, f(\bar{x})) \xrightarrow{\text{rewriting}} \Phi[\bar{x}]$ — matrix $R(\bar{x}, f(\bar{x}))$ rewrites to term $\Phi[\bar{x}]$

① $\Phi[\bar{x}] = T$ — matrix rewrites to true \Rightarrow no constraints

$\boxed{RW} \vdash R(\bar{x}, f(\bar{x})) = T$ — rewriting theorem

$\hat{f}(\bar{x}) \triangleq \dots$ — determined solution (can be anything)

$\boxed{SOL} \vdash S(\hat{f})$ — solution solves

$\left[\begin{array}{l} RW \xrightarrow{f := \hat{f}} \forall \bar{x}. R(\bar{x}, \hat{f}(\bar{x})) \xrightarrow{\delta_S} S(\hat{f}) \\ QED \end{array} \right.$

② $\Phi[\bar{x}] = [f(\bar{x}) = \Psi[\bar{x}]]$ — matrix rewrites to equality of f to something

$\boxed{RW} \vdash R(\bar{x}, f(\bar{x})) = [f(\bar{x}) = \Psi[\bar{x}]]$ — rewriting theorem

$\hat{f}(\bar{x}) \triangleq \Psi[\bar{x}]$ — determined solution

$\boxed{SOL} \vdash S(\hat{f})$

$\left[\begin{array}{l} RW \xrightarrow{f := \hat{f}} \forall \bar{x}. [R(\bar{x}, \hat{f}(\bar{x})) = [\hat{f}(\bar{x}) = \Psi[\bar{x}]]] \xrightarrow{\delta_{\hat{f}}} \forall \bar{x}. R(\bar{x}, \hat{f}(\bar{x})) \xrightarrow{\delta_S} S(\hat{f}) \\ QED \end{array} \right.$

③ $\Phi[\bar{x}] = \text{if } \dots \text{ then } T \text{ else } [f(\bar{x}) = \Psi[\bar{x}]]$ - matrix rewrites to if tree (representative shown)

one leaf is equality } \Rightarrow same as ② - T leaves do not contribute constraints
other leaves are T

$$S'(f) \triangleq [f = \hat{f}]$$

$$\boxed{SS'} \vdash S'(f) \Rightarrow S(f)$$

} as in solution by user, in all cases ①, ②, ③