# Introduction to Rippling

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# Outline

- What is Rippling?
- Rippling-Out
- Rippling-In
- Rippling-Sideways
- Rippling-Across
- Conclusions

# What is Rippling?

- A way to control rewriting during induction
- Based on “rippling-out” the differences between the IH and IC
- Extended to achieve other rewriting goals
- Uses annotated rewrite rules to guide rewriting and ensure termination

# Rippling-Out

- IH $x + (y + z) = (x + y) + z$
- IC $\boxed{s(\underline{x})} + (y + z) = (\boxed{s(\underline{x})} + y) + z$
- $s(...)$ is a *wave-front*
- $x$ *is a* wave-hole
- $x + (y + z) = (x + y) + z$ is the *skeleton*

# Wave Rules

- General Form: $\eta(\boxed{\xi(\underline{\mu})}) \rightarrow \boxed{\zeta(\underline{\eta(\mu)})}$
- $\boxed{s(\underline{U})} + V \rightarrow \boxed{s(\underline{U + V})}$
- $\boxed{s(\underline{U})} \times V \rightarrow \boxed{\underline{U \times V} + V}$
- $even(\boxed{s(s(\underline{U}))}) \rightarrow even(U)$
- $U + (\boxed{\underline{V} + W}) \rightarrow \boxed{(\underline{U + V}) + W}$
- $(\boxed{U + \underline{V}}) + W \rightarrow \boxed{U + (\underline{V + W})}$

# Rippling-Out Example

- IH $x + (y + z) = (x + y) + z$
- IC $\boxed{s(\underline{x})} + (y + z) = (\boxed{s(\underline{x})} + y) + z$
- $\boxed{s(\underline{x + (y + z)})} = \boxed{s(\underline{x + y})} + z$
- $\boxed{s(\underline{x + (y + z)})} = \boxed{s(\underline{(x + y) + z})}$

# Rippling-In

- Useful for when one side of an equality is missing a wave rule
- IH $half(x + x) = x$
- IC $half(\boxed{s(\underline{x})} + \boxed{s(\underline{x})}) = \boxed{s(\underline{x})}$
- $half(\boxed{s(\underline{x + \boxed{s(\underline{x})}})}) = \boxed{s(\underline{x})}$
- Missing: $U + \boxed{s(\underline{V})} \rightarrow \boxed{s(\underline{U + V})}$
- XF $half(s(x + s(x))) = \boxed{s(\underline{half(x + x)})}^{\downarrow}$
- $half(s(x + s(x))) = half(\boxed{s(s(\underline{x + x}))}^{\downarrow})$
- $x + s(x) = s(x + x)$

# Rippling-Sideways

- Unmeasured (free) induction variables can be used as “sinks”
- Rippling-sideways Attempts to ripple wave-front into a sink
- General form: $\eta(\boxed{\xi(\underline{\mu})}^{\uparrow}, \nu) \rightarrow \eta(\mu, \boxed{\zeta(\underline{nu})}^{\downarrow})$
- IH $rev(l) <> M = qrev(l, M)$
- IC $rev(\boxed{h :: \underline{l}}^{\uparrow}) <> \lfloor m \rfloor = qrev(\boxed{h :: \underline{l}}^{\uparrow}, \lfloor m \rfloor)$
- $rev(\boxed{h :: \underline{l}}^{\uparrow}) <> \lfloor m \rfloor = qrev(l, \left\lfloor \boxed{h :: \underline{m}}^{\downarrow} \right\rfloor)$
- $\boxed{\underline{rev(l)} <> (h :: nil)}^{\uparrow}) <> \lfloor m \rfloor = qrev(l, \left\lfloor \boxed{h :: \underline{m}}^{\downarrow} \right\rfloor)$
- $rev(l) <> (\left\lfloor \boxed{(h :: nil) <> \underline{m}}^{\downarrow} \right\rfloor) = qrev(l, \left\lfloor \boxed{h :: \underline{m}}^{\downarrow} \right\rfloor)$

# Rippling-Across

- Adapts rippling to destructor inductions
- $U + V = \text{ if } U = 0 \text{ then } V \text{ else } s(p(U) + V)$
- Creational Rule: $U \neq 0 \implies U + V \to \boxed{s(\underline{\boxed{p(\underline{U})}^{-} + V})}^{\uparrow}$
- $\boxed{p(\underline{x})}^{\uparrow} + (y + z) = (\boxed{p(\underline{x})}^{\uparrow} + y) + z \vdash x + (y + z) = (x + y) + z$
- $\boxed{p(\underline{x})}^{\uparrow} + (y + z) = (\boxed{p(\underline{x})}^{\uparrow} + y) + z \vdash$

  $\boxed{s(\underline{\boxed{p(\underline{x})}^{-} + (y + z)})}^{\uparrow} = \boxed{s(\underline{\boxed{p(\underline{x})}^{-} + y})}^{\uparrow} + z$
- $p(\underline{x}) + (y + z) = (p(\underline{x}) + y) + z \vdash \boxed{s(\underline{p(\underline{x}) + (y + z)})}^{\uparrow} =$

  $\boxed{s(\underline{p(\underline{x}) + y})}^{\uparrow} + z$

# Conclusions

- Pros: Terminating rewrites, ability to use rules right to left, goal directed
- Cons: A little bit complicated
- Doesn't address generalization, lemma generation, or choosing induction scheme
- Some techniques address these problems by asking "how can I make this choice so that rippling will be facilitated?"
- Formalizes informal strategies for rewriting