# State-of-the-art SAT Solving

Marijn J. H. Heule

University of Texas

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# The Satisfiability (SAT) problem

$$
\begin{array}{l}
(x_5 \vee x_8 \vee \bar{x}_2) \wedge (x_2 \vee \bar{x}_1 \vee \bar{x}_3) \wedge (\bar{x}_8 \vee \bar{x}_3 \vee \bar{x}_7) \wedge (\bar{x}_5 \vee x_3 \vee x_8) \wedge \\
(\bar{x}_6 \vee \bar{x}_1 \vee \bar{x}_5) \wedge (x_8 \vee \bar{x}_9 \vee x_3) \wedge (x_2 \vee x_1 \vee x_3) \wedge (\bar{x}_1 \vee x_8 \vee x_4) \wedge \\
(\bar{x}_9 \vee \bar{x}_6 \vee x_8) \wedge (x_8 \vee x_3 \vee \bar{x}_9) \wedge (x_9 \vee \bar{x}_3 \vee x_8) \wedge (x_6 \vee \bar{x}_9 \vee x_5) \wedge \\
(x_2 \vee \bar{x}_3 \vee \bar{x}_8) \wedge (x_8 \vee \bar{x}_6 \vee \bar{x}_3) \wedge (x_8 \vee \bar{x}_3 \vee \bar{x}_1) \wedge (\bar{x}_8 \vee x_6 \vee \bar{x}_2) \wedge \\
(x_7 \vee x_9 \vee \bar{x}_2) \wedge (x_8 \vee \bar{x}_9 \vee x_2) \wedge (\bar{x}_1 \vee \bar{x}_9 \vee x_4) \wedge (x_8 \vee x_1 \vee \bar{x}_2) \wedge \\
(x_3 \vee \bar{x}_4 \vee \bar{x}_6) \wedge (\bar{x}_1 \vee \bar{x}_7 \vee x_5) \wedge (\bar{x}_7 \vee x_1 \vee x_6) \wedge (\bar{x}_5 \vee x_4 \vee \bar{x}_6) \wedge \\
(\bar{x}_4 \vee x_9 \vee \bar{x}_8) \wedge (x_2 \vee x_9 \vee x_1) \wedge (x_5 \vee \bar{x}_7 \vee x_1) \wedge (\bar{x}_7 \vee \bar{x}_9 \vee \bar{x}_6) \wedge \\
(x_2 \vee x_5 \vee x_4) \wedge (x_8 \vee \bar{x}_4 \vee x_5) \wedge (x_5 \vee x_9 \vee x_3) \wedge (\bar{x}_5 \vee \bar{x}_7 \vee x_9) \wedge \\
(x_2 \vee \bar{x}_8 \vee x_1) \wedge (\bar{x}_7 \vee x_1 \vee x_5) \wedge (x_1 \vee x_4 \vee x_3) \wedge (x_1 \vee \bar{x}_9 \vee \bar{x}_4) \wedge \\
(x_3 \vee x_5 \vee x_6) \wedge (\bar{x}_6 \vee x_3 \vee \bar{x}_9) \wedge (\bar{x}_7 \vee x_5 \vee x_9) \wedge (x_7 \vee \bar{x}_5 \vee \bar{x}_2) \wedge \\
(x_4 \vee x_7 \vee x_3) \wedge (x_4 \vee \bar{x}_9 \vee \bar{x}_7) \wedge (x_5 \vee \bar{x}_1 \vee x_7) \wedge (x_5 \vee \bar{x}_1 \vee x_7) \wedge \\
(x_6 \vee x_7 \vee \bar{x}_3) \wedge (\bar{x}_8 \vee \bar{x}_6 \vee \bar{x}_7) \wedge (x_6 \vee x_2 \vee x_3) \wedge (\bar{x}_8 \vee x_2 \vee x_5)
\end{array}
$$

Does there exist an assignment satisfying all clauses?

# Search for a satisfying assignment (or proof none exists)

$$
\begin{aligned}
&(x_5 \vee x_8 \vee \bar{x}_2) \wedge (x_2 \vee \bar{x}_1 \vee \bar{x}_3) \wedge (\bar{x}_8 \vee \bar{x}_3 \vee \bar{x}_7) \wedge (\bar{x}_5 \vee x_3 \vee x_8) \wedge \\
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&(x_2 \vee \bar{x}_3 \vee \bar{x}_8) \wedge (x_8 \vee \bar{x}_6 \vee \bar{x}_3) \wedge (x_8 \vee \bar{x}_3 \vee \bar{x}_1) \wedge (\bar{x}_8 \vee x_6 \vee \bar{x}_2) \wedge \\
&(x_7 \vee x_9 \vee \bar{x}_2) \wedge (x_8 \vee \bar{x}_9 \vee x_2) \wedge (\bar{x}_1 \vee \bar{x}_9 \vee x_4) \wedge (x_8 \vee x_1 \vee \bar{x}_2) \wedge \\
&(x_3 \vee \bar{x}_4 \vee \bar{x}_6) \wedge (\bar{x}_1 \vee \bar{x}_7 \vee x_5) \wedge (\bar{x}_7 \vee x_1 \vee x_6) \wedge (\bar{x}_5 \vee x_4 \vee \bar{x}_6) \wedge \\
&(\bar{x}_4 \vee x_9 \vee \bar{x}_8) \wedge (x_2 \vee x_9 \vee x_1) \wedge (x_5 \vee \bar{x}_7 \vee x_1) \wedge (\bar{x}_7 \vee \bar{x}_9 \vee \bar{x}_6) \wedge \\
&(x_2 \vee x_5 \vee x_4) \wedge (x_8 \vee \bar{x}_4 \vee x_5) \wedge (x_5 \vee x_9 \vee x_3) \wedge (\bar{x}_5 \vee \bar{x}_7 \vee x_9) \wedge \\
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&(x_3 \vee x_5 \vee x_6) \wedge (\bar{x}_6 \vee x_3 \vee \bar{x}_9) \wedge (\bar{x}_7 \vee x_5 \vee x_9) \wedge (x_7 \vee \bar{x}_5 \vee \bar{x}_2) \wedge \\
&(x_4 \vee x_7 \vee x_3) \wedge (x_4 \vee \bar{x}_9 \vee \bar{x}_7) \wedge (x_5 \vee \bar{x}_1 \vee x_7) \wedge (x_5 \vee \bar{x}_1 \vee x_7) \wedge \\
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\end{aligned}
$$

Play the SAT game:

`http://www.cril.univ-artois.fr/~roussel/satgame/satgame.php`

# Motivation

From 100 variables, 200 constraints (early 90s)
to 1,000,000 vars. and 20,000,000 cls. in 20 years.

Applications:
Hardware and Software Verification, Planning, Scheduling, Optimal Control, Protocol Design, Routing, Combinatorial problems, Equivalence Checking, etc.

SAT used to solve many other problems!

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# Overview

## Search for Lemmas

*Depth-first search*

- Learning Lemmas
- Data-structures
- Heuristics

## Search for Simplification

*Breadth-first search*

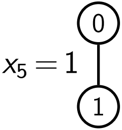
- Variable elimination
- Blocked clause elimination
- Unhiding redundancy

# Conflict-driven SAT solvers: Search and Analysis

$(x_1 \vee x_4) \wedge$
$(x_3 \vee \bar{x}_4 \vee \bar{x}_5) \wedge$
$(\bar{x}_3 \vee \bar{x}_2 \vee \bar{x}_4) \wedge$
$\mathcal{F}_{\text{extra}}$

# Conflict-driven SAT solvers: Search and Analysis

$$
\begin{aligned}
&(x_1 \lor x_4) \land \\
&(x_3 \lor \bar{x}_4 \lor \bar{x}_5) \land \\
&(\bar{x}_3 \lor \bar{x}_2 \lor \bar{x}_4) \land \\
&\mathcal{F}_{\text{extra}}
\end{aligned}
$$



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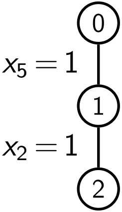

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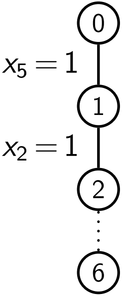

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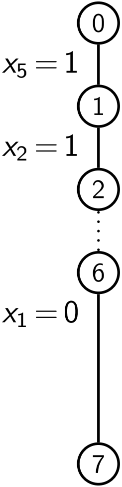

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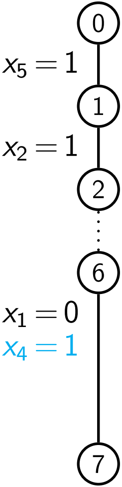

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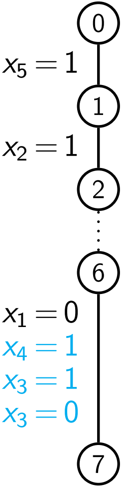

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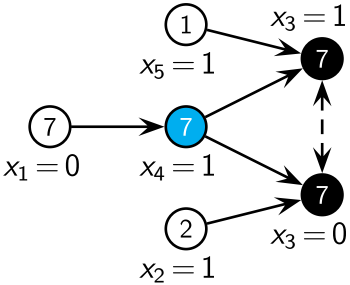


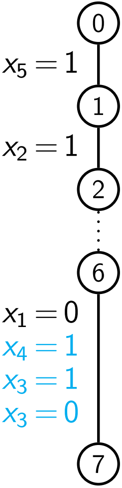

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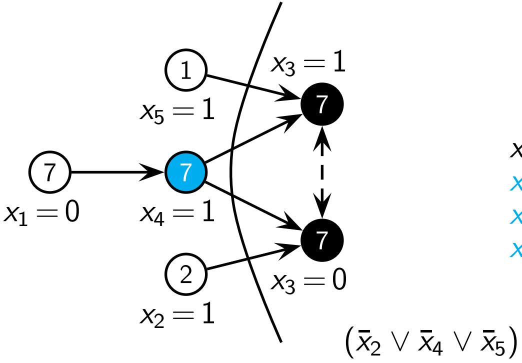


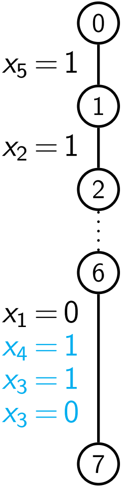

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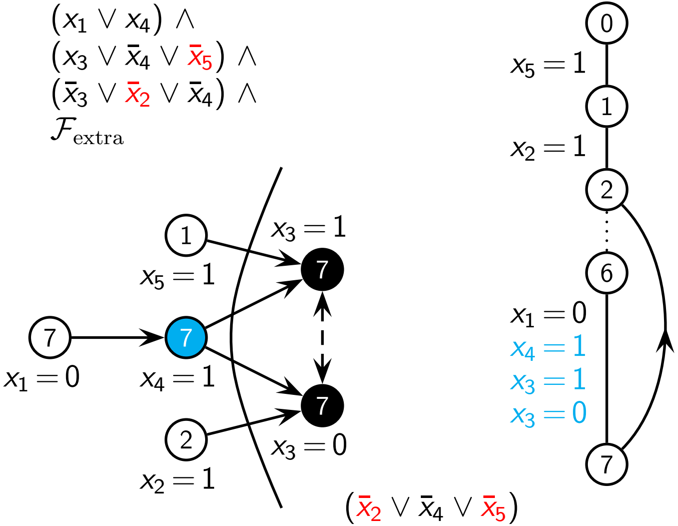

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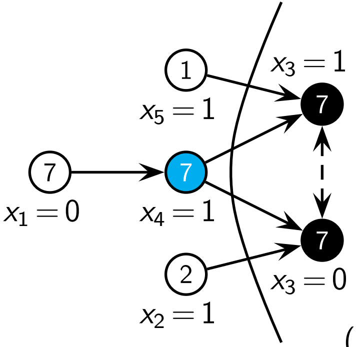
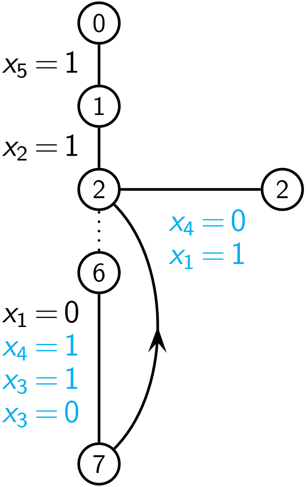



$(\bar{x}_2 \vee \bar{x}_4 \vee \bar{x}_5)$



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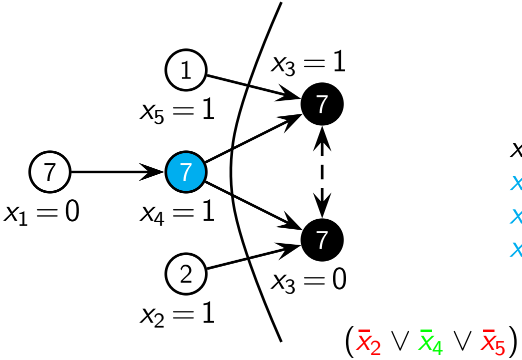
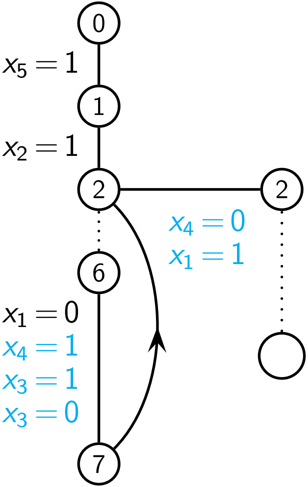
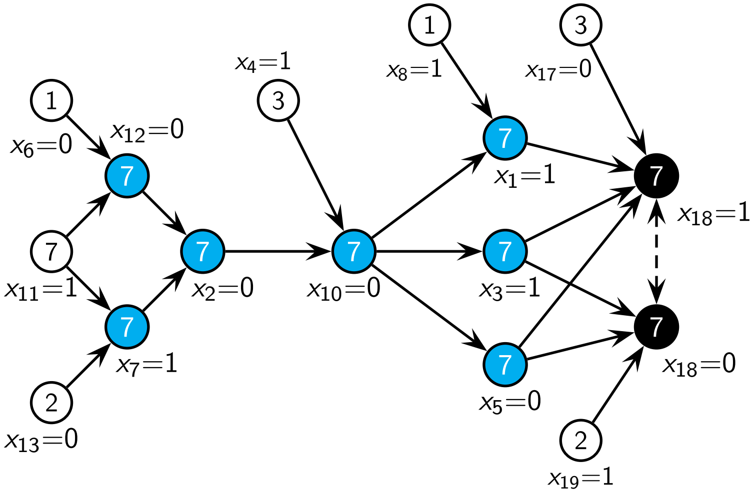


# Conflict-driven SAT solvers: Pseudo-code

**while** TRUE **do**
  $l_{\text{decision}} := \textsc{GetDecisionLiteral}(\ )$
  **If** no $l_{\text{decision}}$ **then return** `satisfiable`
  $\mathcal{F} := \textsc{Simplify}(\ \mathcal{F}(l_{\text{decision}} \leftarrow 1)\ )$
  **while** $\mathcal{F}$ contains $C_{\text{falsified}}$ **do**
    $C_{\text{conflict}} := \textsc{AnalyzeConflict}(\ C_{\text{falsified}}\ )$
    **If** $C_{\text{conflict}} = \emptyset$ **then return** `unsatisfiable`
    $\textsc{BackTrack}(\ C_{\text{conflict}}\ )$
    $\mathcal{F} := \textsc{Simplify}(\ \mathcal{F} \cup \{C_{\text{conflict}}\}\ )$
  **end while**
**end while**

# Learning conflict clauses (lemma's)



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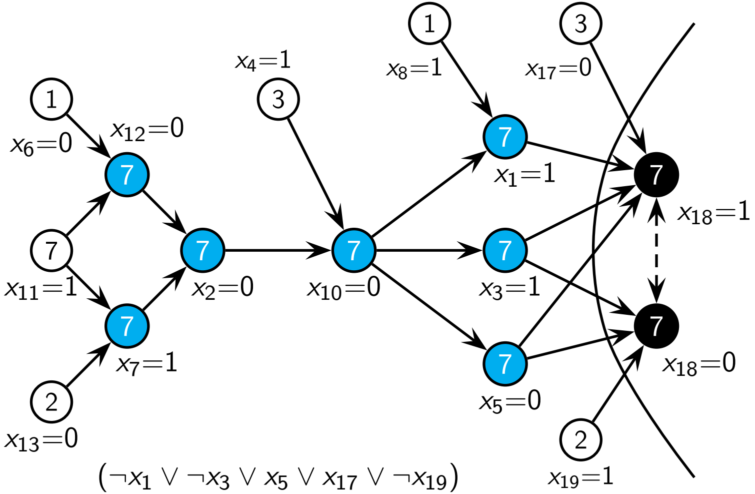
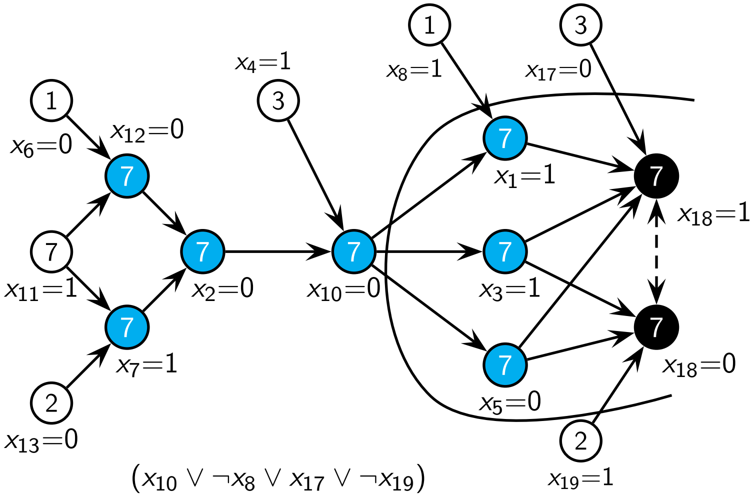



$(\neg x_1 \vee \neg x_3 \vee x_5 \vee x_{17} \vee \neg x_{19})$

tri-asserting clause

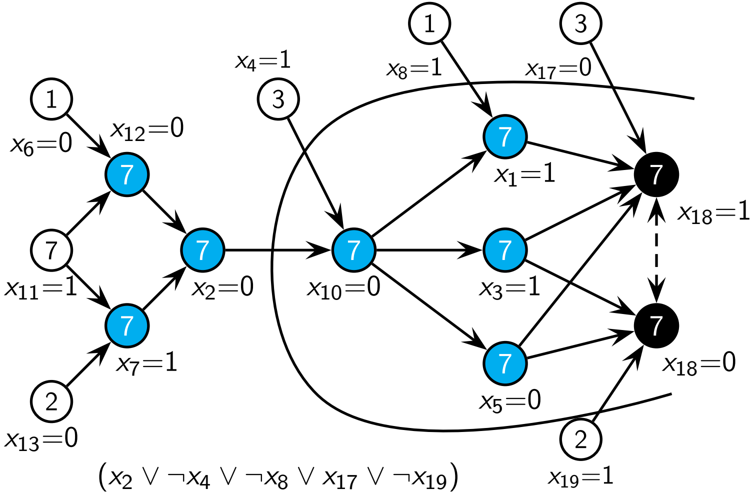
# Learning conflict clauses (lemma's)




$(x_{10} \vee \neg x_8 \vee x_{17} \vee \neg x_{19})$

first unique implication point

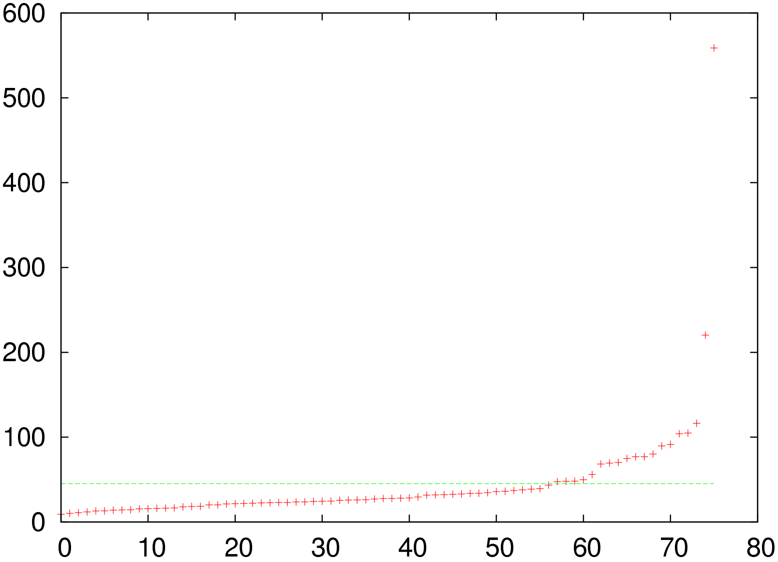
# Learning conflict clauses (lemma's)




$(x_2 \vee \neg x_4 \vee \neg x_8 \vee x_{17} \vee \neg x_{19})$

second unique implication point

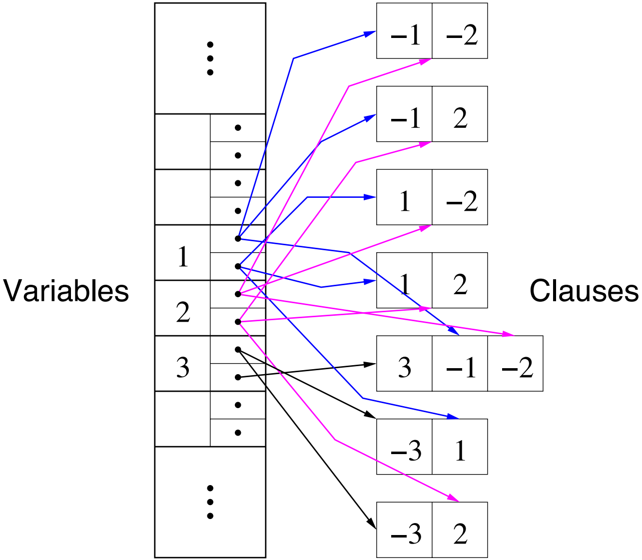
# Average Learned Clause Length



# Data-structures

Watch pointers

# Simple data structure for unit propagation



# Conflict-driven: Watch pointers (1)

$$\varphi = \{x_1 = *, x_2 = *, x_3 = *, x_4 = *, x_5 = *, x_6 = *\}$$

| | | | | |
|---|---|---|---|---|
| $\neg x_1$ | $x_2$ | $\neg x_3$ | $\neg x_5$ | $x_6$ |

| | | | | |
|---|---|---|---|---|
| $x_1$ | $\neg x_3$ | $x_4$ | $\neg x_5$ | $\neg x_6$ |

# Conflict-driven: Watch pointers (1)

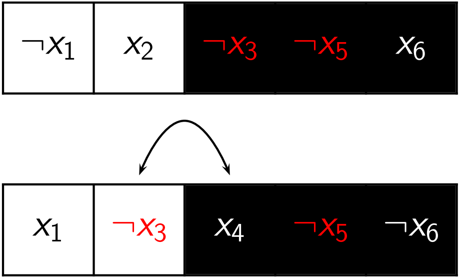
$$\varphi = \{x_1 = *, x_2 = *, x_3 = *, x_4 = *, x_5 = \mathbf{1}, x_6 = *\}$$

| | | | | |
|---|---|---|---|---|
| $\neg x_1$ | $x_2$ | $\neg x_3$ | $\neg x_5$ | $x_6$ |

| | | | | |
|---|---|---|---|---|
| $x_1$ | $\neg x_3$ | $x_4$ | $\neg x_5$ | $\neg x_6$ |

# Conflict-driven: Watch pointers (1)

$$\varphi = \{x_1 = *, x_2 = *, x_3 = \mathbf{1}, x_4 = *, x_5 = 1, x_6 = *\}$$



# Conflict-driven: Watch pointers (1)

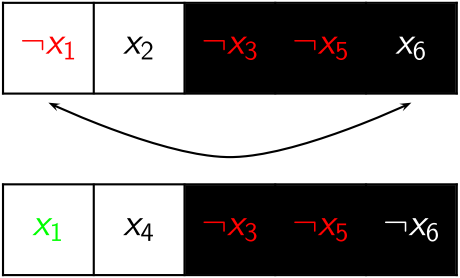
$$\varphi = \{x_1 = *, x_2 = *, x_3 = 1, x_4 = *, x_5 = 1, x_6 = *\}$$

| | | | | |
|---|---|---|---|---|
| $\neg x_1$ | $x_2$ | $\neg x_3$ | $\neg x_5$ | $x_6$ |

| | | | | |
|---|---|---|---|---|
| $x_1$ | $x_4$ | $\neg x_3$ | $\neg x_5$ | $\neg x_6$ |

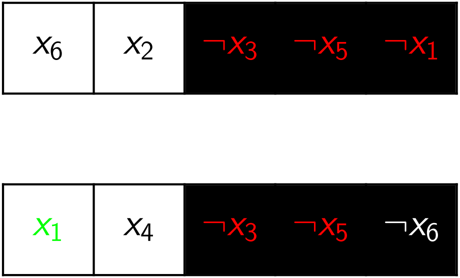
# Conflict-driven: Watch pointers (1)

$$\varphi = \{x_1 = \mathbf{1}, x_2 = *, x_3 = 1, x_4 = *, x_5 = 1, x_6 = *\}$$



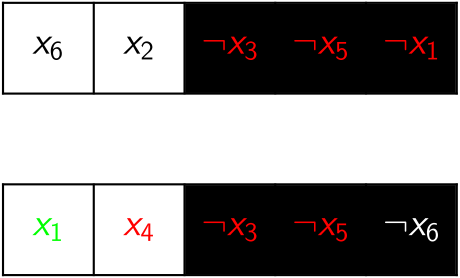
# Conflict-driven: Watch pointers (1)

$$\varphi = \{x_1 = 1, x_2 = *, x_3 = 1, x_4 = *, x_5 = 1, x_6 = *\}$$



# Conflict-driven: Watch pointers (1)

$$\varphi = \{x_1 = 1, x_2 = *, x_3 = 1, x_4 = \mathbf{0}, x_5 = 1, x_6 = *\}$$



# Conflict-driven: Watch pointers (1)

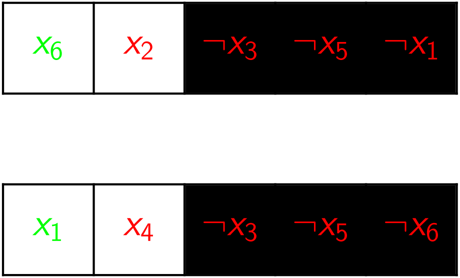
$$\varphi = \{x_1 = 1, x_2 = \mathbf{0}, x_3 = 1, x_4 = 0, x_5 = 1, x_6 = \ * \}$$

| | | | | |
|---|---|---|---|---|
| $x_6$ | $x_2$ | $\neg x_3$ | $\neg x_5$ | $\neg x_1$ |

| | | | | |
|---|---|---|---|---|
| $x_1$ | $x_4$ | $\neg x_3$ | $\neg x_5$ | $\neg x_6$ |

# Conflict-driven: Watch pointers (1)

$$\varphi = \{x_1 = 1, x_2 = 0, x_3 = 1, x_4 = 0, x_5 = 1, x_6 = \mathbf{1}\}$$



# Conflict-driven: Watch pointers (1)

$$\varphi = \{x_1 = 1, x_2 = 0, x_3 = 1, x_4 = 0, x_5 = 1, x_6 = 1\}$$

| | | | | |
|---|---|---|---|---|
| $x_6$ | $x_2$ | $\neg x_3$ | $\neg x_5$ | $\neg x_1$ |

| | | | | |
|---|---|---|---|---|
| $x_1$ | $x_4$ | $\neg x_3$ | $\neg x_5$ | $\neg x_6$ |

# Conflict-driven: Watch pointers (2)

Only examine (get in the cache) a clause when both

- a watch pointer gets falsified
- the other one is not satisfied

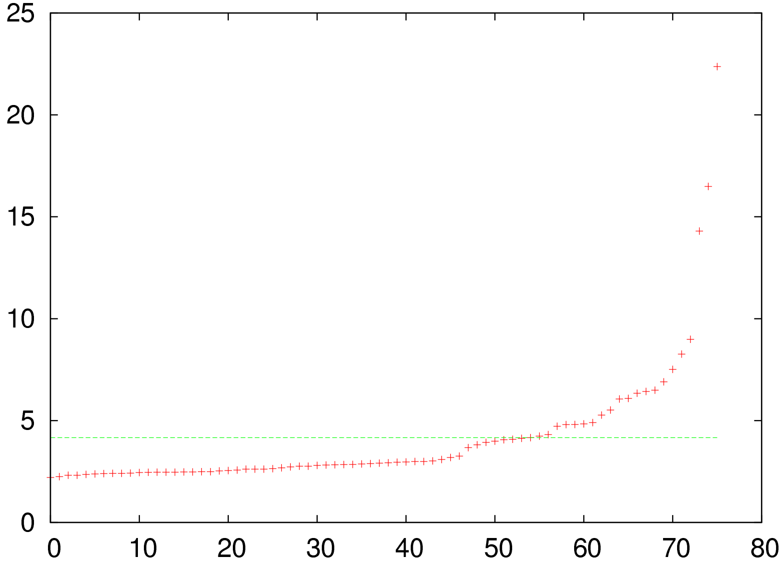
While backjumping, just unassign variables

Conflict clauses $\rightarrow$ watch pointers

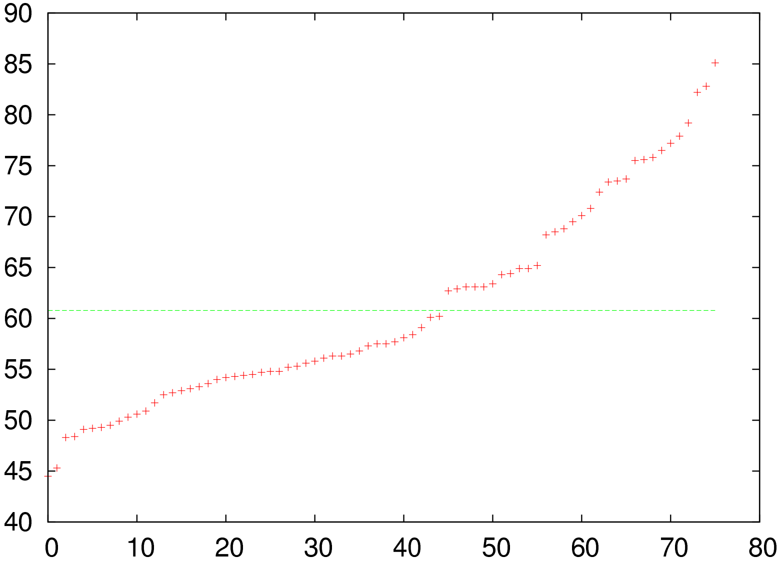
No detailed information available

Not used for binary clauses

# Average Number Clauses Visited Per Propagation



# Percentage visited clauses with other watched literal true



# Heuristics

# Most important CDCL heuristics

## Variable selection heuristics

- aim: minimize the search space
- plus: could compensate a bad value selection

## Value selection heuristics

- aim: guide search towards a solution (or conflict)
- plus: could compensate a bad variable selection, cache solutions of subproblems [PipatsrisawatDarwiche'07]

## Restart strategies

- aim: avoid heavy-tail behavior [GomesSelmanCrato'97]
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Based on the occurrences in the (reduced) formula

- examples: Jeroslow-Wang, Maximal Occurrence in clauses of Minimal Size (MOMS), look-aheads
- not practical for CDCL solver due to watch pointers

Variable State Independent Decaying Sum (VSIDS)

- original idea (zChaff): for each conflict, increase the score of involved variables by 1, half all scores each 256 conflicts [MoskewiczMZZM2001]
- improvement (MiniSAT): for each conflict, increase the score of involved variables by $\delta$ and increase $\delta := 1.05\delta$ [EenSörensson2003]

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# Visualization of VSIDS in PicoSAT

`http://www.youtube.com/watch?v=MOjhFywLre8`

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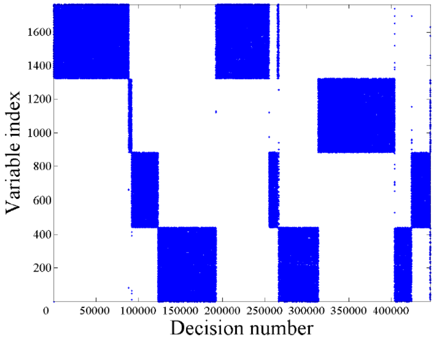
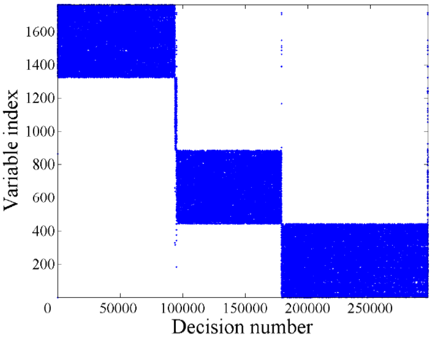
# Heuristics: Phase-saving

Selecting the last implied value remembers solved components




negative branching




phase-saving

# Restarts

Restarts in CDCL solvers:

- Counter heavy-tail behavior [GomesSelmanCrato'97]
- Unassign all variables but keep the (dynamic) heuristics

Restart strategies: [Walsh'99, LubySinclairZuckerman'93]

- Geometrical restart: e.g. 100, 150, 225, 333, 500, 750, ...
- Luby sequence: e.g. 100, 100, 200, 100, 100, 200, 400, ...

Rapid restarts by reusing trail: [vanderTakHeuleRamos'11]

- Partial restart same effect as full restart
- Optimal strategy Luby-1: 1, 1, 2, 1, 1, 2, 4, ...

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# Preliminary CDCL solver in ACL2

"Don't be smart"

# Removal of false literals in ACL2

```
(defun neg (literal) (* -1 literal))

(defun false-literal (assignment literal)
  (member (neg literal) assignment))

(defun one-not-false-literal (assignment clause)
  (cond ((atom clause) nil)
        ((false-literal assignment (car clause))
         (one-not-false-literal assignment (cdr clause)))
        (t clause)))

(defun two-not-false-literals (assignment clause)
  (cond ((atom clause) nil)
        ((false-literal assignment (car clause))
         (two-not-false-literals assignment (cdr clause)))
        (t (cons (car clause)
                 (one-not-false-literal assignment (cdr clause))))))
```

# Unit clause is member of all-lits in ACL2

```
(defun all-lits (formula)
  (if (atom formula)
    nil
    (append (car formula) (all-lits (cdr formula)))))

(defthm reduced-clause-implies-member-car-reduced-clause
  (implies (two-not-false-literals assignment clause)
      (member (car (two-not-false-literals assignment clause)) clause)))

(defthm member-append-member-or
        (iff (member x (append y z))
             (or (member x y) (member x z))))

(defthm reduced-clause-implies-member-car-all-lits
  (implies (and (two-not-false-literals assignment clause)
                (member clause formula))
           (member (car (two-not-false-literals assignment clause))
                   (all-lits formula))))
```

# The new get-unit procedure in ACL2

```
(defun get-unit (formula assignment)
  (if (atom formula)
    (mv nil nil)
    (let ((reduced-clause (two-not-false-literals assignment
                                                  (car formula))))
      (cond ((not reduced-clause) (mv (car formula) nil))
            ((and (car reduced-clause)
                  (not (cdr reduced-clause))
                  (not (member (car reduced-clause) assignment)))
             (mv (car formula) (car reduced-clause)))
            (t (get-unit (cdr formula) assignment))))))

(defthm get-unit-returns-member-of-all-lits
  (implies (cadr (get-unit formula assignment))
           (member (cadr (get-unit formula assignment))
                   (all-lits formula))))
```

# Old unit propagation code in ACL2

```
(defun neg (literal) (* -1 literal))

(defun reduce-clause (assignment clause unassigned)
  (cond ((atom clause) unassigned)
        ((member (neg (car clause)) assignment)
         (reduce-clause assignment (cdr clause) unassigned))
        (unassigned (append unassigned clause))
        (t (reduce-clause assignment (cdr clause) (list (car clause))))))

(defun get-unit (formula assignment)
  (if (atom formula)
    (mv nil nil)
    (let ((reduced-clause (reduce-clause assignment (car formula) nil)))
      (if (and (not (cdr reduced-clause)) ; if unit and not satisfied
               (not (member (car reduced-clause) assignment)))
        (mv (car formula) (car reduced-clause))
        (get-unit (cdr formula) assignment)))))
```

# Reduction theorem and some defuns in ACL2

```
(defthm new-element-reduces-difference
        (implies (and (member e y)
                      (not (member e x)))
                 (< (len (set-difference-equal y (cons e x)))
                    (len (set-difference-equal y x)))))

(defun remove-literal (clause literal)
  (cond ((atom clause) clause)
        ((eql (car clause) literal) (cdr clause))
        (t (cons (car clause) (remove-literal (cdr clause) literal)))))

(defun resolve (clause resolvent literal)
  (union-equal (remove-literal clause literal)
               (remove-literal resolvent (neg literal))))

(defun unit-under-assignment (assignment clause)
  (and (car (two-not-false-literals assignment clause))
       (not (cdr (two-not-false-literals assignment clause)))))
```

# First unique implication point in ACL2

```
(defun implications-or-resolvent (formula assignment implications)
  (declare (xargs :measure (nfix (len
                  (set-difference-equal (all-lits formula) implications)))))
  (mv-let (clause literal)
        (get-unit formula (append assignment implications))
        (if (not literal) ; end recursion
          (if clause (mv nil clause) (mv implications nil))
          (mv-let (more-implications resolvent)
                  (implications-or-resolvent formula assignment
                                             (cons literal implications))
                  (if more-implications
                    (mv more-implications nil)
                    (if (or (unit-under-assignment assignment resolvent)
                            (not (member (neg literal) resolvent)))
                      (mv nil resolvent)
                      (mv nil (resolve clause resolvent literal))))))))
```

# Old code of first unique implication point in ACL2

```
(defun implications−or−resolvent (formula assignment implications)
  (mv−let (clause literal)
          (get−unit formula (append assignment implications))
          (if (not literal) ; no unit means either conflict or done
            (mv implications clause)
            (mv−let (more−implications resolvent)
                    (implications−or−resolvent formula assignment
                                               (cons literal implications))
                    (if (and (member (neg literal) resolvent)
                             (cadr (two−not−false−literals assignment
                                                           resolvent)))
                      (mv nil (resolve clause resolvent literal))
                      (mv more−implications resolvent))))))
```

# get-decision in ACL2

```
(defun get-decision (heuristics assignment)
  (if (atom heuristics)
    nil
    (if (or (member (car heuristics) assignment)
            (member (neg (car heuristics)) assignment))
      (get-decision (cdr heuristics) assignment)
      (list (car heuristics)))))

(defthm get-decision-returns-not-member-assignment
  (implies (get-decision heuristics assignment)
           (not (member (car (get-decision
                                 heuristics
                                 assignment))
                        assignment))))
```

# car get-decision member of implications in ACL2

```
(defthm cons−subsetp−lemma
        (implies (subsetp x lst)
                 (subsetp x (cons y lst))))

(defthm decision−subsetp−of−implications
        (implies (car (implications−or−resolvent f a d))
                 (subsetp d (car (implications−or−resolvent f a d)))))

(defthm subsetp−car−member
        (implies (and (consp x)
                      (subsetp x y))
                 (member (car x) y)))

(defthm car−get−decision−member−car−implications
      (implies (and (consp d)
                    (car (implications−or−resolvent f a d)))
               (member (car d) (car (implications−or−resolvent f a d)))))
```

# get-decision-and-implication-reduce-set-difference in ACL2

```
(defthm member-not-member-reduce-set-difference
        (implies (and (member (car get-d) h)
                      (member (car get-d) i)
                      (not (member (car get-d) a)))
                 (< (len (set-difference-equal h (append a i)))
                    (len (set-difference-equal h a)))))

(defthm get-decision-and-implication-reduce-set-difference
    (implies (and (get-decision h a)
                  (car (implications-or-resolvent f a (get-decision h a))))
   (and (member (car (get-decision h a)) h)
        (member (car (get-decision h a))
                (car (implications-or-resolvent f a (get-decision h a))))
        (not (member (car (get-decision h a)) a))
        (< (len (set-difference-equal h (append a
                (car (implications-or-resolvent f a (get-decision h a))))))
           (len (set-difference-equal h a))))))
```

# Solution or conflict clause in ACL2

```
(defun assign-rec (f h a)
  (declare (xargs :measure (nfix (len (set-difference-equal h a)))))
  (let ((decision (get-decision h a)))
    (if (not decision)
      (mv assignment nil) ; found a solution -> satisfiable
      (mv-let (implications resolvent)
              (implications-or-resolvent f a decision)
              (if implications
                (assign-rec f h (append a implications))
                (mv nil resolvent))))))

(defun solution-or-resolvent (formula heuristics)
  (mv-let (assignment resolvent)
          (implications-or-resolvent formula nil nil)
          (if resolvent
            (mv nil nil) ; found refutation -> unsatisfiable
            (assign-rec formula heuristics assignment))))
```

# Top level structure CDCL in ACL2

```
(defun heuristics-init (formula)
  (all-lits formula))

(skip-proofs
  (defun cdcl-rec (formula heuristics) ; returns solution or unsatisfiable
    (mv-let (solution resolvent)
            (solution-or-resolvent formula heuristics)
            (cond (resolvent (cdcl-rec (cons resolvent formula) heuristics))
                  (solution solution) ; found solution
                  (t 'unsatisfiable)))) ; found refutation
)

(defun cdcl (formula)
  (cdcl-rec formula (heuristics-init formula)))
```

# Search for Simplification

# Variable Elimination

# Variable Elimination [DavisPutnam'60]

**Definition (Resolution)**

Given two clauses $C = (x \vee a_1 \vee \cdots \vee a_i)$ and $D = (\bar{x} \vee b_1 \vee \cdots \vee b_j)$, the *resolvent* of $C$ and $D$ on variable $x$ (denoted by $C \otimes_x D$) is $(a_1 \vee \cdots \vee a_i \vee b_1 \vee \cdots \vee b_j)$

Resolution on sets of clauses $F_x$ and $F_{\bar{x}}$ (denoted by $F_x \otimes_x F_{\bar{x}}$) generates all (non-tautological) resolvents of $C \in F_x$ and $D \in F_{\bar{x}}$.

**Definition (Variable elimination (VE))**

Given a CNF formula $F$, *variable elimination* (or DP resolution) removes a variable $x$ by replacing $F_x$ and $F_{\bar{x}}$ by $F_x \otimes_x F_{\bar{x}}$

**Proof procedure [DavisPutnam60]**

VE is a complete proof procedure. Applying VE until fixpoint results in the empty formula (satisfiable) or empty clause (unsatisfiable)

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# Example VE by clause distribution [DavisPutnam'60]

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## Example of clause distribution

| | $F_x$: $(x \vee c)$ | $F_x$: $(x \vee \bar{d})$ | $F_x$: $(x \vee \bar{a} \vee \bar{b})$ |
|---|---|---|---|
| $F_{\bar{x}}$: $(\bar{x} \vee a)$ | $(a \vee c)$ | $(a \vee d)$ | $(a \vee \bar{a} \vee \bar{b})$ |
| $F_{\bar{x}}$: $(\bar{x} \vee b)$ | $(b \vee c)$ | $(b \vee d)$ | $(b \vee \bar{a} \vee \bar{b})$ |
| $F_{\bar{x}}$: $(\bar{x} \vee \bar{e} \vee f)$ | $(c \vee \bar{e} \vee f)$ | $(d \vee \bar{e} \vee f)$ | $(\bar{a} \vee \bar{b} \vee \bar{e} \vee f)$ |

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| | $F_x$ | | |
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| $F_{\bar{x}}$: $(\bar{x} \lor b)$ | $(b \lor c)$ | $(b \lor d)$ | ~~$(b \lor \bar{a} \lor \bar{b})$~~ |
| $F_{\bar{x}}$: $(\bar{x} \lor \bar{e} \lor f)$ | $(c \lor \bar{e} \lor f)$ | $(d \lor \bar{e} \lor f)$ | $(\bar{a} \lor \bar{b} \lor \bar{e} \lor f)$ |

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# VE by substitution [EenBiere07]

## General idea

Detect gates (or definitions) $x = \mathrm{GATE}(a_1, \ldots, a_n)$ in the formula and use them to reduce the number of added clauses

## Possible gates

| gate | $G_x$ | $G_{\bar{x}}$ |
|---|---|---|
| $\mathrm{AND}(a_1, \ldots, a_n)$ | $(x \vee \bar{a}_1 \vee \cdots \vee \bar{a}_n)$ | $(\bar{x} \vee a_1), \ldots, (\bar{x} \vee a_n)$ |
| $\mathrm{OR}(a_1, \ldots, a_n)$ | $(x \vee \bar{a}_1), \ldots, (x \vee \bar{a}_n)$ | $(\bar{x} \vee a_1 \vee \cdots \vee a_n)$ |
| $\mathrm{ITE}(c, t, f)$ | $(x \vee \bar{c} \vee \bar{t}), (x \vee c \vee \bar{f})$ | $(\bar{x} \vee \bar{c} \vee t), (\bar{x} \vee c \vee f)$ |

## Variable elimination by substitution [EenBiere07]

Let $R_x = F_x \setminus G_x$; $R_{\bar{x}} = F_{\bar{x}} \setminus G_{\bar{x}}$.

Replace $F_x \wedge F_{\bar{x}}$ by $G_x \otimes_x R_{\bar{x}} \wedge G_{\bar{x}} \otimes_x R_x$.

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# VE by substitution [EenBiere07]

## General idea

Detect gates (or definitions) $x = \mathrm{GATE}(a_1, \ldots, a_n)$ in the formula and use them to reduce the number of added clauses

## Possible gates

| gate | $G_x$ | $G_{\bar{x}}$ |
|---|---|---|
| $\mathrm{AND}(a_1, \ldots, a_n)$ | $(x \vee \bar{a}_1 \vee \cdots \vee \bar{a}_n)$ | $(\bar{x} \vee a_1), \ldots, (\bar{x} \vee a_n)$ |
| $\mathrm{OR}(a_1, \ldots, a_n)$ | $(x \vee \bar{a}_1), \ldots, (x \vee \bar{a}_n)$ | $(\bar{x} \vee a_1 \vee \cdots \vee a_n)$ |
| $\mathrm{ITE}(c, t, f)$ | $(x \vee \bar{c} \vee \bar{t}), (x \vee c \vee \bar{f})$ | $(\bar{x} \vee \bar{c} \vee t), (\bar{x} \vee c \vee f)$ |

## Variable elimination by substitution [EenBiere07]

Let $R_x = F_x \setminus G_x$; $R_{\bar{x}} = F_{\bar{x}} \setminus G_{\bar{x}}$.

Replace $F_x \wedge F_{\bar{x}}$ by $G_x \otimes_x R_{\bar{x}} \wedge G_{\bar{x}} \otimes_x R_x$.

# VE by substitution [EenBiere'07]

## Example of gate extraction: $x = \mathrm{AND}(a, b)$

$$F_x = (x \vee c) \wedge (x \vee \bar{d}) \wedge (x \vee \bar{a} \vee \bar{b})$$
$$F_{\bar{x}} = (\bar{x} \vee a) \wedge (\bar{x} \vee b) \wedge (\bar{x} \vee \bar{e} \vee f)$$

## Example of substitution

| | | $R_x$ | | $G_x$ |
|---|---|---|---|---|
| | | $(x \vee c)$ | $(x \vee \bar{d})$ | $(x \vee \bar{a} \vee \bar{b})$ |
| $G_{\bar{x}}$ | $(\bar{x} \vee a)$ | $(a \vee c)$ | $(a \vee d)$ | |
| | $(\bar{x} \vee b)$ | $(b \vee c)$ | $(b \vee d)$ | |
| $R_{\bar{x}}$ | $(\bar{x} \vee \bar{e} \vee f)$ | | | $(\bar{a} \vee \bar{b} \vee \bar{e} \vee f)$ |

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## Example of gate extraction: $x = \mathrm{AND}(a, b)$

$$F_x = (x \lor c) \land (x \lor \bar{d}) \land \color{red}{(x \lor \bar{a} \lor \bar{b})}$$
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## Example of substitution

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# Blocked Clause Elimination

# Blocked Clauses [Kullmann'99]

**Definition (Blocking literal)**

A literal $l$ in a clause $C$ of a CNF $F$ blocks $C$ w.r.t. $F$ if for every clause $C' \in F$ with $\bar{l} \in C'$, the resolvent $(C \setminus \{l\}) \cup (C' \setminus \{\bar{l}\})$ obtained from resolving $C$ and $C'$ on $l$ is a tautology.

With respect to a fixed CNF and its clauses we have:

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A clause is blocked if it contains a literal that blocks it.

**Example**

Consider the formula $(a \vee b) \wedge (a \vee \bar{b} \vee \bar{c}) \wedge (\bar{a} \vee c)$.
First clause is not blocked.
Second clause is blocked by both $a$ and $\bar{c}$. Third clause is blocked by $c$

**Proposition**

Removal of an arbitrary blocked clause preserves satisfiability.

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# Blocked Clause Elimination (BCE)

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While there is a blocked clause $C$ in a CNF $F$, remove $C$ from $F$.

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After removing either $(a \vee \bar{b} \vee \bar{c})$ or $(\bar{a} \vee c)$,
the clause $(a \vee b)$ becomes blocked (*no clause with either* $\bar{b}$ *or* $\bar{a}$).
An extreme case in which BCE removes all clauses of a formula!

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BCE converts the Tseiting encoding to Plaisted Greenbaum
BCE simulates Pure literal elimination, Cone of influence and much more

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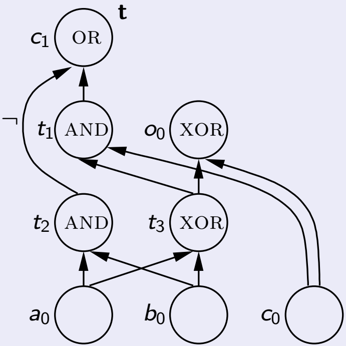

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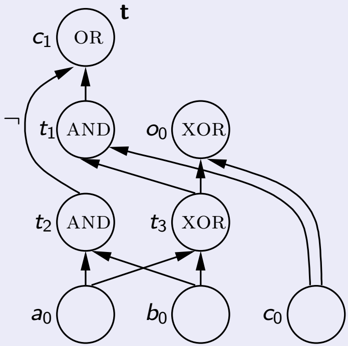

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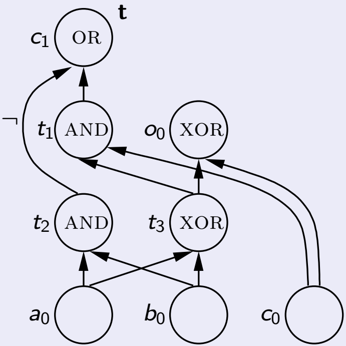

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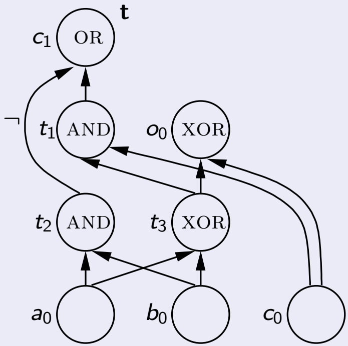

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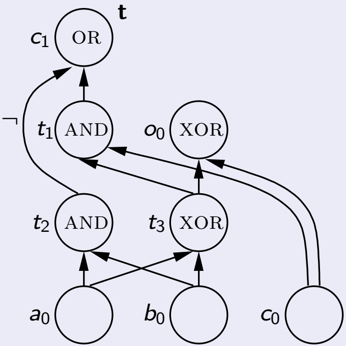

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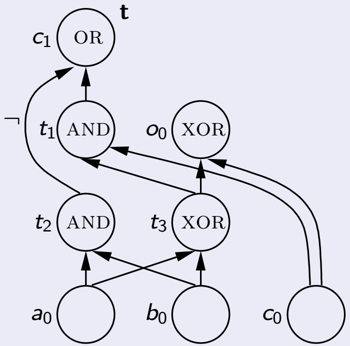

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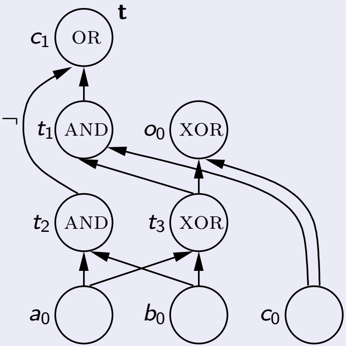

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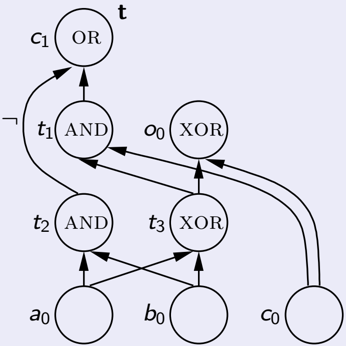

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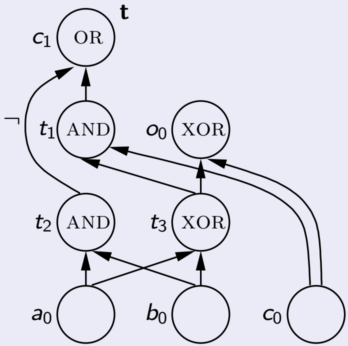

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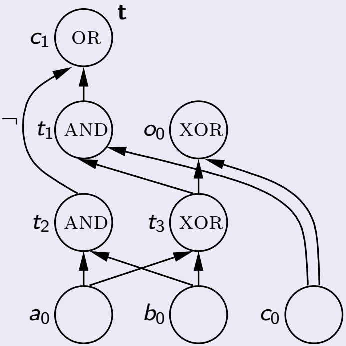

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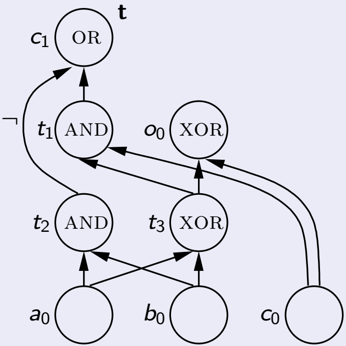

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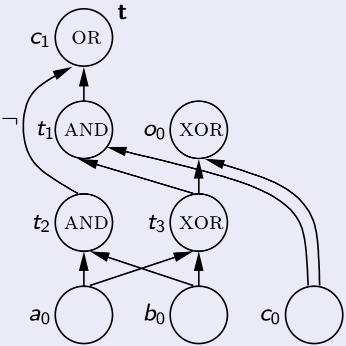

# Unhiding redundancy

# Redundancy

## Redundant clauses:

- Removal of $C \in F$ preserves unsatisfiability of $F$
- Assign $l \in C$ to false and check for a conflict in $F \setminus \{C\}$

## Redundant literals:

- Removal of $l \in C$ preserves satisfiability of $F$
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## Redundancy elimination during pre- and in-processing

- Distillation [JinSomenzi2005]
- ReVivAl [PietteHamadiSaïs2008]
- Unhiding [HeuleJärvisaloBiere2011]

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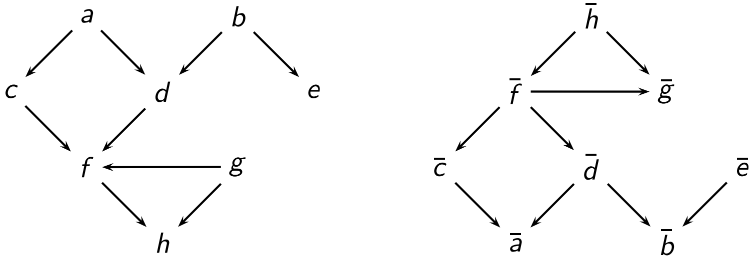
# Unhide: Binary implication graph (BIG)

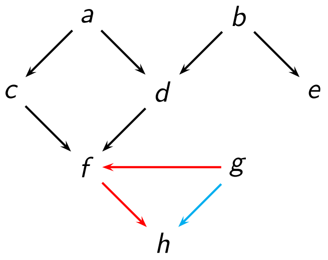
unhide: use the binary clauses to detect redundant clauses and literals

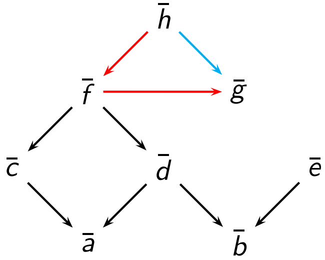



$$(\bar{a} \vee c) \wedge (\bar{a} \vee d) \wedge (\bar{b} \vee d) \wedge (\bar{b} \vee e) \wedge$$
$$(\bar{c} \vee f) \wedge (\bar{d} \vee f) \wedge (\bar{g} \vee f) \wedge (\bar{f} \vee h) \wedge$$
$$(\bar{g} \vee h) \wedge \underbrace{(\bar{a} \vee \bar{e} \vee h) \wedge (\bar{b} \vee \bar{c} \vee h) \wedge (a \vee b \vee c \vee d \vee e \vee f \vee g \vee h)}_{\text{non binary clauses}}$$

# Unhide: Transitive reduction (TRD)

transitive reduction: remove shortcuts in the binary implication graph



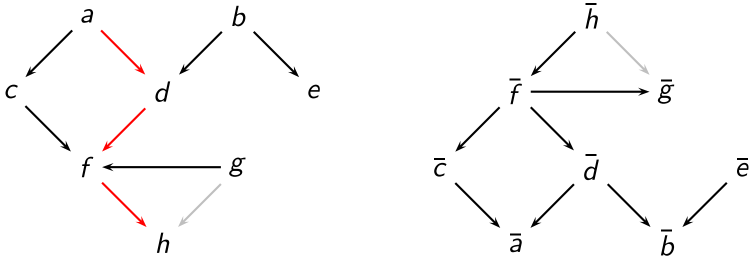

$(\bar{a} \vee c) \wedge (\bar{a} \vee d) \wedge (\bar{b} \vee d) \wedge (\bar{b} \vee e) \wedge$
$(\bar{c} \vee f) \wedge (\bar{d} \vee f) \wedge (\bar{g} \vee f) \wedge (\bar{f} \vee h) \wedge$
~~$(\bar{g} \vee h)$~~ $\wedge (\bar{a} \vee \bar{e} \vee h) \wedge (\bar{b} \vee \bar{c} \vee h) \wedge (a \vee b \vee c \vee d \vee e \vee f \vee g \vee h)$

TRD

$g \rightarrow f \rightarrow h$

# Unhide: Hidden tautology elimination (HTE) (1)

HTE removes clauses that are subsumed by an implication in BIG




$(\bar{a} \vee c) \wedge (\bar{a} \vee d) \wedge (\bar{b} \vee d) \wedge (\bar{b} \vee e) \wedge$

$(\bar{c} \vee f) \wedge (\bar{d} \vee f) \wedge (\bar{g} \vee f) \wedge (\bar{f} \vee h) \wedge$

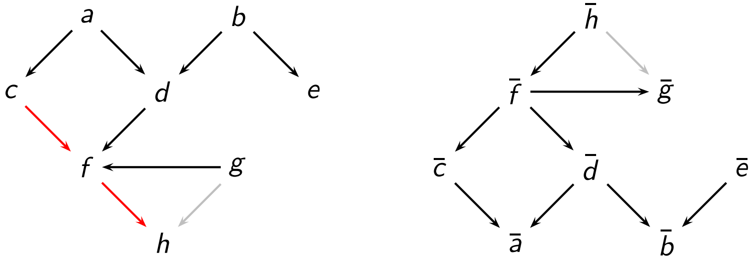
~~$(\bar{a} \vee \bar{e} \vee h)$~~ $\wedge (\bar{b} \vee \bar{c} \vee h) \wedge (a \vee b \vee c \vee d \vee e \vee f \vee g \vee h)$

HTE

$a \rightarrow d \rightarrow f \rightarrow h$

# Unhide: Hidden tautology elimination (HTE) (2)

HTE removes clauses that are subsumed by an implication in BIG




$$(\bar{a} \vee c) \wedge (\bar{a} \vee d) \wedge (\bar{b} \vee d) \wedge (\bar{b} \vee e) \wedge$$
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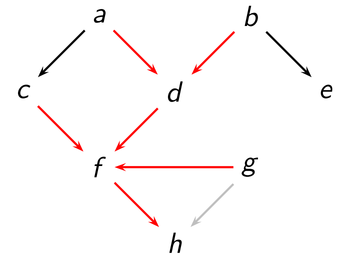
$$\cancel{(\bar{b} \vee \bar{c} \vee h)} \wedge (a \vee b \vee c \vee d \vee e \vee f \vee g \vee h)$$

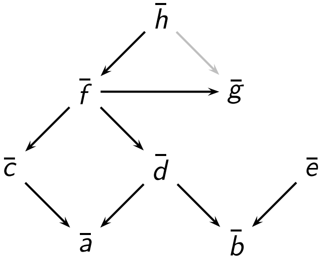
HTE

$c \rightarrow f \rightarrow h$

# Unhide: Hidden literal elimination (HLE)

HLE removes literal using the implication in BIG




$$(\bar{a} \lor c) \land (\bar{a} \lor d) \land (\bar{b} \lor d) \land (\bar{b} \lor e) \land$$
$$(\bar{c} \lor f) \land (\bar{d} \lor f) \land (\bar{g} \lor f) \land (\bar{f} \lor h) \land$$

( ~~$a \lor b \lor c \lor d \lor$~~ $e \lor$ ~~$f \lor g \lor$~~ $h$ )

HLE

all but $e$ imply $h$

also $b$ implies $e$

# Conclusions: state-of-the-art SAT solver

Key contributions to SAT search engine:

- adding conflict clauses (grasp) [Marques-Silva'96]
- restart strategies [GomesSC'97,LubySZ'93]
- 2-watch pointers and VSIDS (zChaff) [MoskewiczMZZM'01]
- efficient implementation (Minisat) [EenSörensson'03]
- variable elimination (SatElite) [EenBiere'05]
- phase-saving (Rsat) [PipatsrisawatDarwiche'07]

Recent progress: pre- and in-processing

- removal of redundant clauses and literals [JinSomenzi'05]
- removal of blocked clauses [JärvisaloBiereHeule'10]
- unhiding redundancy [HeuleJärvisaloBiere'11]

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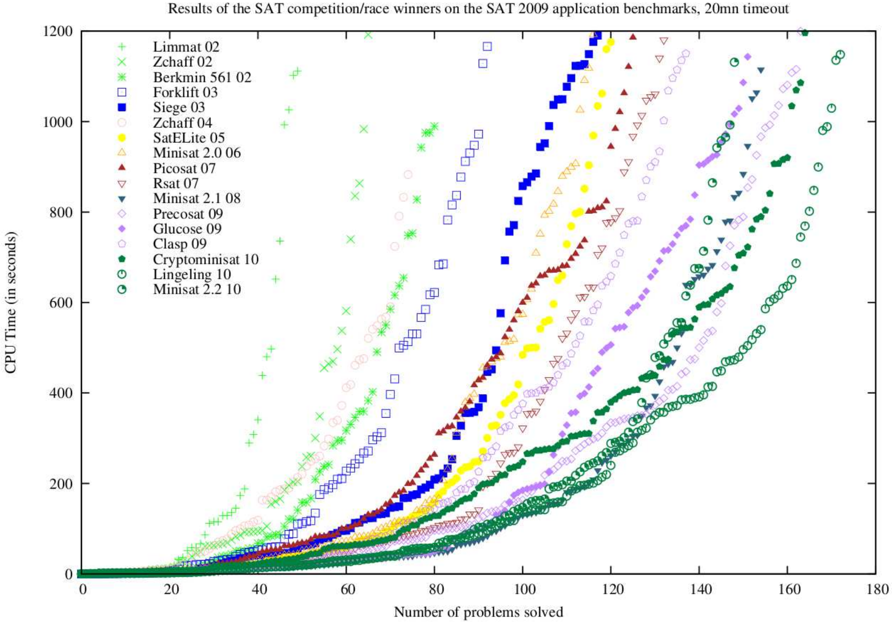
# Cactus plot: Lingeling [Biere'10] contains all features



# State-of-the-art SAT Solving

Marijn J. H. Heule

University of Texas

April 16, 2012 @ ACL2