# Trimming while Checking Clausal Proofs

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Formal Methods in Computer-Aided Design (FMCAD)
Portland, Oregon
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# Outline

- Motivation and Contributions
- Resolution versus Clausal Proofs
- Checking Clausal Proofs Efficiently
- Experimental Evaluation
- Conclusion

# Motivation

SAT solvers are used in many tools and applications.

- Counter-examples (satisfiable) using symbolic simulation;
- Equivalence-checking (unsatisfiable) using miters;
- Small explanations (unsatisfiable core) for diagnosis;
- Small (trimmed) proofs to validate with a verified checker.

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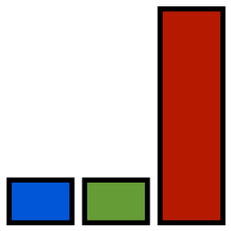
***We developed a tool that can efficiently validate the results of SAT solvers and produce trimmed formulas and trimmed proofs***

# Contributions and Related Work

Easy to Emit

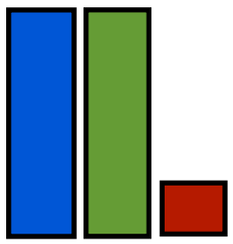
Compact

Checked Efficiently



Resolution Proofs

Zhang and Malik, 2003
Van Gelder, 2008; Biere, 2008



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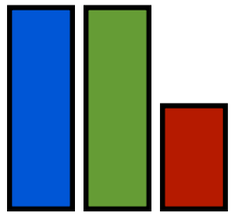
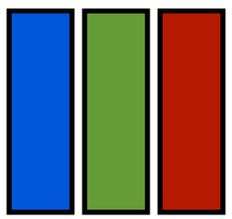


Clausal proofs + clause deletion

Heule, Hunt, Jr., and Wetzler [STVR 201X]



A fast clausal proof checker, called DRUP-trim

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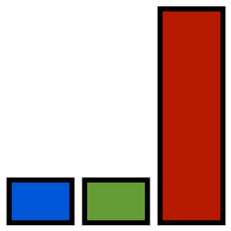

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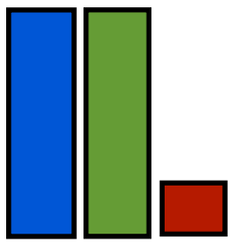

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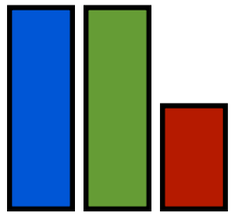

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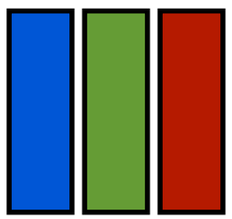

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*All approaches can be used for applications such as minimal unsatisfiable core extraction, computing interpolants, reduce proofs*

# Satisfiability and Resolution

Given a Boolean formula *F*, is there an assignment to variables in *F* such that the formula evaluates to TRUE?

$\bar{b}c$ $ac$ $\bar{a}b$ $\bar{a}\bar{b}$ $a\bar{b}$

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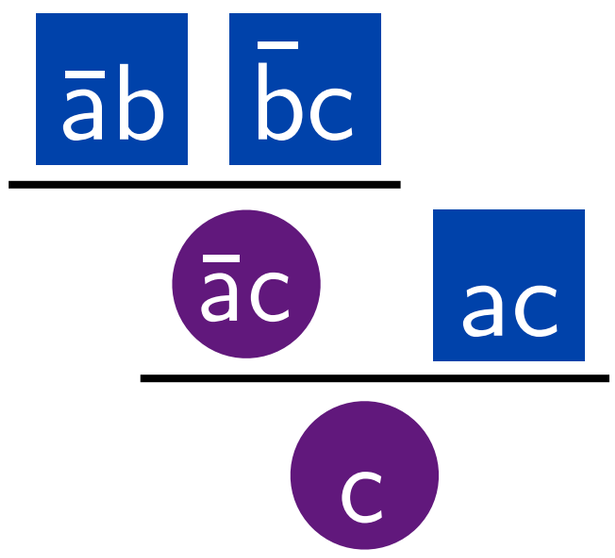
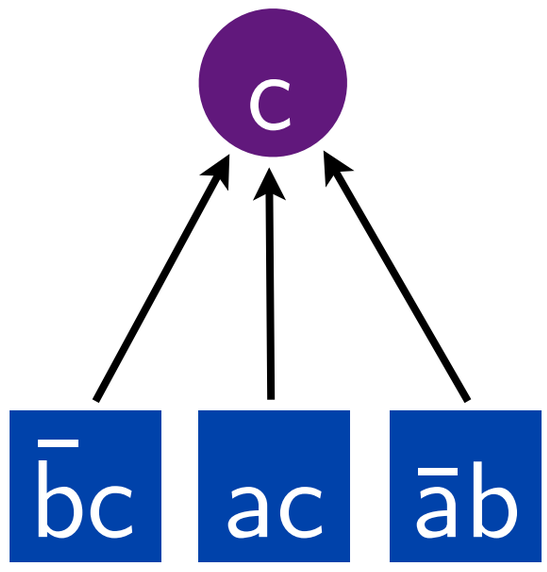
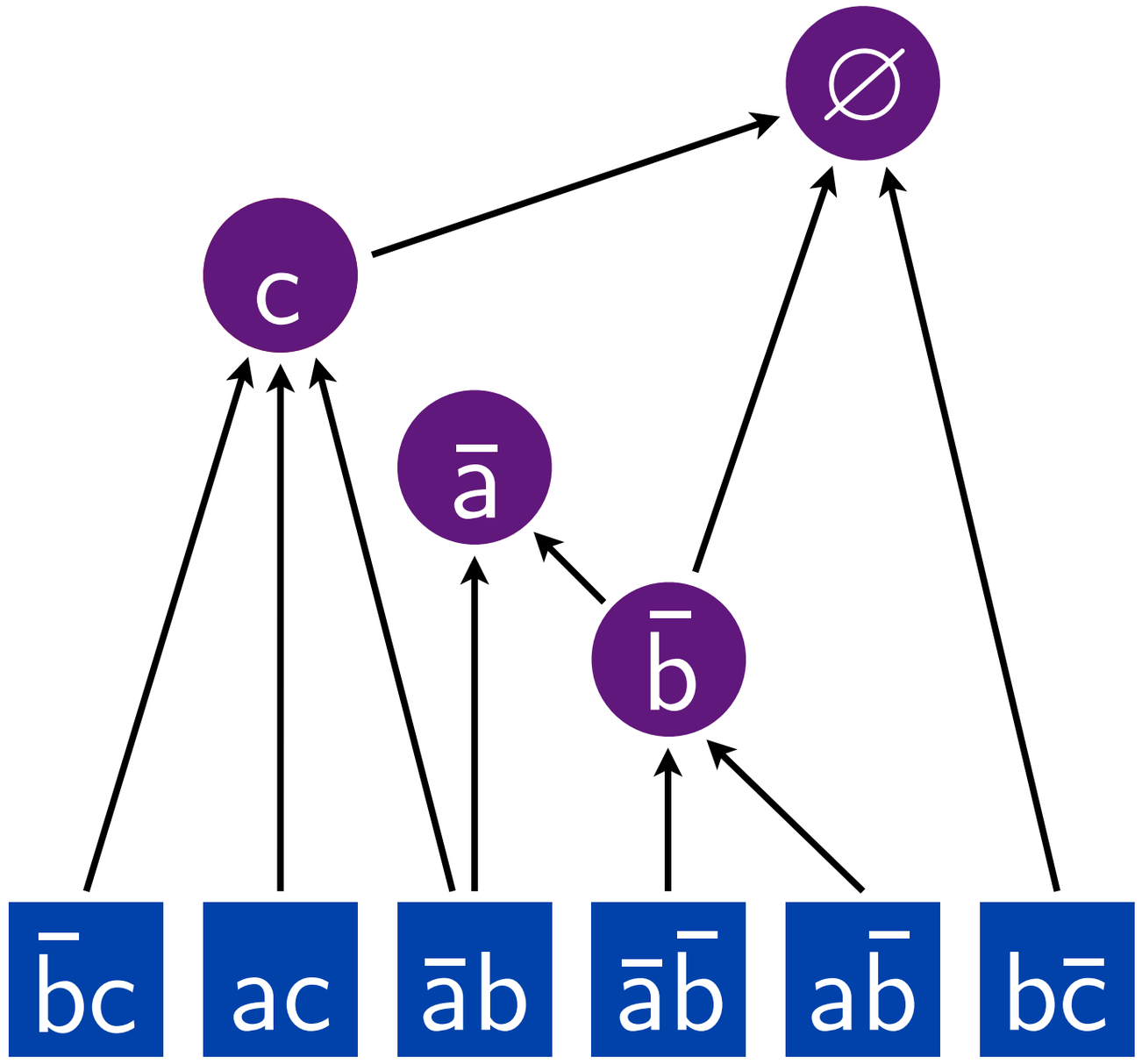
Unsatisfiability proofs use lemmas (resolvents):



# Resolution Graph / Proof and Core

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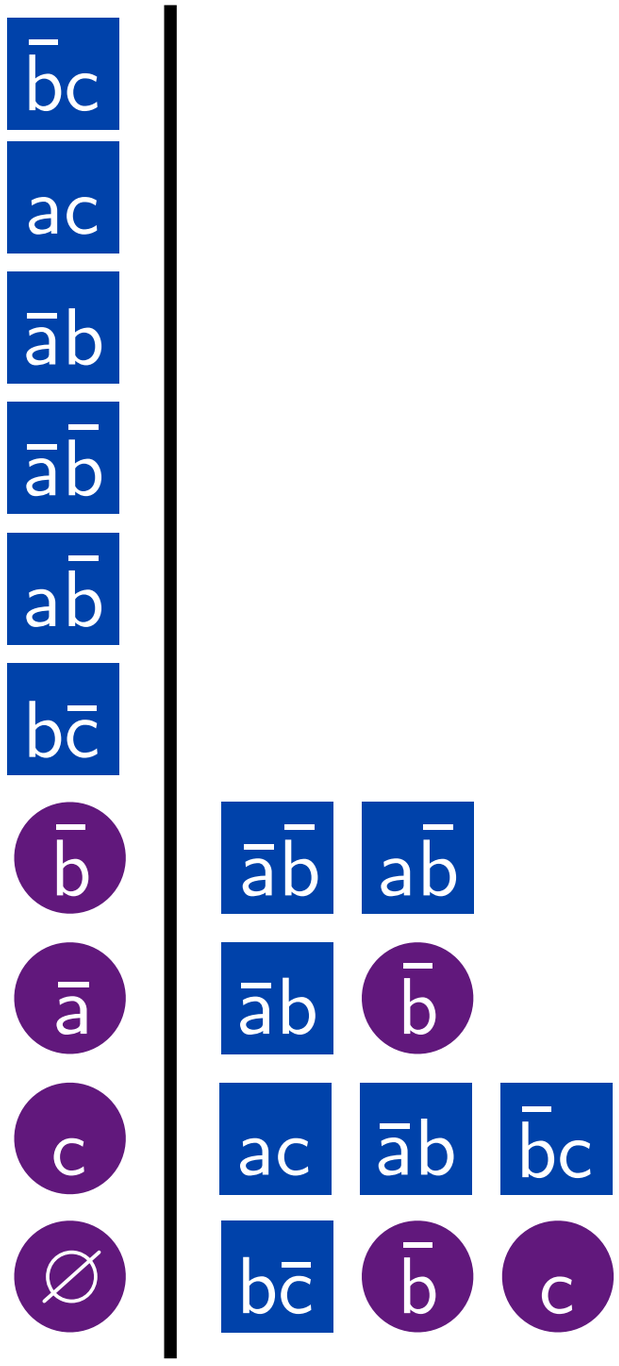
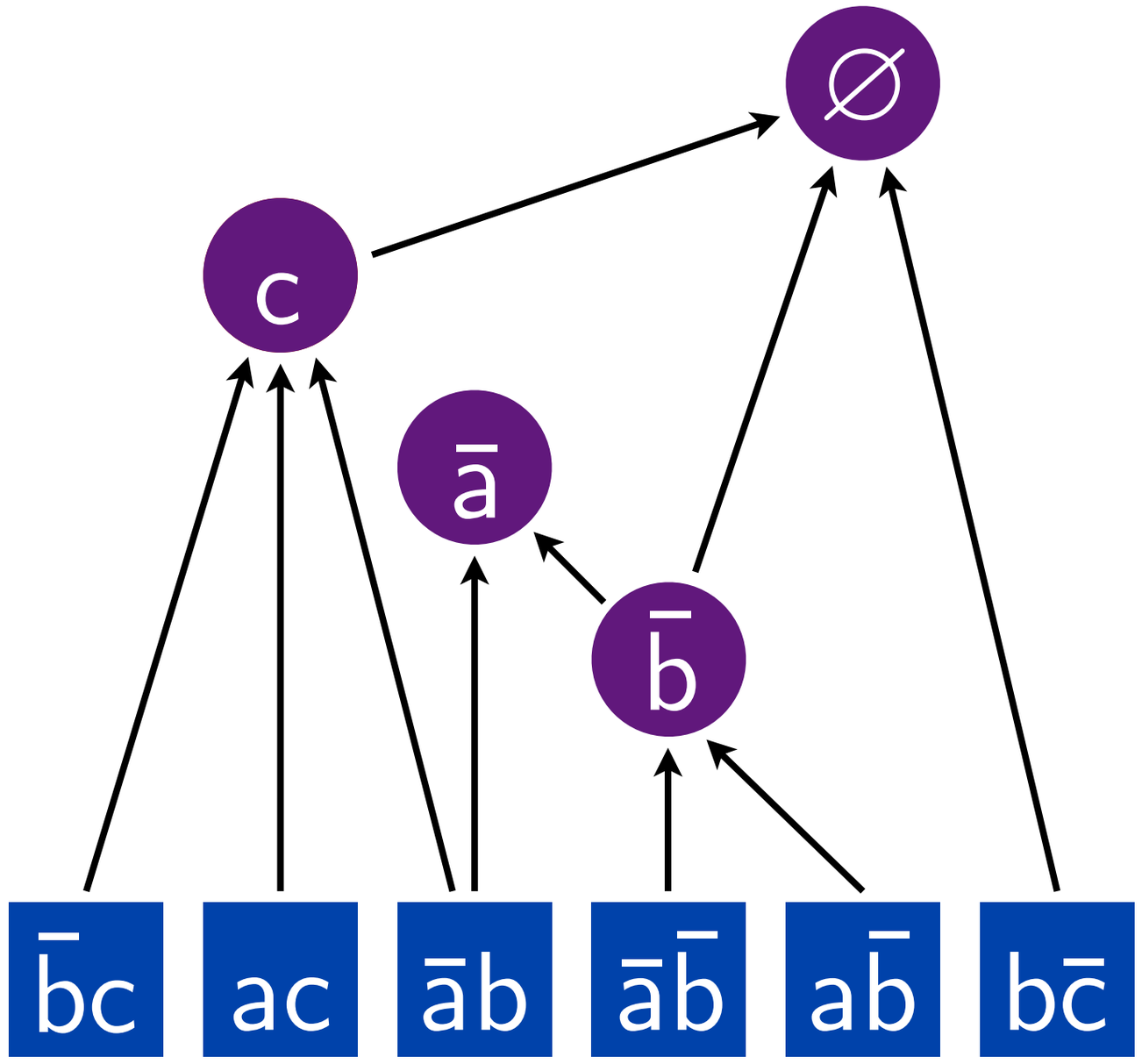



resolution proof

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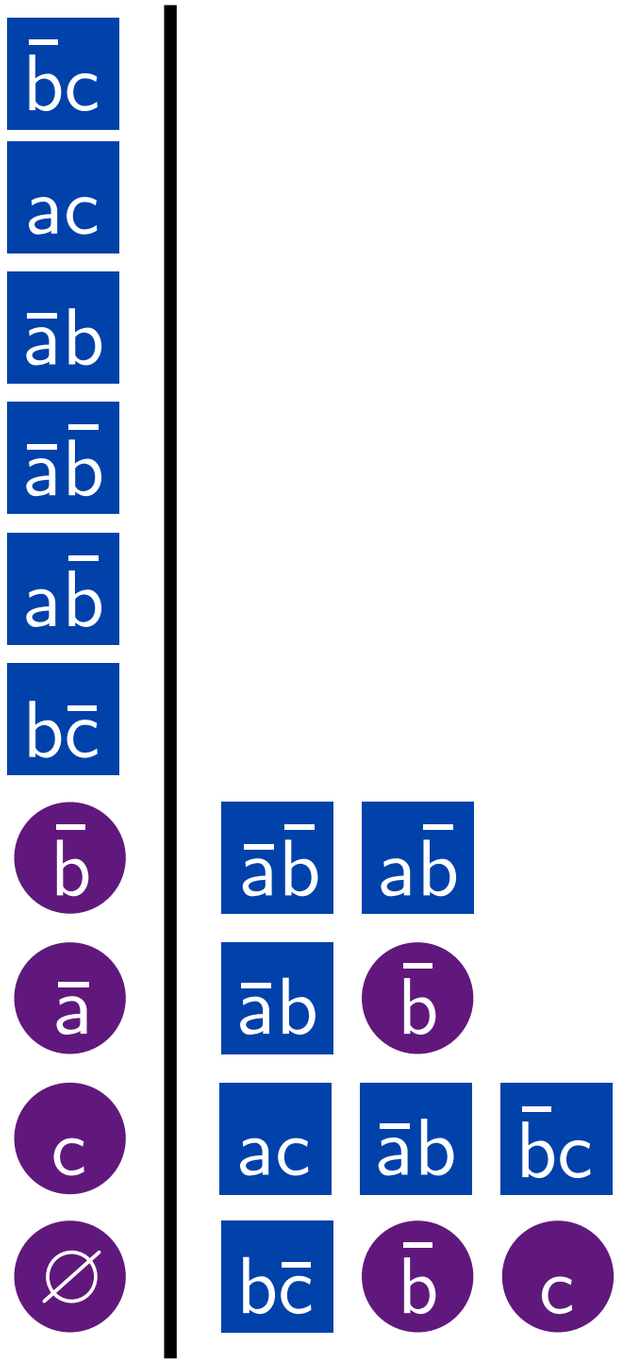


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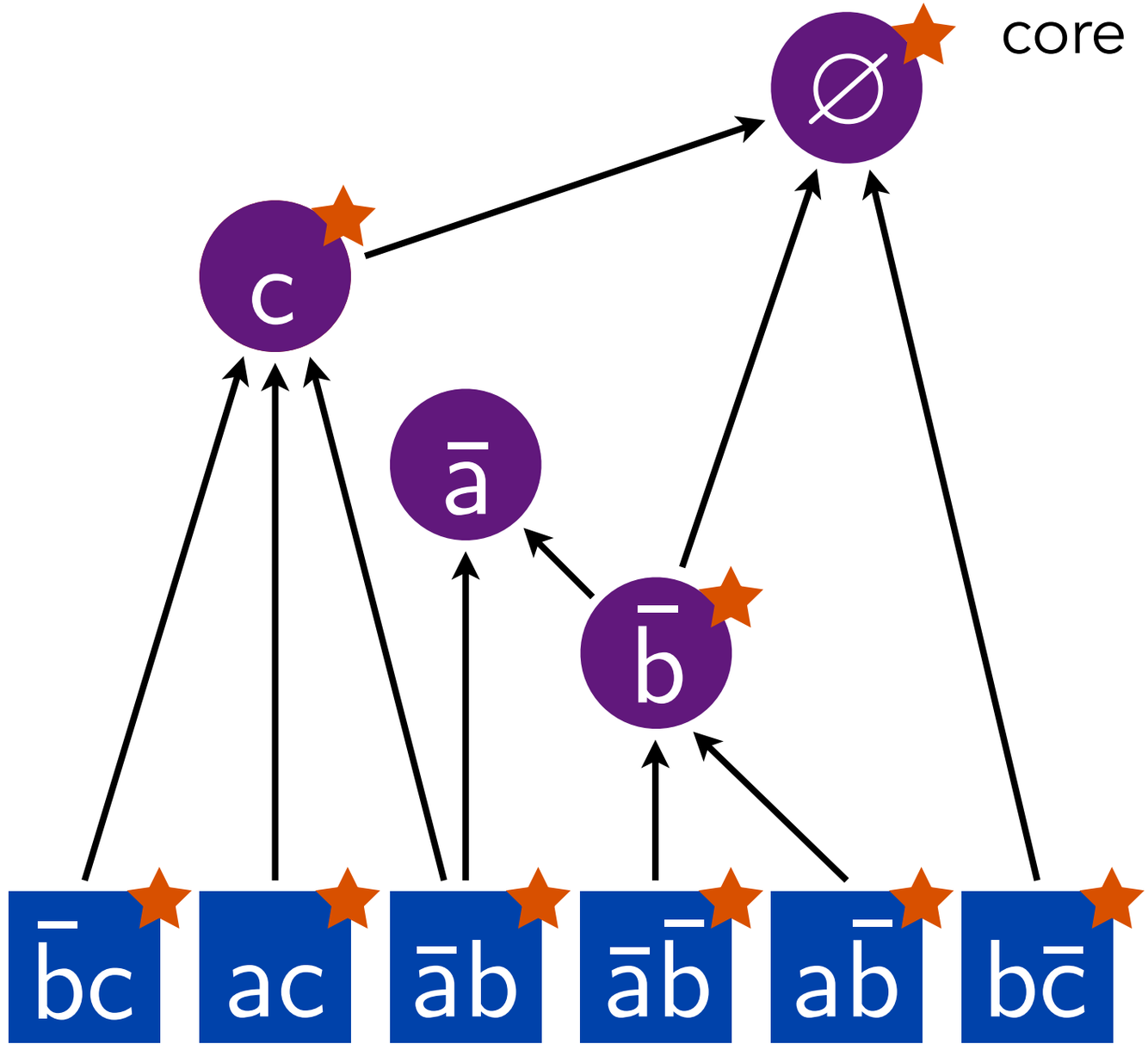


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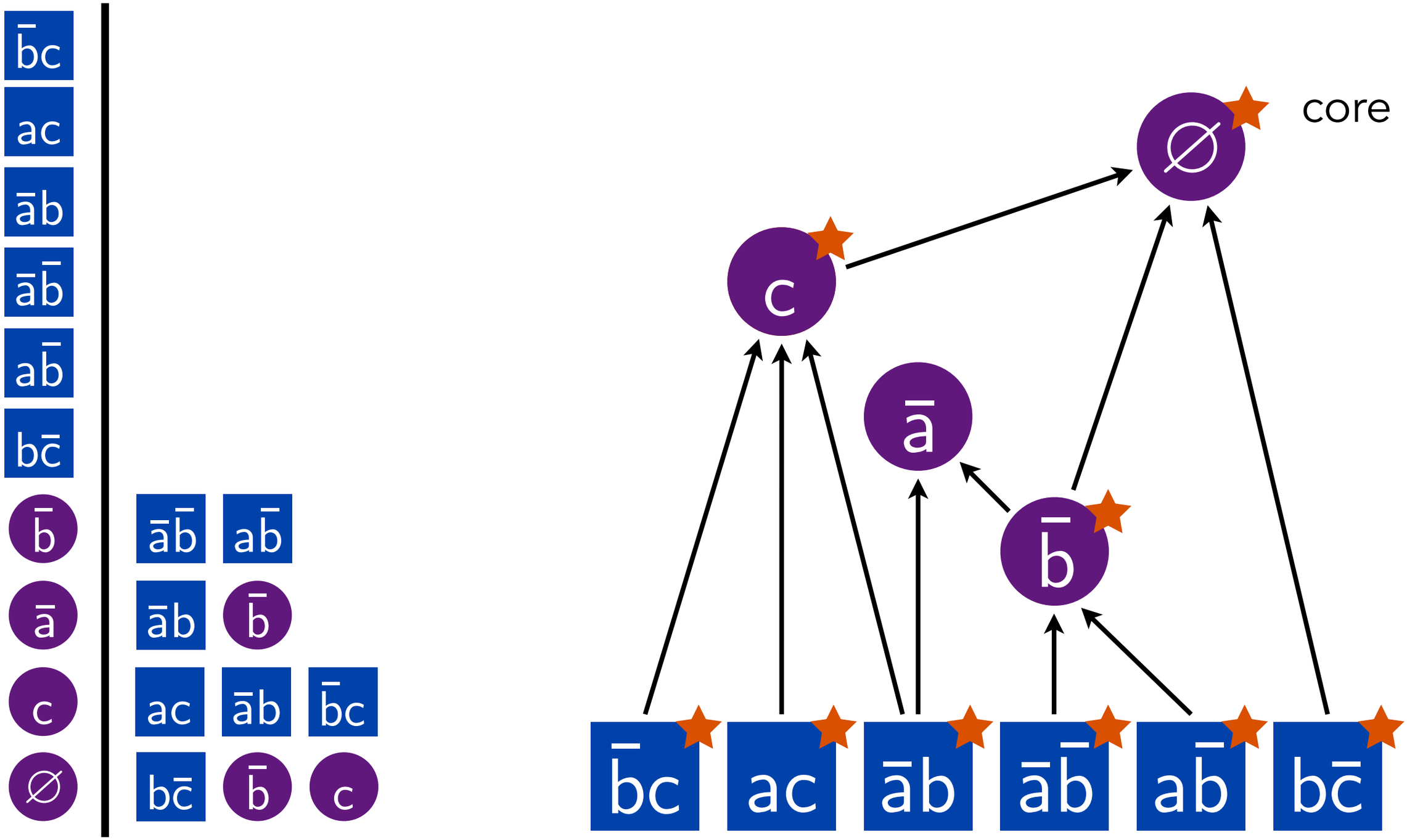


***resolution proofs are HUGE***

# Checking Lemmas by Unit Propagation

A clause is **unit** with respect to an assignment if all literals in the clause are falsified except for one literal, which is unassigned.

Unit propagation:

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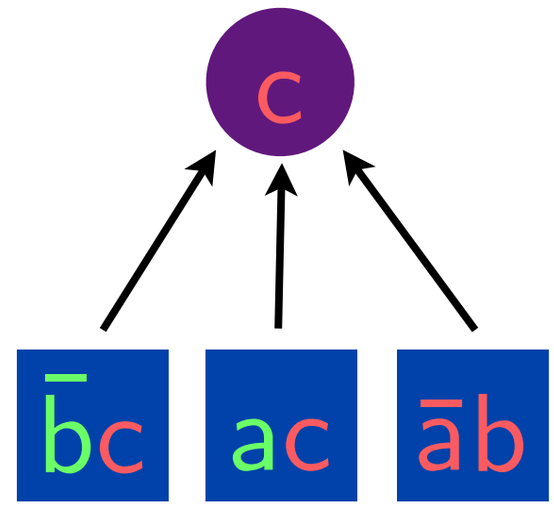
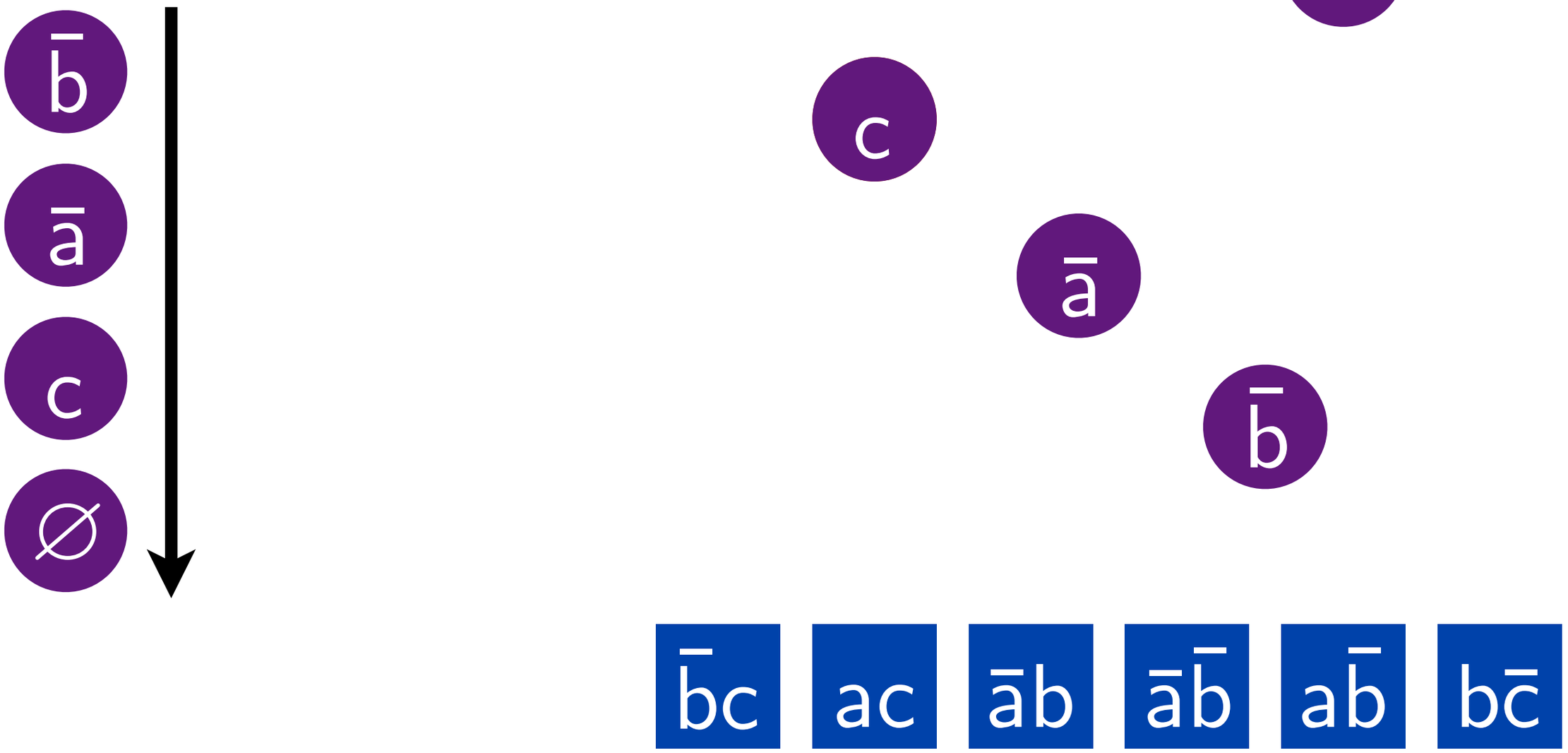
Reverse Unit Propagation (RUP) of a lemma:

- Assign all literals in the lemma to false and apply unit propagation
- If another clause / lemma becomes falsified, then the lemma is valid



# Clausal Proof: Check using Unit Propagation



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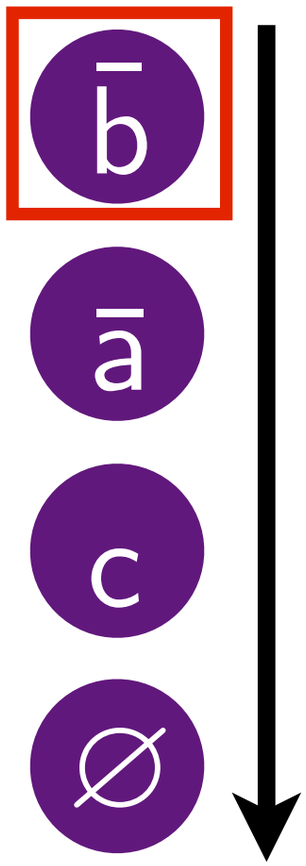


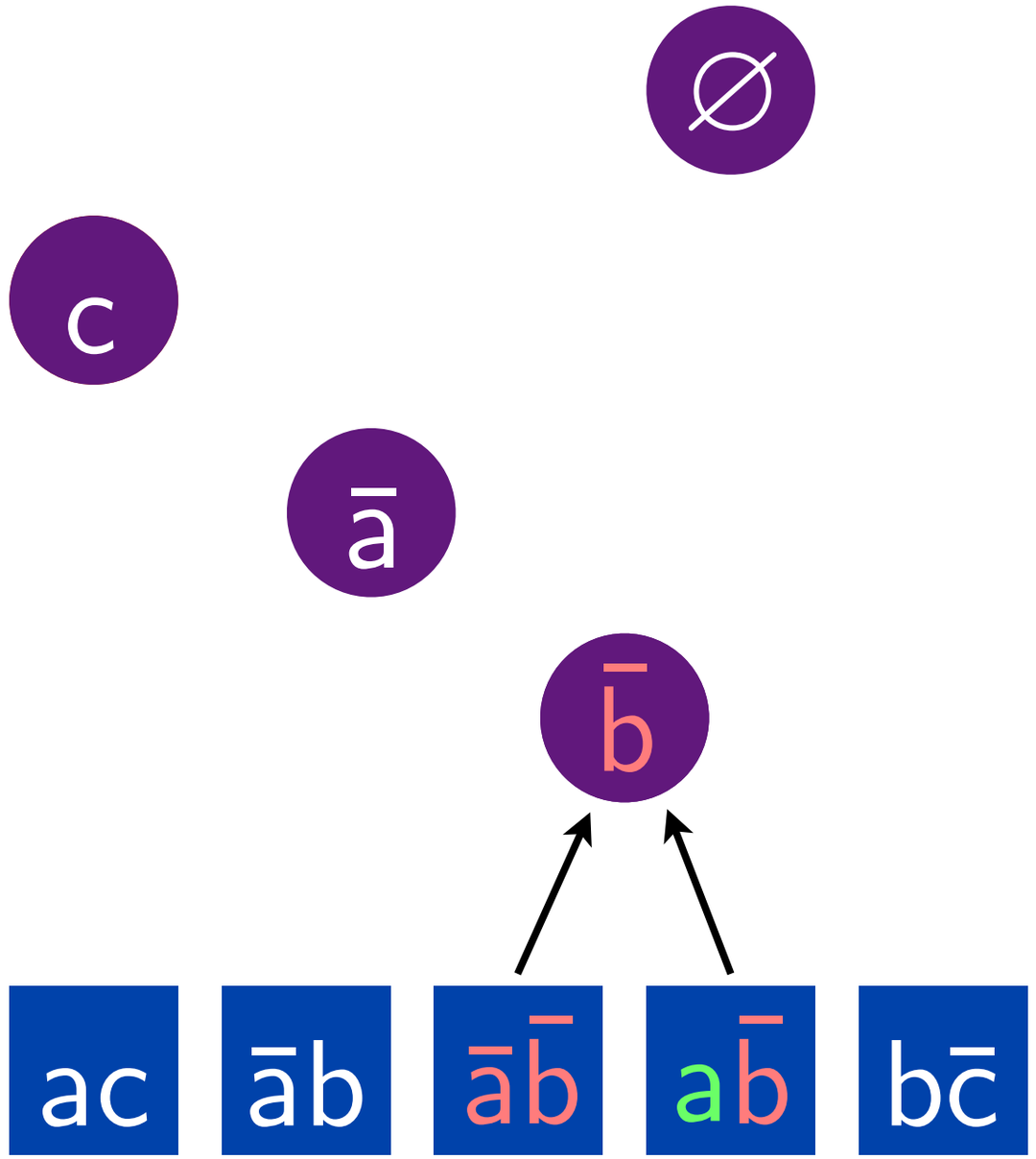

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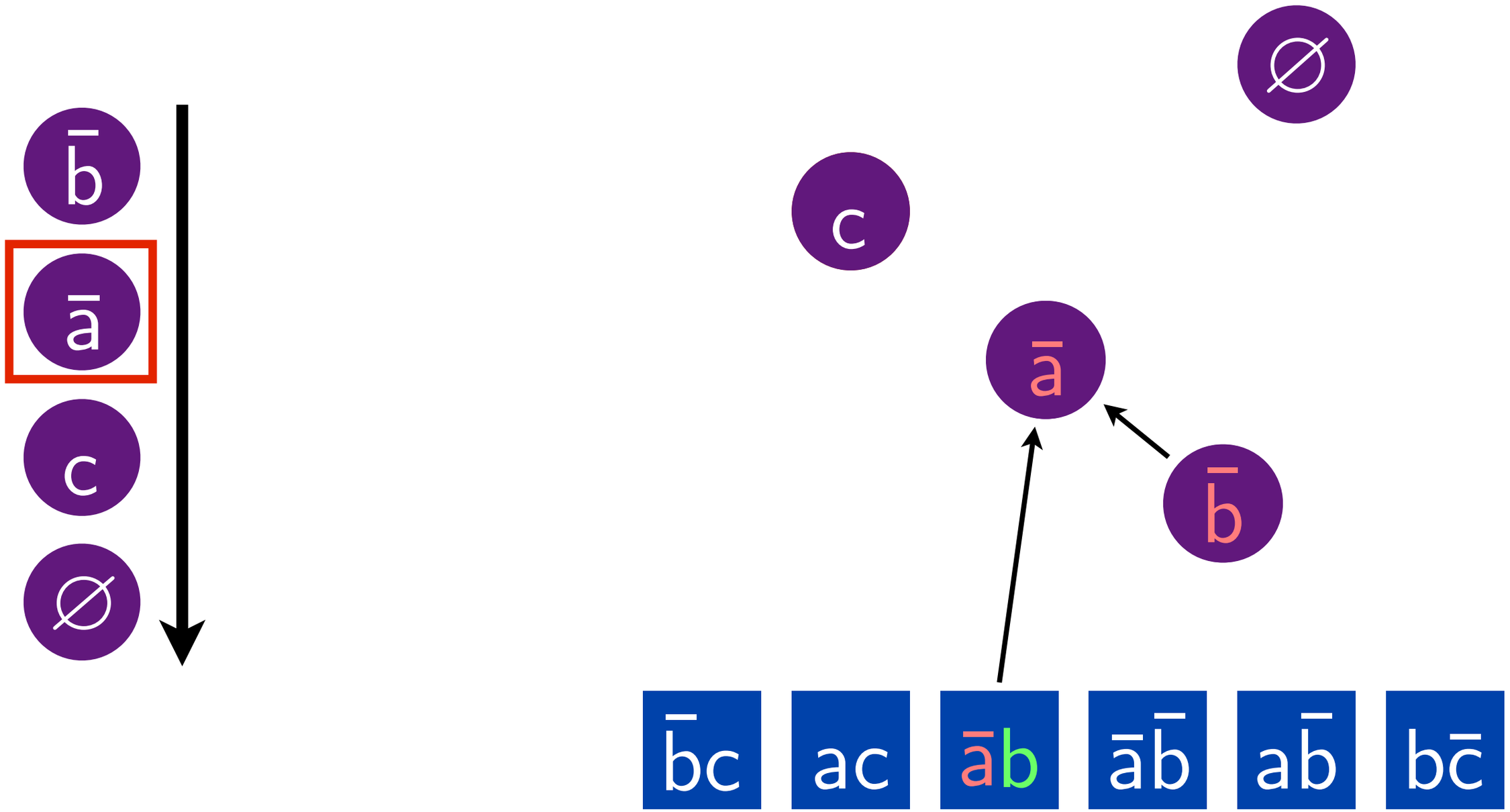

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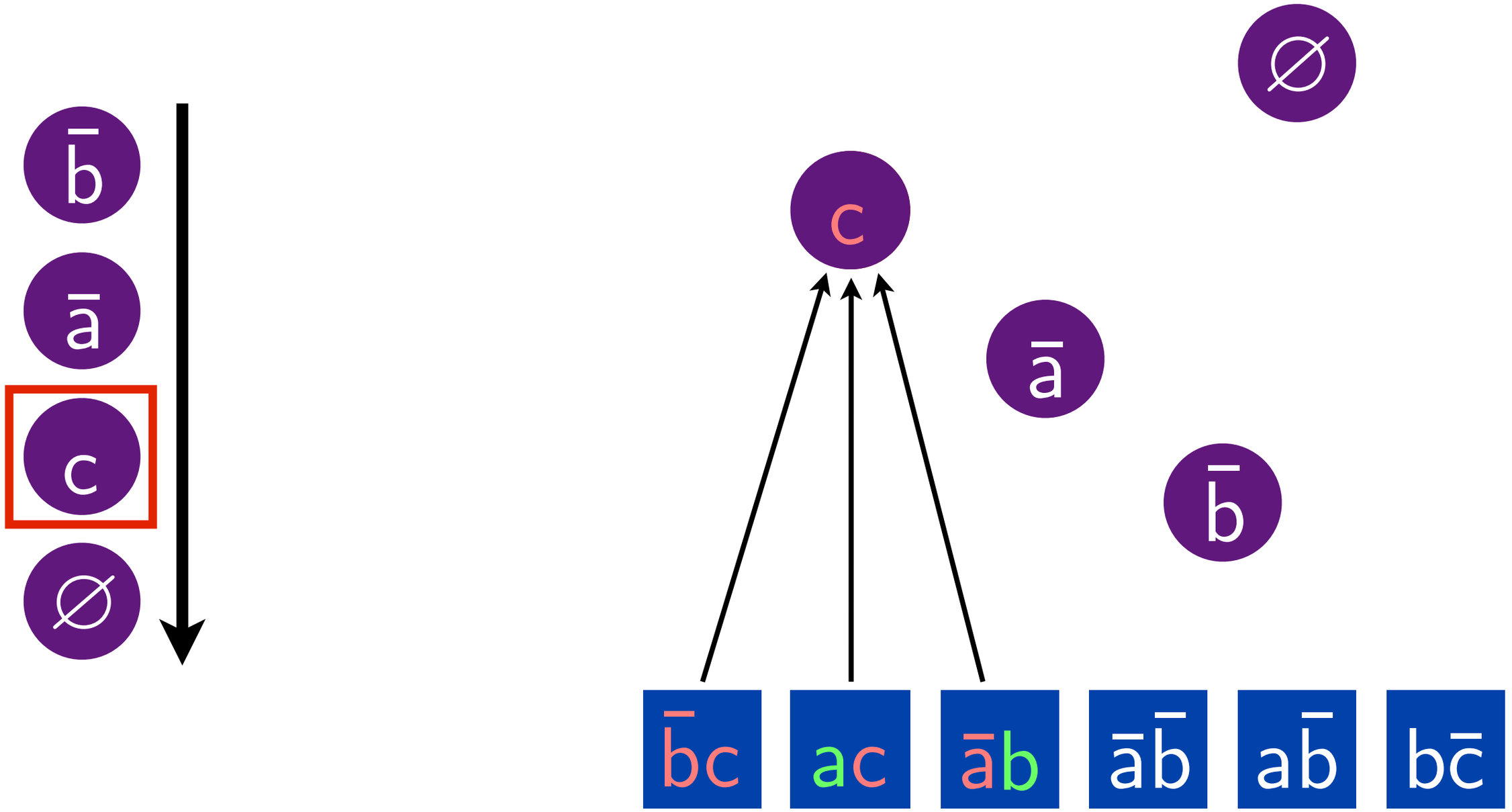

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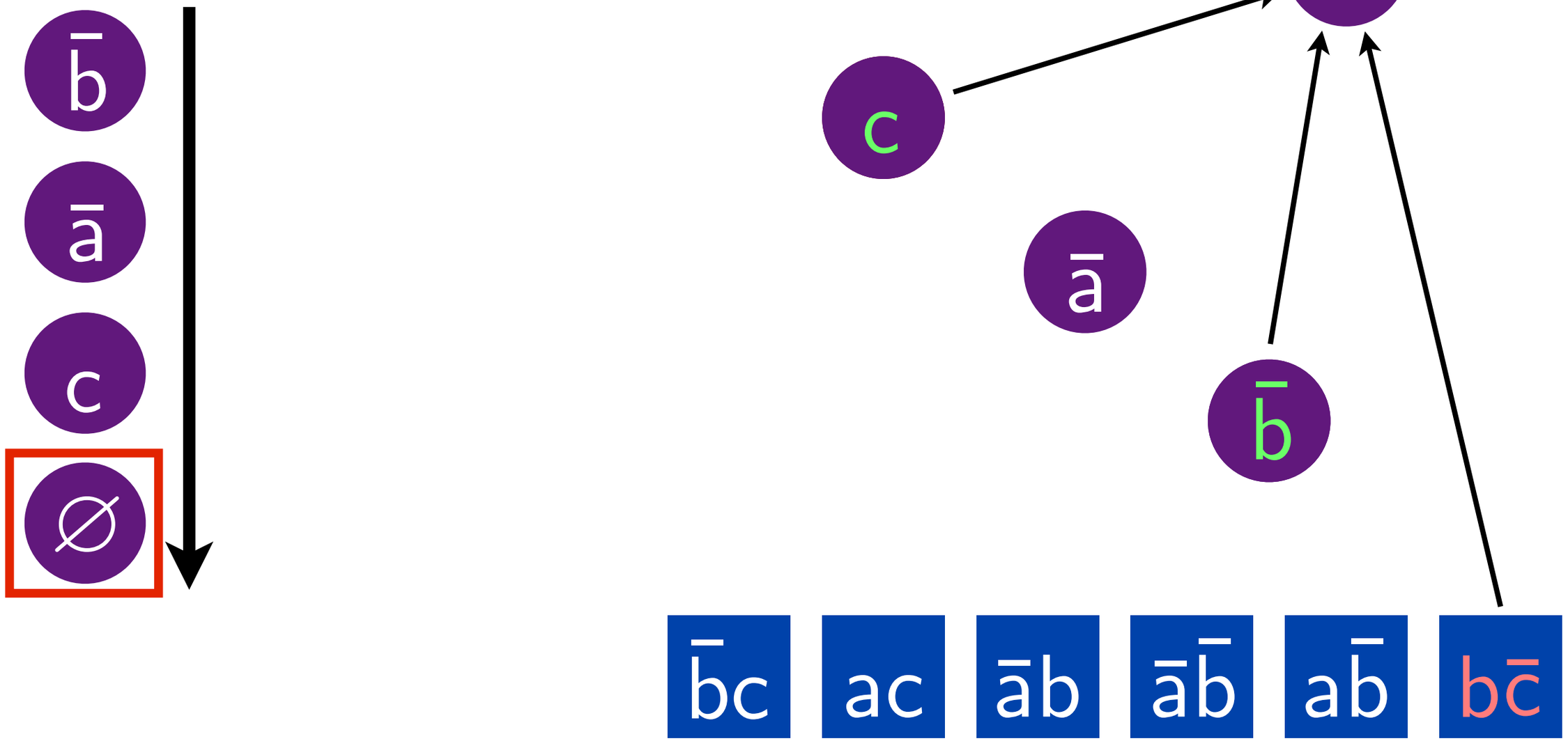

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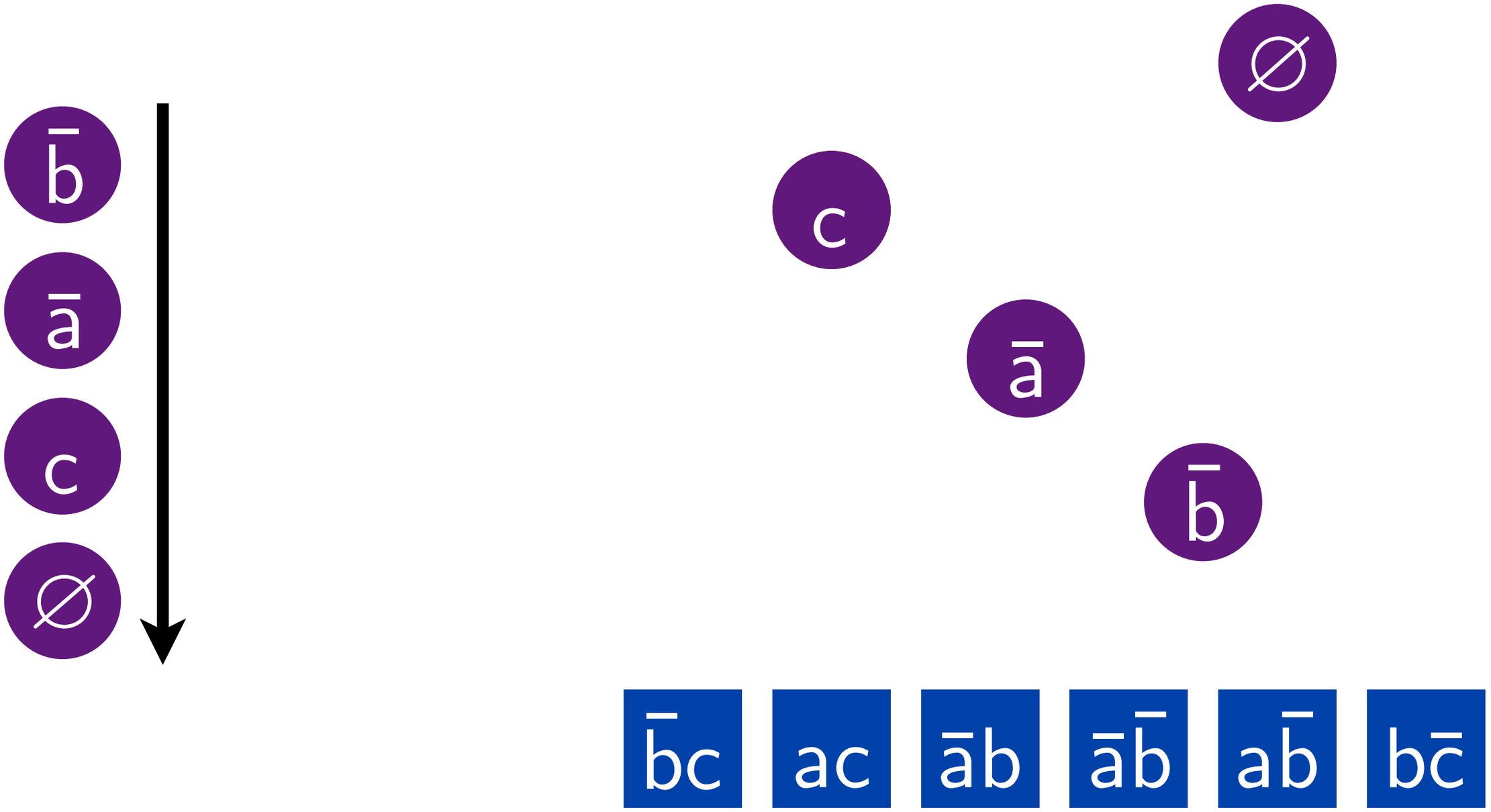
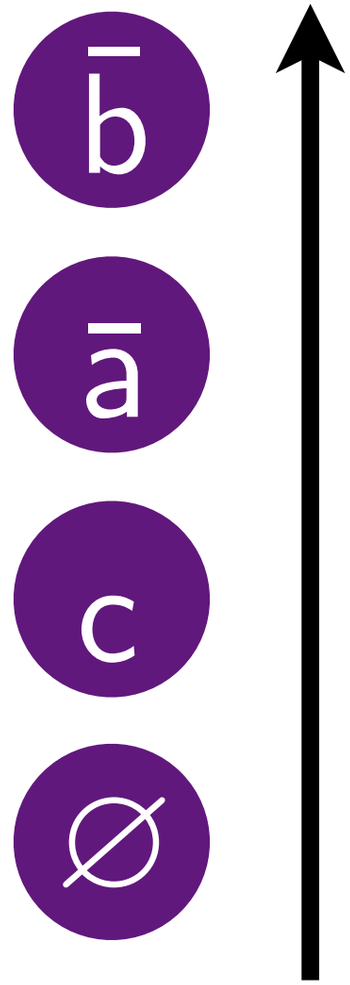



***clausal proofs are expensive to validate***

# Improvement I: Backwards Checking

Goldberg and Novikov proposed checking the refutation backwards [DATE 2003]:

- start by validating the empty clause;
- mark all lemmas using conflict analysis;
- only validate marked lemmas.




Advantage: validate fewer lemmas.

Disadvantage: more complex.

We provide a fast open source implementation of this procedure.

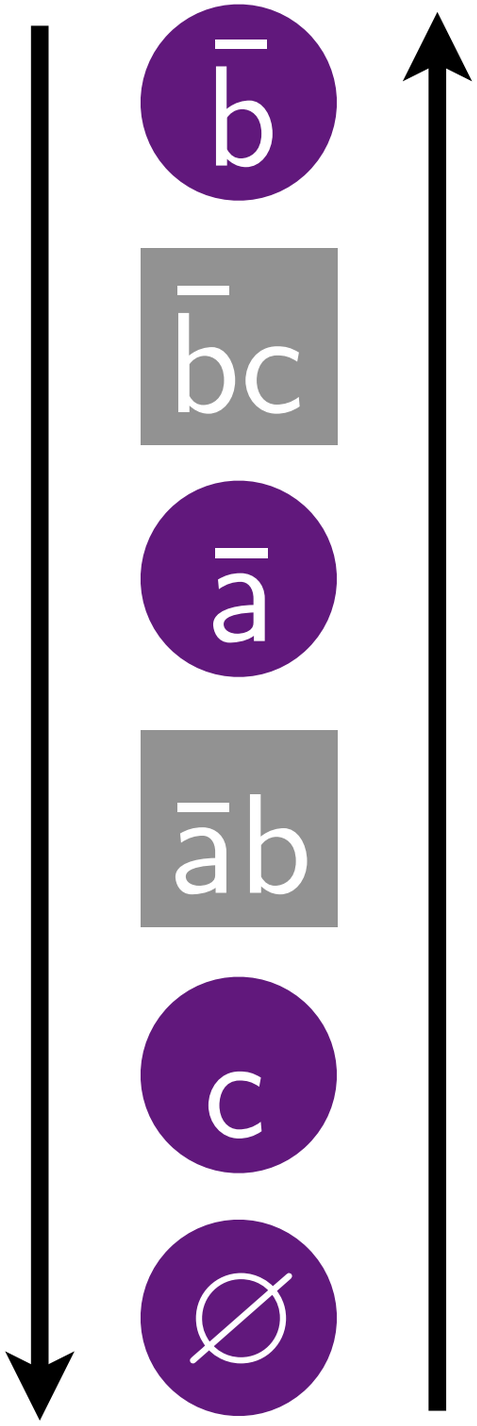
# Improvement II: Clause Deletion

We proposed to extend clausal proofs with deletion information [STVR 201X]:

- clause deletion is crucial for efficient solving;
- emit learning and deletion information;
- proof size might double;
- checking speed can be reduced significantly.

Clause deletion can be combined with backwards checking [FMCAD 2013]:

- ignore deleted clauses earlier in the proof;
- optimize clause deletion for trimmed proofs.



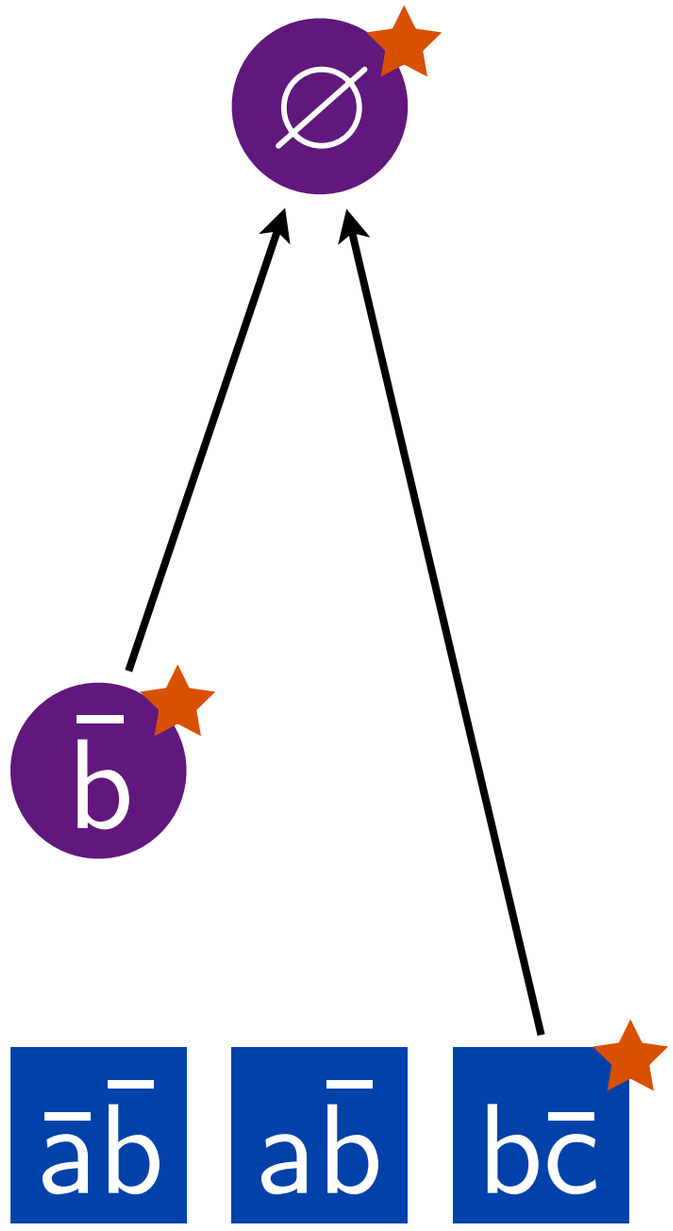
# Improvement III: Core-first Unit Propagation

We propose a new unit propagation variant:

1) propagate using clauses already in the core;
2) examine non-core clauses only at fixpoint;
3) if a non-core unit clause is found, goto 1);
4) otherwise terminate.

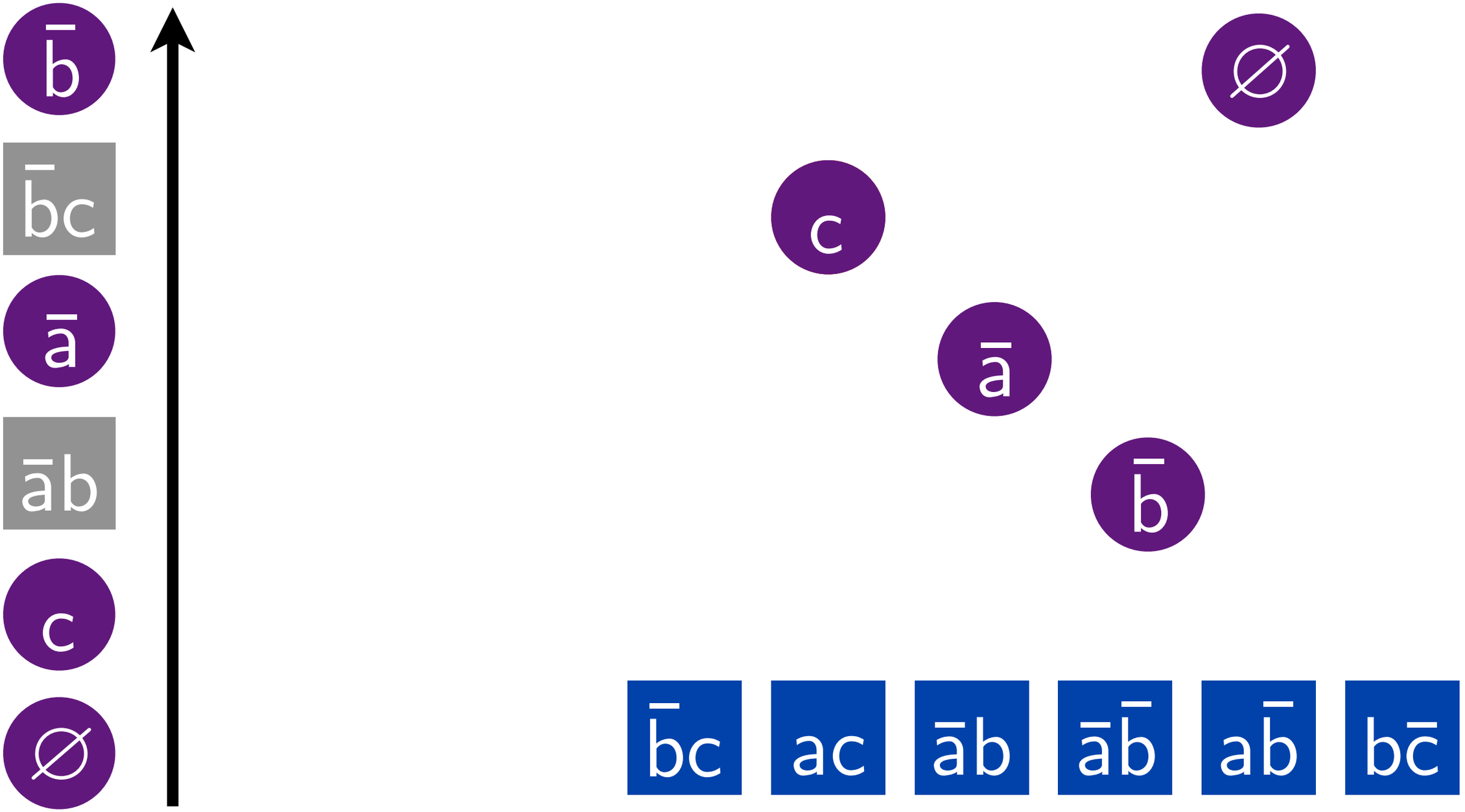
Our variant, called *Core-first Unit Propagation*, can reduce checking costs considerably.

Fast propagation in a checker is different than fast propagation in a SAT solver.




***Also, the resulting core and proof are smaller***

# Checking: Backwards + Core-first + Deletion



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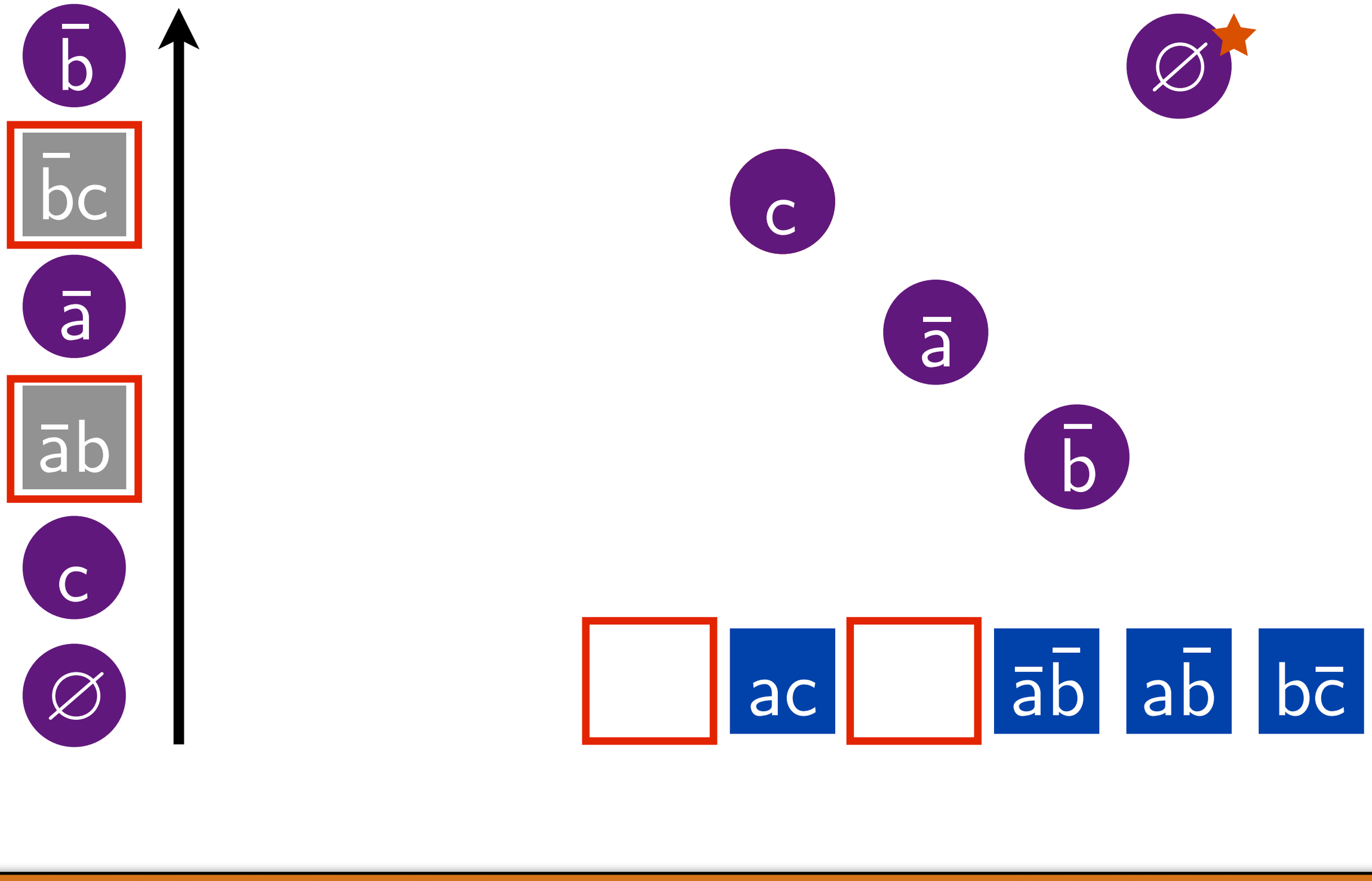

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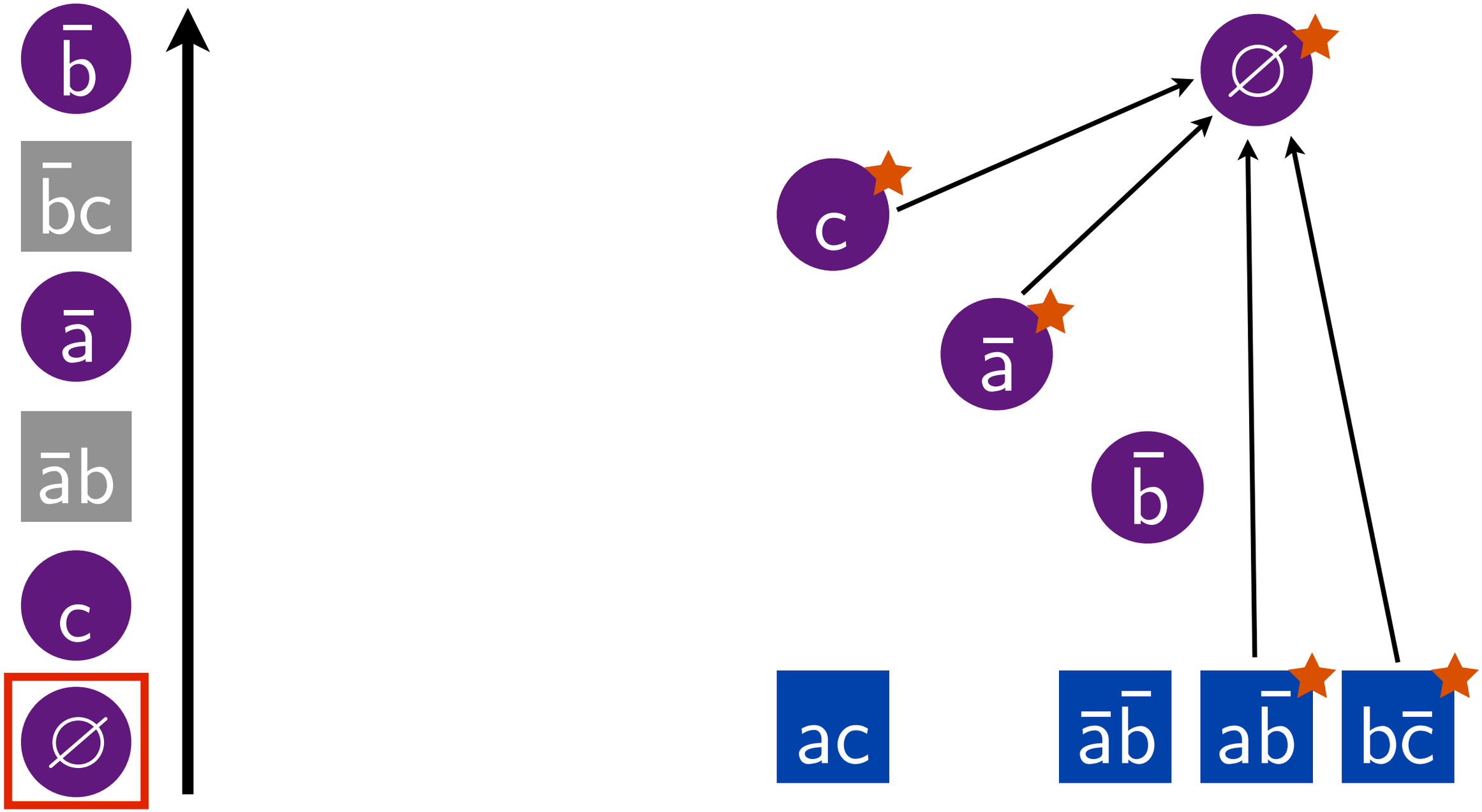

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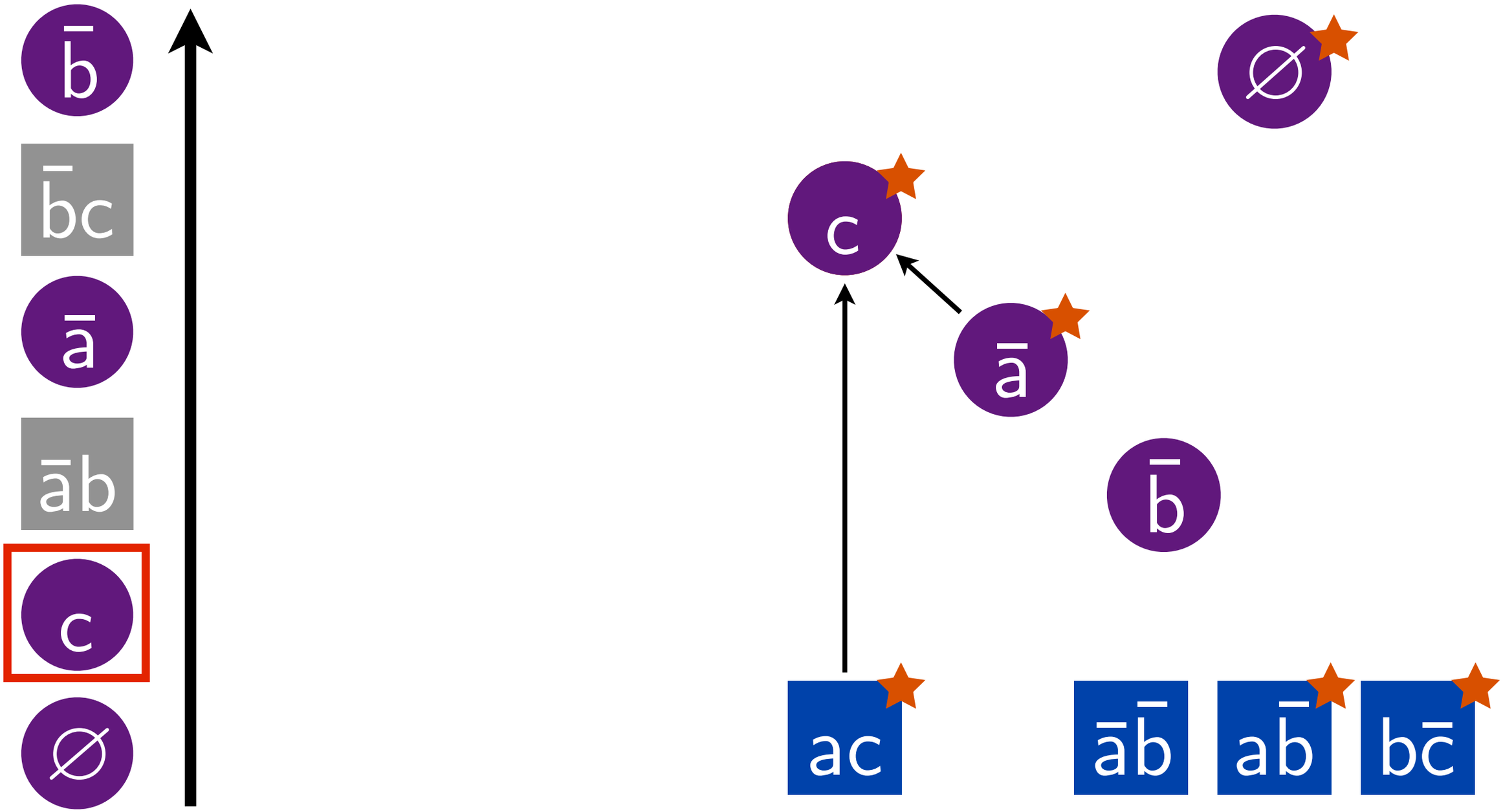

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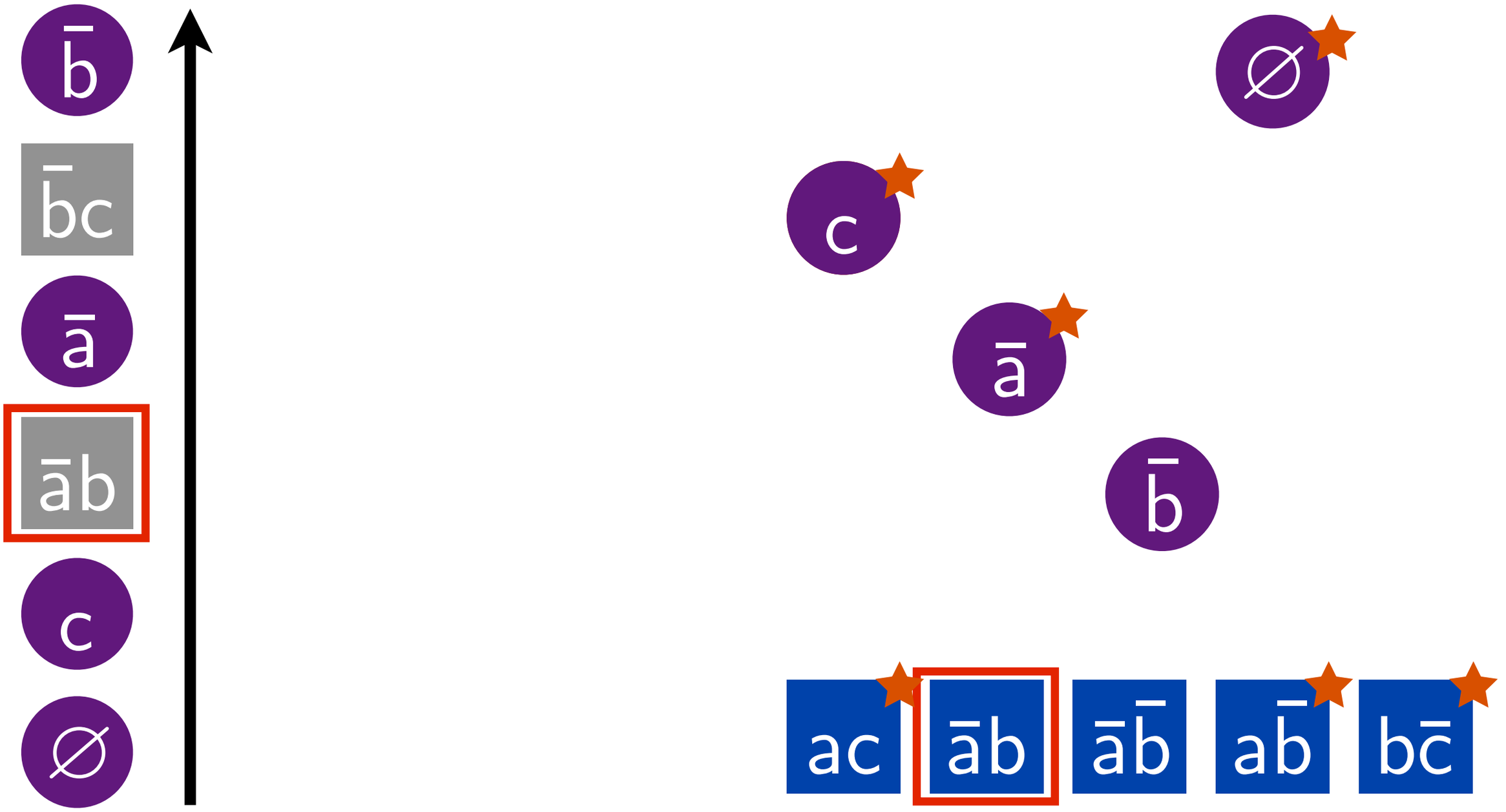

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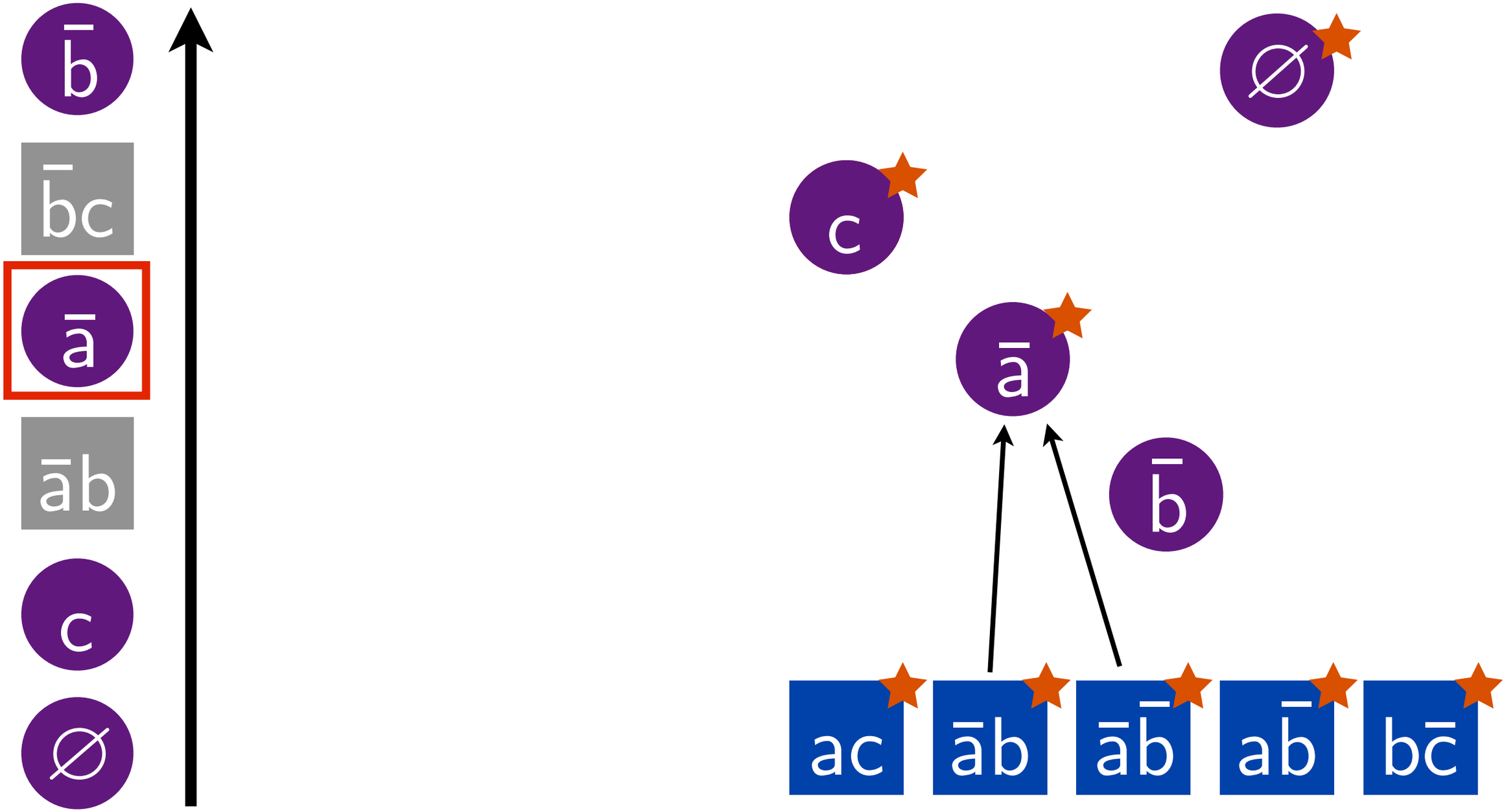

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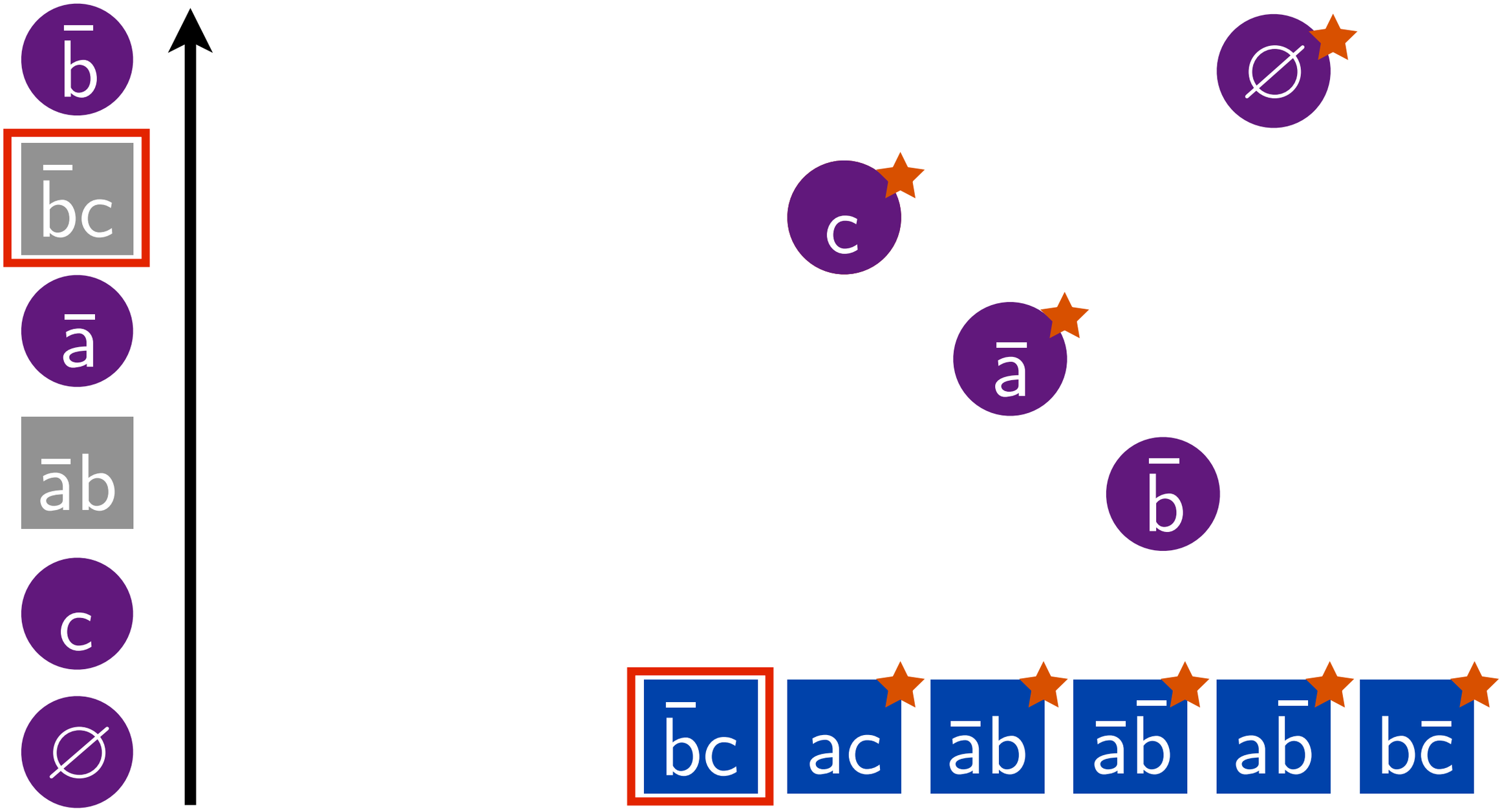

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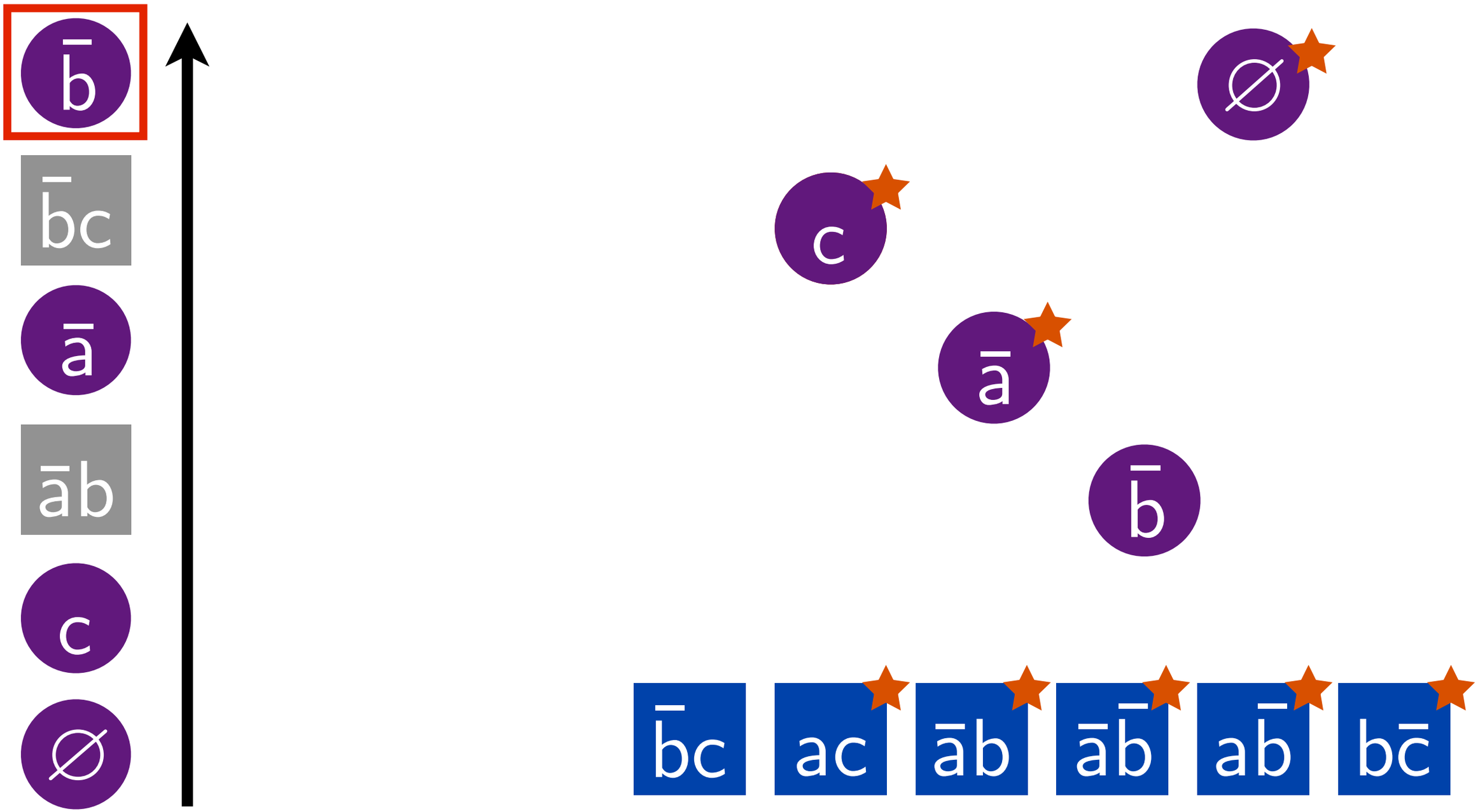

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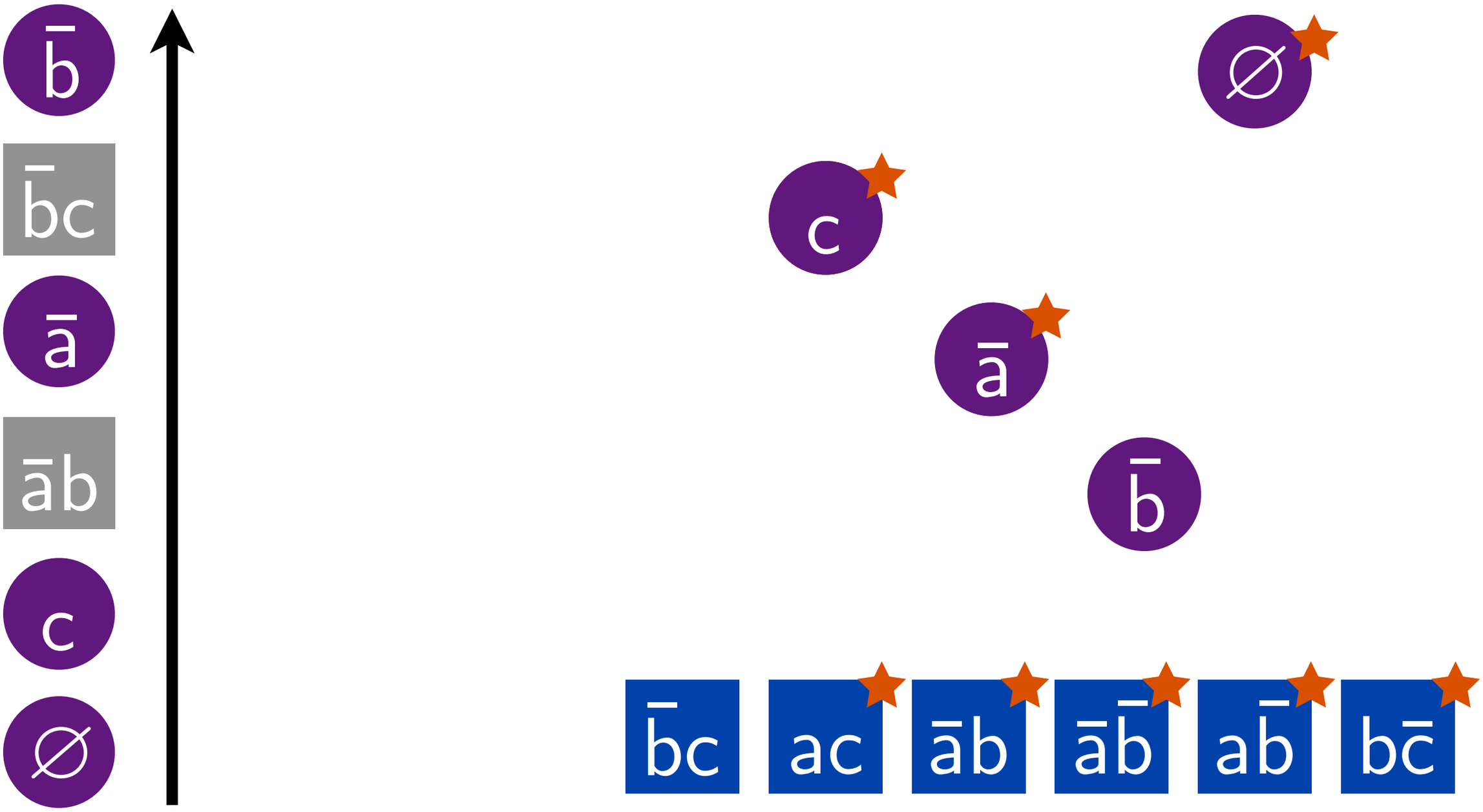



***Core-first unit propagation results in smaller cores and proofs***

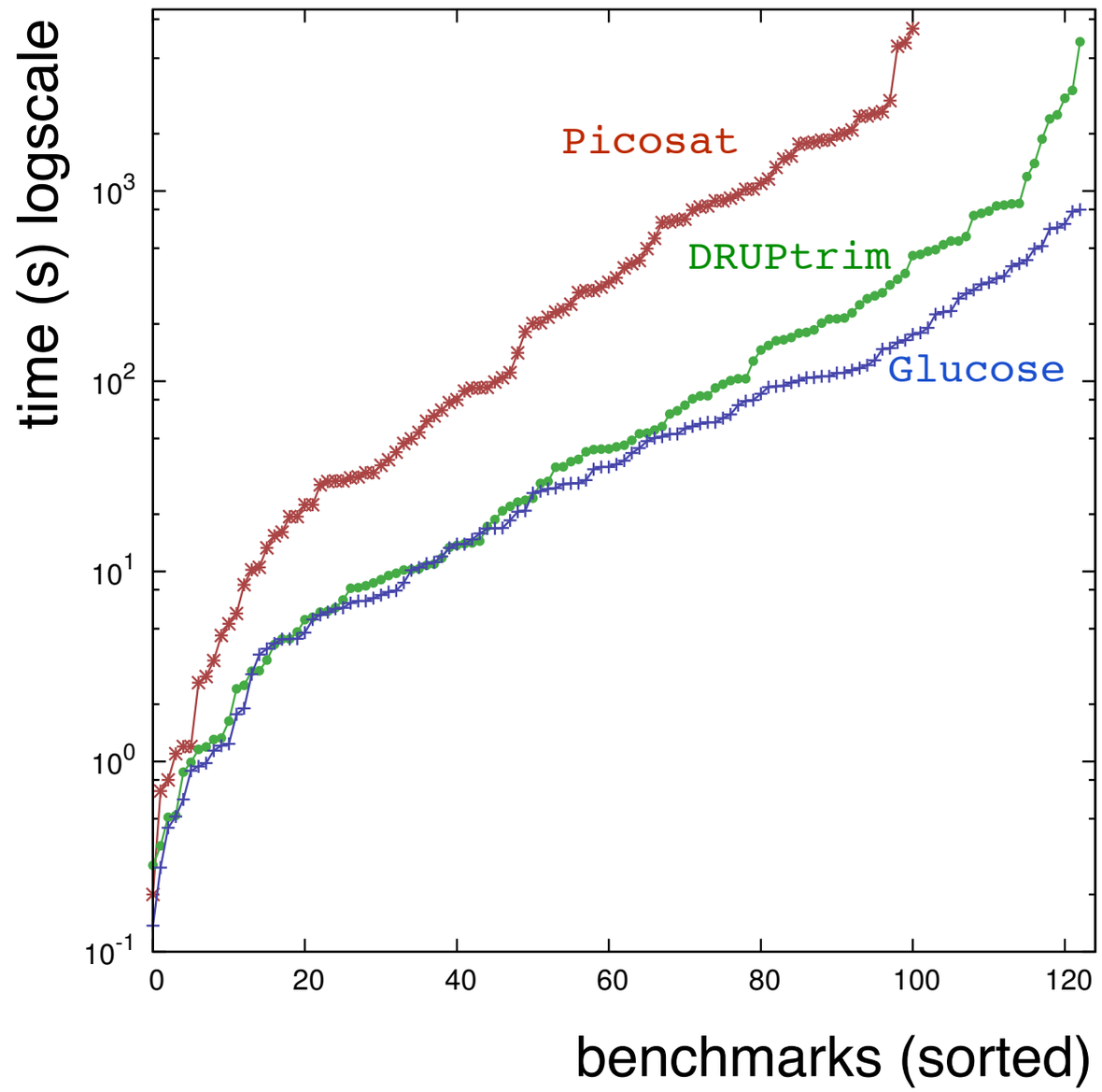
# Experimental Evaluation

We implemented DRUP logging into `Glucose` using only 40 LoC.

`Glucose` (DRUP) solves about twice as many benchmarks as compared to `Picosat` (resolution).

Resolution proof logging increased memory usage up to a factor 100.

`DRUPtrim` validated clausal proofs in a time similar to the solving time.



# DRUPtrim in SAT Competition 2013

***Unsatisfiable tracks required certificates.*** Allowed formats:

- TraceCheck (resolution);
- DRUP, Delete Reverse Unit Propagation.

Timeout : 5,000 seconds for solving and 20,000 seconds for checking

Submissions with proof logging:

- 11 application solvers (9 DRUP, 2 RUP);
- 9 hard-combinatorial solvers (7 DRUP, 2 RUP);
- Most submissions were certified unsatisfiable versions of top-tier solvers.

Statistics:

- 98% of DRUP proofs of top-tier solvers were checked within the time limit;
- Checking most RUP proofs (i.e., no clause deletion) results in a timeout.

# Conclusion

Our `DRUPtrim` tool:

- makes it feasible to check the results of state-of-the-art solvers efficiently (demonstrated at SAT Competition 2013);
- validates learning, preprocessing, and inprocessing techniques;
- and produces trimmed proofs and trimmed formulas.

**Our next goal** is to increase confidence in **all** SAT solvers by efficiently checking proofs with a mechanically-verified proof checker.

**Discussion:** should UNSAT proof logging be mandatory for tools participating in competitive events (e.g., SAT Competition, HWMCC)?

# Recent Work

*Bridging the Gap Between Easy Generation and Efficient Verification of Unsatisfiability Proofs*

Marijn J.H. Heule, Warren A. Hunt, Jr., and Nathan Wetzler

Accepted: Software Testing, Verification, and Reliability (STVR 201X)

*Verifying Refutations with Extended Resolution*

Marijn J.H. Heule, Warren A. Hunt, Jr., and Nathan Wetzler

Published: Conference on Automated Deduction (CADE 2013)

*Mechanical Verification of SAT Refutations with Extended Resolution*

Nathan Wetzler, Marijn J.H. Heule, and Warren A. Hunt, Jr.

Published: Interactive Theorem Proving (ITP 2013)

*Trimming while Checking Clausal Proofs*

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Accepted: Formal Methods in Computer-Aided Design (FMCAD 2013)

**Thank you for your attention!** **Questions?**