ACL2: Implementation of a Computational Logic

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Dept. of Computer Science

June 10, 2015

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Let's start with some context.

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- UT Austin: x86 interpreter defined in ACL2, validation by co-simulation, proofs about x86 machine code

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REMARK (thanks to J Moore for this):

> *All industrial-scale deduction tools are, in a deep sense, interactive, even the ones that claim to be automatic. The issue is HOW MUCH interaction is required to do interesting things.*

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That talk mentions this link to several demos and their logs:

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  - *Boyer-Moore Theorem Provers* go back to the start of their collaboration in 1971.

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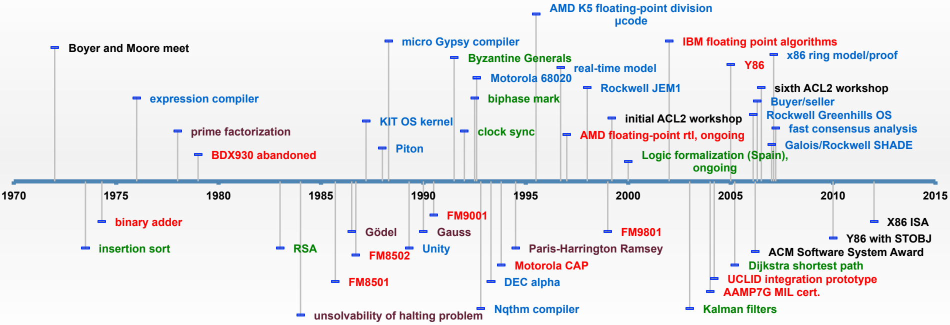
- Interfaces include Emacs, ACL2 Sedan (Eclipse-based), none.

Partial timeline



Some ACL2 features *not* discussed further today:

- Prover algorithms
  - Waterfall, linear arithmetic, Boolean reasoning, . . .
  - Rewriting: Conditional, congruence-based, rewrite cache, syntaxp, bind-free, . . .
- Using the system effectively
  - The-method and introduction-to-the-theorem-prover
  - Theories, hints, rule-classes, . . .
  - Accumulated-persistence, brr, proof-checker, dmr, . . .
- Programming support, including (just a few):
  - Guards
  - Hash-cons and function memoization
  - Packages
  - Mutable State, stobjs, arrays, applicative hash tables, . . .
- System-level: Emacs support, books and certification, abbreviated printing, parallelism (ACL2(p)), . . .

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- characters,
- strings,
- symbols,
- complex numbers with rational coefficients, and
- closure under a pairing operation (`cons`).

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  - M. Kaufmann and J Moore, “Structured Theory Development for a Mechanized Logic.” *Journal of Automated Reasoning* 26, no. 2 (2001) 161-203.
- Importance: One may want to introduce new concepts to carry out some proofs, but this must be done conservatively in order to believe the results.

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We'll look at just a few on the next slides.

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For details, including issues pertaining to evaluation, see the *Essay on Defattach* comment in the ACL2 sources.

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Conservatively introduce $w(y,z)$ and $r(y,z)$ *using local witness*
$w(y,z) = (\varepsilon\ x)(p(x,y,z) \wedge q(x,y,z))$
to prove these axioms:

- $r(y,z) = (p(w(y,z),y,z) \wedge q(w(y,z),y,z))$
- $(p(x,y,z) \wedge q(x,y,z)) \implies r(y,z)$

QUANTIFICATION, CHOICE, & INDUCTION (2)

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Conservativity *with* induction follows from a model-theoretic forcing argument.

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- direct its use during later proofs.

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ACL2 supports a notion of “eval”, together with this sort of *meta* theorem, directing the use of `fn` to transform terms that are calls of `nth` or of `foo`.

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- One can specify a *measure* in order to admit a recursive definition. But what if the measure is defined in terms of a function whose definition is `LOCAL`?

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THANK YOU!